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FISH'S

ARITHMETIC

NUMBER TWO

ORAL AND WRITTEN

UPON

THE INDUCTIVE METHOD

BY

DANIEL W. FISH, A.M.

EDITOR OF ROBINSON'S "PROGRESSIVE SERIES" AND "SHORTER COURSE"

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PREFACE.

IN the preparation of this work, the aim of the author has been to furnish a brief and comprehensive text-book, in which no more of theory shall be introduced than is necessary for the illustration of such principles and processes of arithmetic as are needed in the common business of everyday life.

The work is thoroughly *inductive* in its methods. The principles and processes of computation are developed by inductive questions and exercises; the order of arrangement and gradation of the topics and applications is logical and progressive, hence, *practical;* the definitions and rules are clear and concise; the applications, both oral and written, are numerous and varied, familiarizing the pupil with ordinary business transactions.

The *drill* and *oral* exercises presented in the fundamental rules are peculiarly adapted to secure rapidity and accuracy in the use of numbers, and to pupils who have not been thoroughly drilled in similar exercises, either *orally* or in any *first book,* these are of the first importance, and are a sufficient preparation for the written work that follows in each topic. Hence, pupils who have received only *oral* instruction in the lower primary grades, may take up this book without first using a more elementary work.

Those who are already familiar with such exercises should not pass over them without a *thorough review* of the same, the *written* work of each topic being quite as full, both of *abstract* and of *applied* examples, as any other similar book.

Many new features have been introduced, which will be acknowledged improvements. Some *new methods* and *illustrations*, favorable to *clearness* and *brevity*, will be noticed; *rules* have been reduced to the *minimum* number; a *single rule*, each, is given for Multiplication, and for Division of Fractions; only *two*, for all cases of Reduction of Denominate Numbers, whether integral or fractional; *formulas* are substituted for rules, whenever they clearly indicate the steps to be taken in the solution of examples; and when the principles, explanations, and processes make the rule obvious, it is omitted, and the pupil left to construct one in his own language.

Cancellation is made very prominent, especially in Fractions, Interest, and Proportion. Its application in working Interest, Partial Payments, and Proportion presents some new, valuable, and practical features.

Such topics, rules, and applications as are of minor importance to the majority of Grammar School pupils, who are qualifying themselves for the ordinary business of life, have been placed in an "Addenda," and may be taken up in their proper connection with kindred matter in the body of the book, or at its close, or omitted entirely, and thus the course completed independent of them, at the option of the pupil or teacher.

The endeavor of the author has been to present a clear, systematic, and comprehensive text-book, sufficiently full for the student and ordinary business man, introducing the latest improvements, discarding obsolete terms, repetitions and needless theories, and so in *quantity* and *quality* of matter to economize *time*, *labor*, and *money*, and at the same time give *unity*, *system*, and *practical utility* to the science and art of computation. How nearly this end has been attained can be determined by the only *true test* of a text-book—its use in the class-room by the intelligent and experienced teacher.

D. W. F.

BROOKLYN, July, 1883.

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FISH'S ARITHMETIC.

NUMBER TWO.

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1. ***Arithmetic*** is the science of numbers and the art of computation.

As a *science*, arithmetic treats of the nature and properties of numbers. As an *art*, it teaches how to apply a knowledge of numbers to practical and business purposes.

2. A ***unit*** is *one* thing, or a group of things regarded as *one*.

Thus, *one* day, *one* man, *one* ten, *one* dozen, are units.

3. A ***number*** is a unit, or a collection of units.

Thus, *one, six; five* books, *eight* days, are numbers.
A number answers to the question, "*How many?*"

4. An ***integral number*** or ***integer*** is a number representing whole things.

Thus, 5, 7; 12 books, 9 men, 20 days, are integers or whole things.

5. The ***unit of a number*** is one of the collection forming that number.

Thus, 1 is the unit of 4; 1 dollar is the unit of 5 dollars.

6. ***Like numbers*** are numbers having the same kind of unit, or that express the same kind of quantity.

Thus, 3 and 5, 2 *days* and 6 *days*, 5 *cents* and 10 *cents* are like numbers.

NOTATION AND NUMERATION.

7. ***Notation*** is a method of *writing* numbers.

Numbers are expressed by *words* or *characters.*

The characters may be either *figures* or *letters.*

8. ***Numeration*** is the method of *reading* numbers expressed by characters.

9. ***Arabic notation*** employs *ten* different characters called *figures,* to express numbers, viz.:

0	1	2	3	4	5	6	7	8	9
Naught,	***one,***	***two,***	***three,***	***four,***	***five,***	***six,***	***seven,***	***eight,***	***nine.***

The first character, *naught,* is also called *cipher,* or *zero,* and when standing alone, has no value.

The other nine are called *significant* figures, because each has a value of its own. They are also called *digits.*

These ten characters, when combined according to certain principles, can be made to express any number.

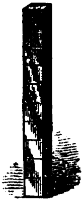
10. In representing numbers, objects are supposed to be arranged in *groups* of *ten,* each group being *ten times* as great as the next lower group, and having a different name. Hence, we have single things, or *units;* next, *groups* containing *ten* units, or *one ten;* next, groups containing *ten* tens, or *one hundred;* and again, *groups* containing *ten* hundreds, or *one thousand,* etc.

11. This method of grouping is called the ***decimal system,*** from the Latin *decem,* which signifies *ten.*

The successive order of units in a number form a ***scale;*** and when the increase or decrease is *uniformly ten,* it is called a ***decimal scale.***

1 Unit.

12. The first nine numbers are each expressed by a *single figure,* and are called units of the *first* order.



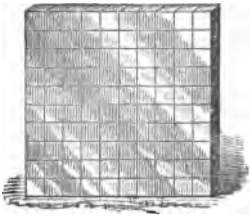
1 Ten.

13. Since there is no single character to express the number *ten,* by grouping ten ones or units of the *first* order into a larger collection, there is formed a unit of the *second* order, called *ten,* written ***10.***

In the same manner are expressed,

2 tens, or **Twenty,** by **20.**
3 tens, or **Thirty,** by **30.**
4 tens, or **Forty,** by **40.**
5 tens, or **Fifty,** by **50.**
6 tens, or **Sixty,** by **60.**
7 tens, or **Seventy,** by **70.**
8 tens, or **Eighty,** by **80.**
9 tens, or **Ninety,** by **90.**

The greatest number that can be expressed by *two* figures is 99.



1 Hundred.

14. By grouping ten units of the *second* order or *ten tens* into a larger collection, there is formed a unit of the *third* order, called *one hundred,* written ***100.***

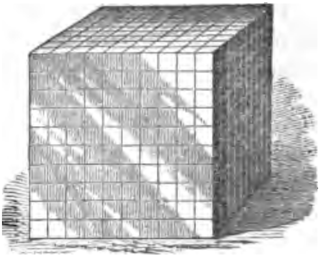
In like manner are expressed,

2 hundred, by **200.**
3 hundred, by **300.**
4 hundred, by **400.**
5 hundred, by **500.**
6 hundred, by **600.**
7 hundred, by **700.**
8 hundred, by **800.**
9 hundred, by **900.**

The numbers from one hundred to nine hundred ninety-nine inclusive are expressed by writing *hundreds* in the *third* place from the *right, tens* in the *second* place, and *units* in the *first* place.

The greatest number that can be expressed by *three* figures is 999.

15. By grouping ten units of the *third* order or *ten hundreds* into a larger collection, there is formed a unit of the *fourth* order, called *one thousand*, written ***1000.***



1 Thousand.

In like manner are expressed,

2 thousand, by **2000.**
3 thousand, by **3000.**
4 thousand, by **4000.**
5 thousand, by **5000.**
6 thousand, by **6000.**
7 thousand, by **7000.**
8 thousand, by **8000.**
9 thousand, by **9000.**

The numbers from one thousand to nine thousand nine hundred ninety-nine are expressed by writing *thousands* in the *fourth* place from the right, *hundreds* in the *third* place, *tens* in the *second* place, and *units* in the *first* place.

The greatest number that can be expressed by four figures is 9999.

16. In the same manner, other *new* orders are formed to represent larger numbers, by grouping *ten* units of the *fourth* order to form the *fifth* order, or *tens* of thousands; and *ten* units of the *fifth* order to form the *sixth* order, or *hundreds* of thousands, etc.

Thus, 432076 represents 6 units of the *first* order, 7 units of the *second* order, 0 units of the *third* order, 2 units of the *fourth* order, 3 units of the *fifth* order, and 4 units of the *sixth* order, and is read, *Four hundred thirty-two thousand seventy-six.*

TABLE.

10 Units (*1*)	make	1 Ten,	written,	***10.***
10 Tens (*10*)	"	1 Hundred,	"	***100.***
10 Hundreds (*100*)	"	1 Thousand,	"	***1000.***
10 Thousands (*1000*)	"	1 Ten-thousand,	"	***10000.***

17. ***Orders of units*** are denoted by the *position* of the figures used in expressing a number.

Thus, 532 represents 2 units of the *first order*, 3 units of the *second order*, or 3 *tens*, and 5 units of the *third order* or 5 *hundreds*, and is read *five hundred thirty-two.*

PRINCIPLES.—I. *Ten units of any order in a number make one unit of the next higher order.*

II. *When a figure is moved one place to the left, it represents units of ten times the value it did before.*

III. *When a figure is moved one place to the right, it represents units of one tenth the value it did before.*

When any order of units in a number is vacant, the place should be filled with a cipher.

DRILL EXERCISES.

18. Copy and read the following numbers, naming in each the number of units of *each order :*

231
426
1542
3473
128
561
1020
1256
727
636
32507
53106
810
244
7638
62732
2190
1927
52803
60032

Write in figures and read :

1. Seven units of the fourth order, eight of the second order, and three of the first.

2. Three units of the fifth order, two of the third, and one of the first.

3. Two units of the sixth order, nine of the fifth, four of the third, one of the second, and seven of the first.

4. Forty-seven thousand six hundred forty.
5. One hundred forty-six thousand five hundred.
6. Ninety-five thousand three hundred ten.
7. Four hundred seventy thousand two hundred.
8. Eight hundred five thousand one hundred three.

19. A ***period*** is a group of *three orders* of units, counting from right to left.

The first *period* on the right is called UNITS' PERIOD, the second, THOUSANDS' PERIOD, the third, MILLIONS' PERIOD, etc., as shown in the following

TABLE.

PERIODS.	4th. Billions.			3d. Millions.			2d. Thousands.			1st. Units.		
NAMES OF ORDERS OF UNITS.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.
Numbers to be read.	5	0	4,	1	2	7	3	6	4,	5	4	9
			2,	3	0	4,	2	5	1,	7	3	0
				3	4	5,	7	0	5,	2	4	9
						6,	3	2	0,	7	5	1

Beginning at the *right*, the order of *simple units* occupies the first place; *tens*, the second place; *hundreds*, the third; *thousands*, the fourth; *tens of thousands*, the fifth; *hundreds of thousands*, the sixth; *millions*, the seventh, etc.

Each period is read like a number of *three* figures, giving it the *name* of the period; thus, the first number above is read, 504 *billion* 127 *million* 364 *thousand* 549.

20. RULE FOR NOTATION.—*Begin at the left, and write the hundreds, tens, and units of each period in their proper order, filling all vacant orders and periods with ciphers.*

21. RULE FOR NUMERATION.—1. *Begin at the right, and separate the number into periods of three figures each.*

2. *Begin at the left, and read each period as if it were units, giving each its name.*

DRILL EXERCISES.

22. Express the following numbers by figures:

1. Twenty-six thousand twenty-six.
2. Fourteen thousand two hundred eighty.
3. One hundred seventy-six thousand.
4. Four hundred fifty thousand thirty-nine.
5. Seven hundred twenty thousand fifty-six.
6. Two hundred forty-nine thousand five hundred.
7. One million ninety-six thousand two hundred ten.
8. Seven million thirty-six.
9. Five hundred sixty-three thousand four.
10. One million ninety-six thousand.
11. Ten million ten thousand ten hundred ten.
12. Four hundred eighty-three million eight hundred sixteen thousand one hundred forty-nine.
13. Seven hundred fifty million six thousand nine hundred.
14. Ninety-nine billion thirty-seven thousand four.
15. Five billion eighty million one hundred ten thousand seventy-five.
16. Ten million two hundred forty thousand seventeen.
17. Four hundred twenty-six billion twelve million eight hundred sixty.
18. Four hundred eighty-three million eight hundred sixteen thousand one hundred forty-nine.

Copy, point off, and read the following numbers:

19. 24835.
20. 500037.
21. 20304.
22. 600317.
23. 108300.
24. 100103.
25. 310420.
26. 6423001.
27. 120897.
28. 4058205.
29. 31628045.
30. 84076301.
31. 53000008.
32. 8900406.
33. 250708590.
34. 466270035.
35. 3730016000.
36. 42020871400.

ADDITION.

INDUCTIVE EXERCISES.

23. 1. How many are 2 pounds and 3 pounds? 3 pounds and 4 pounds? 5 pounds and 3 pounds?

2. Can you unite 6 books and 4 pounds? Why not?

3. Can you unite 3 pounds and 7 pounds? Why?

4. How many are 6 units and 4 units?

5. How many are 3 tens and 4 tens? 5 tens and 3 tens?

6. How many are 4 hundreds and 5 hundreds?

7. What is the unit of 3 men? Of 12 rods? Of 8 tens?

8. What is the unit of 4 tens? Of 6 hundreds? Can these units be united? Why not?

9. What kind of *numbers* only can be united? What *orders* of units?

10. How many are 9, 3, and 6? 6, 3, and 9? 3, 6, and 9?

11. Is the result changed by uniting the same numbers in a different order?

12. A farmer sold 6 sheep to one man, and 9 to another. How many did he sell to both?

13. If a coat costs 10 dollars, and a pair of boots 8 dollars, what is the cost of both?

14. James gave 4 pears to George, and 5 to Belle, and kept 7. How many pears had he at first?

15. In a shop are 8 men, 6 boys, and 7 girls at work. How many persons in all are at work?

PRINCIPLE.—*Only like numbers and units of the same order can be added.*

DEFINITIONS.

24 ***Addition*** is the process of finding a number equal to two or more *like* numbers.

The **sum** or **amount** is the number found by addition.

25. The ***sign of addition*** is $+$. It is read *plus,* and signifies *more.*

26. The ***sign of equality*** is $=$, It is read, "*equals,*" or "*is equal to.*"

Thus, $5+6=11$ is read, 5 *plus* 6 *equals* 11. It may also be read, 5 *and* 6 *are* 11.

27. An ***equation*** is an expression of *equality* between two numbers or sets of numbers.

Thus, $8+4=12$ is an equation, and is read, 8 *plus* 4 *equals* 12, and means that the sum of 8 and 4 is equal to 12. $8+4$ is the *first member* of the equation, and 12 is the *second member;* and 8, 4, and 12 are called the *terms* of the equation.

Name the *members* and the *terms* of each of the following equations:

1. $7+10=17$.
2. $8+12=20$.
3. $15+6=21$.
4. $30=20+10$.
5. $9+6=10+5$.
6. $12+8=16+4$.
7. $14+6=12+?$
8. $9+20=?+7$.
9. $25+?=5+30$.
10. $36+0=?+20$.
11. $8+9+?=25$.
12. $?+6+8=15+8$.
13. $10+?+5=20+10$.
14. $21+7+6=?+25$.
15. $9+13+10=24+?$

The expression "$=?$" is read "*equals how many?*" or "*equals what?*"

28. DRILL TABLE No. 1.

	A	B	C	D	E	F	G	H	I
1.	2	3	8	6	7	2	4	6	5
2.	3	4	2	7	5	1	6	8	3
3.	4	7	3	9	4	6	3	4	7
4.	1	5	6	3	6	5	8	2	4
5.	7	1	9	2	8	4	2	7	1
6.	6	0	5	6	9	0	7	5	8
7.	0	6	1	4	6	8	4	9	6
8.	5	8	4	0	7	1	9	3	5
9.	8	3	7	3	5	9	3	6	2
10.	3	7	2	5	4	6	5	7	9

The pupil may be exercised on the above table as follows: *first*, by *columns, at sight*, give the sum of each number and the one below it; then, by *lines*, give the sum of each number and the one to the right of it.

In like manner, practice with sets of *three* figures, then of *four*, and so on, until each *column* from bottom to top, and from top to bottom, and each *line* from left to right, and from right to left, can be added with rapidity and accuracy.

Next start with 1, and add to it each column and each line successively; then start with 2, 3, etc., to 10.

The pupil should be required always to add as he counts: thus, in column A, beginning at the bottom, say, 3, 11, 16, 22, 29, 30, 34, 37, 39.

The above and all other subsequent *drill* and *oral exercises* are designed for *daily practice* in numbers. Not more than *five minutes* should be given to this kind of work in connection with the regular lesson.

DRILL EXERCISES.

29. 1. Add or count by 3's from 3 to 36.

Written, 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3, etc.
Read, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, etc.

Next, form a new combination, commencing with 1, and then another, commencing with 2, thus:

2, 5, 8, 11, 14, 17, 20, 23, 26, 29, etc.

Add, or count,

2. By 2's from 4 to 50.
3. By 3's from 1 to 43.
4. By 3's from 6 to 51.
5. By 4's from 1 to 53.
6. By 4's from 5 to 45.
7. By 5's from 7 to 82.

In like manner, treat the remaining digits 6, 7, 8, and 9.

8. Add by 2's and 3's alternately to 32.

Written, 2 + 3 + 2 + 3 + 2 + 3 + 2 + 3 + 2 + 3, etc.
Read, 2, 5, 7, 10, 12, 15, 17, 20, 22, 25, etc.

In like manner add,

9. By 2's and 4's to 48.
10. By 4's and 5's to 72.
11. By 5's and 3's to 69.
12. By 3's and 6's to 75.
13. By 5's and 6's to 88.
14. By 10's and 5's to 100.

Add the following rapidly, from the *open book,* naming only results, thus: 9, 15; 12, 16, 20; etc. Then the teacher may *dictate* the *first* member, and the pupil give the *second.*

9 + 0 + 6 =
12 + 4 + 4 =
4 + 12 + 4 =
6 + 9 + 3 =
12 + 7 + 6 =

15 + 3 + 6 =
3 + 15 + 4 =
2 + 11 + 10 =
16 + 0 + 10 =
7 + 20 + 5 =

8 + 9 + 6 =
9 + 6 + 8 =
10 + 7 + 5 =
21 + 7 + 6 =
9 + 13 + 10 =

The teacher may add to these last exercises, or the pupil be required to place upon the slate or the blackboard, original work of the same kind.

ORAL EXERCISES.

30. 1. How many *units* are 8, 6, and 7? 9, 5, and 8?

2. How many *tens* are 4 tens, 6 tens, and 7 tens?

3. How many *tens* are 40, 60, and 70? 50, 20, and 80?

4. How many *hundreds* are 4 hundreds, 6 hundreds, and 7 hundreds?

5. How many *hundreds* are 400, 600, and 700?

6. How many *thousands* are 4000, 6000, and 7000?

It will be observed that the *sum* of 4, 6, and 7 is the same, whether they represent *tens*, *hundreds*, or *thousands*.

7. How many *tens* and *units* are 30, 25, and 13?

8. How many *hundreds*, *tens*, and *units* are 200, 30, and 8?

Ans. 2 hundreds 3 tens 8 units; read, *two hundred thirty-eight*, 238.

Give the *sum* of each of the following *at sight:*

9. 200+50+7.
10. 400+60+5.
11. 900+40+9.
12. 600+70+4.
13. 1000+ 500+20.
14. 2500+ 300+46.
15. 5000+2100+80.
16. 700+3000+65.

17. A barrel of flour cost 10 dollars, a ton of coal 6 dollars, and a caddy of tea 9 dollars. What was the cost of all?

Analysis.—The *sum* of 10 dollars, 6 dollars, and 9 dollars, which is 25 dollars.

18. There are 9 birds sitting on one limb, 6 on another and 8 on another. How many birds on the three limbs?

19. An orchard contains 50 apple trees and 46 pear trees How many trees in the orchard?

In all the drill and oral exercises, little more than *models* can be given. The teacher is expected to give such additional examples for practice as may be required.

20. George earned 47 cents one day, and 32 cents the next. How many cents did he earn in both days?

21. A man bought a sleigh for 45 dollars, paid 22 dollars for repairing it, and 7 dollars for painting it. What was the entire cost?

22. A tailor bought two pieces of cloth, one containing 37 yards, and the other 29 yards. How many yards in both pieces?

23. There are 66 boys and 45 girls in a school. How many pupils in all?

24. What is the cost of a pound of tea at 53 cents, and a pound of coffee at 27 cents?

25. A lady paid 36 dollars for a shawl, and 24 dollars for dress goods. What did all cost?

31. The ***dollar sign*** is **$**. It is read, *dollars.*

Thus, $45 is read, 45 *dollars;* $9 is read, 9 *dollars.*

32. The character ¢, or the letters *ct.* or *cts.,* are used to denote *cents.*

Thus, 28¢, or 28 *cts.*, is read, 28 *cents.*

33. To distinguish *dollars* from *cents,* when written as *one number,* a point (.) is placed between the dollars and cents.

Thus, $6.25 is read, 6 *dollars* 25 *cents.*

34. Since 100 cents make one dollar, *cents* always occupy *two* places at the right of the point.

Thus, 42 cents may be written, $.42; 18 cents, $.18.

35. If the number of cents is less than 10, a cipher must occupy the first place at the right of the point.

Thus, 8 *cents* is written, $.08; 4 dollars 6 cents, $4.06.

36. The *sign* $ and the point (.) should never be omitted.

37. Copy and read the following numbers:

1. $456.
2. $1250.
3. $25600.
4. $30250.
5. $.42.
6. $.08.
7. $.02.
8. $.10.
9. $75.84.
10. $125.07.
11. $200.40.
12. $401.02.
13. $.05.
14. $.03.
15. $.62.
16. $.70.

Express the following by proper figures and signs:

17. Eight dollars fifty cents.
18. Twelve dollars twenty cents.
19. Forty-one dollars ten cents.
20. Ninety-seven cents.
21. Six dollars eighteen cents.
22. Fifty-five dollars five cents.
23. Eighty-one dollars ten cents.
24. Two hundred fifty cents.
25. Three hundred eight cents.
26. 63 dollars 80 cents.
27. 105 dollars 9 cents.
28. 100 dollars 10 cents.
29. 216 dollars 5 cents.
30. 20 dollars 3 cents.
31. 10 dollars 9 cents.
32. 310 dollars 41 cents.
33. 76 dollars 84 cents.
34. 70 dollars 7 cents.

WRITTEN EXERCISES.

38. To find the sum of two or more numbers.

1. Find the sum of 465, 762, and 238.

$$\begin{array}{rl}465 & \\ 762 & \\ 238 & \\ \hline 1465 & \text{Sum.}\end{array}$$

Explanation.—Arrange the numbers so that units of the same order stand in the same column.

When the sum of any column is 10, or more than 10, it contains one or more units of a *higher* order, which must be added to the next column.

Begin with the lowest order of units, and add each column separately, pronouncing successive results *only*; thus, 8, 10, 15, the sum of the *units*, equal to 1 ten 5 units. Write the 5 units in the units' place, and add the 1 ten to the tens' column; thus, 1, 4, 10, 16, the sum of the *tens*, equal to 1 hundred 6 tens. Write the 6 tens in the tens' place, and add the 1 hundred to the hundreds' column; thus, 1, 3, 10, 14, the sum of the *hundreds*, equal to 1 thousand 4 hundreds, which write in the hundreds' and thousands' places. Hence, the *sum* is 1465.

In like manner, copy and add the following:

(2.)	(3.)	(4.)	(5.)
876 feet.	407 pounds.	715 men.	$2342
325 "	390 "	306 "	760
422 "	678 "	478 "	1479
276 "	536 "	956 "	687

In arranging for addition, dollars should be written under dollars, and cents under cents, in such order that the *points* stand in a *vertical* line.

Copy and add the following:

(6.)	(7.)	(8.)	(9.)
$4.07	$200.35	$20.30	$145.24
2.80	57.84	7.56	36.60
12.39	125.06	13.08	105.08
10.	.65	25.	.75

10. Find the sum of $370.21, $2.49, $3.07, and $.94.

11. If 4 loads of coal weigh respectively 1922, 1609, 2100, and 1873 pounds, what is the entire weight?

12. Gave $325 for a horse, $275.50 for a carriage, $75.75 for a harness, and $20.62 for a robe. What was the cost of the whole?

Rule.—1. *Write the numbers so that figures of the same order stand in the same column.*

2. *Beginning at the right, add each column separately, and write the sum, if expressed by one figure, under the column added.*

3. *If the sum of any column consists of more than one figure, write the unit figure under that column, and add the remaining figure or figures to the next column.*

To test the result, perform the addition in the reverse direction, and, if the results agree, the work is probably correct.

The following is also a good exercise for the slate or board:

Explanation.—Write two numbers, as 342 and 534; add them, and use their sum, 876, as a *third* number; then add the three numbers and use their sum, 1752, for a *fourth* number, and so on, as far as desired.

342
534
876
1752

Add the following, extending each to the *sixth* number:

(13.)	(14.)	(15.)	(16.)	(17.)	(18.)
247	481	506	1407	3028	5630
325	149	97	2086	672	2074

19. 123 + 456 + 785 + 12 + 345 + 901 + 567 + 48 = ?
20. 12345 + 67890 + 8763 + 347 + 1037 + 198760 = ?
21. 172 + 4005 + 3761 + 20472 + 367012 + 19762 = ?
22. \$246.26 + \$85.50 + \$100.09 + \$17.75 + \$.96 = ?
23. \$84.90 + \$760 + \$1235.18 + \$56.73 + \$1000 = ?
24. \$4500 + \$63.19 + \$508.07 + \$24.30 + \$300.05 = ?

25. A merchant bought at public sale 852 yards of broadcloth, 750 yards of muslin, 2140 yards of flannel, and 350 yards of silk. How many yards in all?

26. Bought a pair of boots for \$8.50, an umbrella for \$3.62, a pair of gloves for \$1.25, some collars for \$.75, and a hat for \$4. What was the whole cost?

27. A lady gave \$48.50 for silk for a dress, \$16.75 for the trimmings, and \$15.62 for making. What was the cost of the dress?

28. Find the sum of 5736 dollars 45 cents, 1200 dollars 80 cents, 405 dollars 15 cents, 50 dollars 9 cents, and 84 cents.

29. A lady paid 12 dollars for a scarf, 3 dollars and 25 cents for a fan, two dollars for a pair of gloves, and 42 cents for a collar. How much did she pay for all?

30. A has \$2248, B has \$3677, and C has 800 more than A and B together. How many dollars have all?

31. Paid $3456 for a house, $426.75 for painting it, $2809.48 for furniture. What was the cost of the whole?

32. From Chicago to Detroit is 284 miles; Detroit to Suspension Bridge, 230 miles; Suspension Bridge to Albany, 304 miles; Albany to New York, 143 miles. What is the distance from Chicago to New York?

(33.)	(34.)	(35.)	(36.)	(37.)	(38.)
$99.84	96256	$117.76	98304	1728	$675.84
24.96	6016	29.44	6144	864	168.86
6.24	376	7.36	384	108	10.56
1.56	141	1.84	24576	81	1.32
12.48	188	3.68	3072	5296	.96
.98	1504	58.88	145	3456	2.64
3.12	752	1.38	49152	432	84.48

39. Four loads of coal weigh respectively 2148, 1976, 2209, and 1894 pounds. What is the weight of all?

40. A gentleman gave to each of his five sons $3840.50. What sum did he give to all?

41. A speculator bought, at different times, wheat as follows: 300 bushels, for $480.75; 615 bushels, for $1391.70; 2000 bushels, for $4240; and 1210 bushels, for $2620.80. How much wheat did he purchase, and what did it cost him?

For examples 42 to 48 inclusive, use the numbers in a *line* across the page; for 49 to 52, in *columns* as arranged.

	(49.)	(50.)	(51.)	(52.)
42.	$150.38	$475.00	$8120.60	$30402.55
43.	74.06	600.18	475.17	286.09
44.	349.71	34.60	46.82	5304.62
45.	8.94	109.09	120.08	20463.20
46.	67.08	66.87	2461.50	607.05
47.	503.79	10.07	500.07	6315.46
48.	42.51	246.23	1208.92	73.90

39. DRILL TABLE No. 2.

	A	B	C	D	E	F	G
1.	3	5	6	2	90	500	6000
2.	5	11	3	7	120	400	4000
3.	2	7	5	3	20	700	5000
4.	6	2	7	9	80	800	7000
5.	12	6	6	4	80	200	3000
6.	8	4	4	6	10	600	1000
7.	7	8	11	5	70	900	2000
8.	4	3	7	10	40	100	6000
9.	9	9	12	3	30	500	4000
10.	5	10	9	7	60	600	8000
11.	10	7	8	12	50	300	11000
12.	6	5	6	8	110	200	12000

Specific directions for using the above table for addition may be found under Table No. 1, page 16. Tables Nos. 1 and 2 will afford very profitable drill exercises in all the fundamental rules of Arithmetic.

The teacher will observe that besides the drills in *Addition, Subtraction, Multiplication*, and *Division*, this and Table No. 1 also afford abundant exercise in *Numeration*. The exercises of *adding*, etc., *at sight*, also involve *numerating at sight*. Further exercise may be had, by rapidly reading the columns and lines. Also, by mentally prefixing one or more figures to each number, and then reading rapidly by column and by line.

SUBTRACTION.

INDUCTIVE EXERCISES.

40. 1. How many are 10 cents less 3 cents? 10 cents less 4 cents? 10 cents less 5 cents?

2. Five taken from 10 leave how many? 7 from 10, how many? 8 from 10? 3 from 10?

3. How many tens are 7 *tens* less 3 *tens?* 9 *tens* less 5 *tens?*

4. How many hundreds are 7 *hundreds* less 4 *hundreds?* 9 hundreds less 6 hundreds? 800 less 500?

5. How many thousands are 8 *thousands* less 3 *thousands?* 9 thousands less 5 thousands? 7000 less 4000?

6. What is the unit of 6 miles? Of 12 acres?

7. Can 6 miles be taken from 12 acres? Why not?

8. Can 5 hundreds be taken from 8 thousands? 4 tens from 7 tens? Why?

9. Only what kind of a number can be taken from another?

PRINCIPLE.—*Any number, or any order of units, can be subtracted only from a like number, or a like order of units.*

41. ***Subtraction*** is the process of finding the difference between two *like* numbers.

1. The **minuend** is the number from which the other is taken.
2. The **subtrahend** is the number to be taken from the minuend.
3. The **difference** or **remainder** is the number found by subtraction.

42. The ***sign of subtraction*** is —. It is read *minus*, and signifies *less*.

Thus, $9-6=3$ is read, 9 *minus* 6 *equals* 3, or 9 *less* 6 *equals* 3.

DRILL EXERCISES.

43. Write upon the slate or board a line of figures, thus:

3, 4, 2, 5, 7, 3, 8, 6, 9, 5.

1. *At sight,* promptly give the difference between each two consecutive numbers; thus, 1, 2, 3, 2, 4, 5, 2, etc.

2. Next, give the difference between the *sum* of each two consecutive numbers, and the number next on the right; thus, 3 and 4 less 2 is 5, 4 and 2 less 5 is 1, etc. Then repeat, simply naming the results, 5, 1, 0, 9, 2, 5, etc.

3. Then subtract each number in succession from 10; thus, 7, 6, 8, 5, 3, 7, 2, 4, 1, 5.

4. In the same manner, subtract each number from 11, 12, 13, 14, 15, etc., to 20.

A large number of similar exercises may be obtained by the use of Drill Tables Nos. 1 and 2.

Each *line* and *column* of the table should be used for drills like the above.

Next, give the difference between the *sum* of two consecutive numbers and the number next below in each *column*; then, between the *sum* of two consecutive numbers and the number next on the right, in each *line*.

5. Take 2 from 23 till the remainder is 1.

Written, 23—2—2—2—2—2—2—2—2—2—2 = 1.
Read, 23, 21, 19, 17, 15, 13, 11, 9, 7, 5, 3, 1.

6. Take 3 from 26, till the remainder is 2.

7. Take 4 from 28, till the remainder is 0.

8. Take 4 from 39, till the remainder is 3.

9. Take 5 from 52, till the remainder is 2.

10. Begin with 2, and count 29 by adding 3 successively; then reverse and subtract 3, till the remainder is 2.

11. Begin with 3, and count by 4's to 31; then reverse, and subtract 5, till the remainder is 2.

12. In like manner, beginning with different numbers, add forward and subtract back by 4's, 5's, 6's, 7's, 8's, and 9's.

Equations to be treated in the same manner as those in Art. **29**:

$6+4-3=$	$7+5-6=$	$6+9-7=$
$9+5-6=$	$8+7-4=$	$10+6-8=$
$7-5+6=$	$10-4+8=$	$11-4+9=$

For rapid exercise:

Written, $9+8-5+3+7-2+6-8+4+5-9$, etc.
Read, 9, 17, 12, 15, 22, 20, 26, 18, 22, 27, 18.

8. $7+6+4-8+10-5=?$
9. $12+5+7-3+8-6+4=?$
10. $10+10-8+5-7+9=?$
11. $4+12+8-5-6+7=?$

Similar dictation exercises may be given by the teacher, or put upon the slate or blackboard by the pupil.

ORAL EXERCISES.

44. 1. How many *units* are 9 units less 4 units? $9-4$?

2. How many *tens* are 9 tens less 4 tens? $90-40$?

3. How many *hundreds* are 9 hundreds less 4 hundreds? $900-400$?

4. How many *thousands* are 9 thousands less 4 thousands? $9000-4000$?

It will be observed that the *difference* between 9 and 4 *abstractly* is the same, whether it represents units, tens, hundreds, or thousands.

5. Give the difference of each of the following *at sight:*

50 and 20.	650 and 400.	3000 and 1000.
70 and 30.	500 and 200.	8000 and 3000.
60 and 10.	900 and 300.	6000 and 2000.

6. Also, when arranged in *columns:*

750	900	1200	5500	1600	2500
600	200	800	3000	500	1200

For further exercises of this kind, use Drill Table No. 2.

7. A grocer having 20 boxes of lemons, sold 12 boxes. How many boxes had he left?

Analysis.—He had left the *difference* between 20 boxes and 12 boxes, which is 8 boxes.

8. If a man earns $12 a week and spends $7 for provisions, how many dollars has he left?

9. From a piece of cloth containing 17 yards, 8 yards were cut. How many yards remained?

10. A jeweler bought a watch for $11, and sold it for $18. What was his gain?

11. Clarence had ten cents, and Eloine gave him six more; he then gave 7 cents for a book. How many cents had he left?

Analysis.—He had left the *difference* between 7 cents and the *sum* of 10 cents and 6 cents, which is 9 cents.

12. Lillian bought a shawl for $16, and a pair of gloves for $2, and gave in payment two $10 bills. How much change should she receive?

13. A man having $20, paid $7 for a hat, and $8 for a vest. How many dollars had he left?

14. A boy had 9 cents, he earned 12 cents more, then gave 7 cents to his sister. How many cents had he left?

15. What is the difference between 76 and 39?

Separating the larger number, 76, into 60+16, and the smaller, 39, into 30+9, the *difference* between 76 and 39 is 37.

$$\begin{array}{r}76 = 60 + 16\\ \underline{39 = 30 + 9}\\ 37 = 30 + 7\end{array}$$

16. From a piece of cloth containing 56 yards, 18 yards were cut. How many yards remained?

17. James having 75 cents, gave 12 cents for a book and 15 cents for a slate. How many cents had he left?

18. A man having received $45 for labor, paid $15 for a coat, $7 for a barrel of flour, and $6 for a ton of coal. How much had he left?

19. In a garden are 57 fruit trees; 15 of them are peach trees, 12 plum trees, and the remainder pear trees. How many pear trees are there?

What is the difference between

20. 17 and 4+6?
21. 24 and 9+5?
22. 18 and 7+7?
23. 20 and 6+6?
24. 35 and 9+20?
25. 28 and 9+9?
26. 12+5 and 25+3?
27. 27−8 and 14+8?
28. 30−10 and 9+8?

Find the *omitted* terms in the following equations:

29. 18− 6 = ? +6.
30. 14+ 7 = 18+?
31. 8+ 7 = 17−?
32. 13+ 7 = ? −4.
33. 22−10 = 8+?
34. 24− 8 = 6+?
35. 28+ 6 = 40−?
36. 42−12 = 18+?
37. 30+25 = ? +40.
38. 27−11 = 19−?
39. 36+16 = 60−?
40. 54−12 = ? +12.

WRITTEN EXERCISES.

45. To find the difference between two like numbers.

1. Find the difference between 653 and 476.

	5	14	13
Minuend,	6	5	3
Subtrahend,	4	7	6
Difference,	1	7	7

Explanation.—Write the less number under the greater, so that units of the same order stand in the same column.

Begin at the right hand and subtract each order of units separately.

Since 6 units cannot be taken from 3 units, increase the 3 units by 10 units (1 *ten*) taken from the next higher order, making 13 units; 6 units from 13 units leave 7 units, which write in the units' place.

Since 1 of the tens was united with the units, there are 4 tens left. As 7 tens cannot be subtracted from 4 tens, increase the 4 tens by 10 tens (1 *hundred*) taken from the next higher order, making 14 tens; 7 tens from 14 tens leave 7 tens, which write in tens' place.

Since 1 of the hundreds was united with the tens, there are 5 hundreds left; and 4 hundreds from 5 hundreds leave 1 hundred, which write in hundreds' place. Hence, the *difference* between 653 and 476 is 177.

RULE.—1. *Write the subtrahend under the minuend, placing units of the same order in the same column.*

2. *Begin at the right hand, and take each figure of the subtrahend from the figure above it, and write the result underneath.*

3. *If any order of the subtrahend is greater than the same order of the minuend, add* 10 *to the latter, and subtract.*

4. *Then diminish by 1 the next higher order of the minuend, and proceed as before.*

PROOF.—*Add the remainder to the subtrahend; if the sum is equal to the minuend, the work is correct.*

Instead of *diminishing* by 1 the next higher order of the *minuend*, *practically* we add 1 to the next higher order of the *subtrahend*.

Copy, solve, and prove the following:

	(2.)	(3.)	(4.)	(5.)	(6.)
Min.	439	674	523	834	423
Sub.	172	327	247	263	276
Rem.	267				

How many years from the date of each of the following events to the year 1887?

7. Printing was invented in 1441.
8 The telescope was invented by Galileo in 1610.
9. First use of a locomotive in this country in 1820.
10. First railway in the United States built in 1827.
11. The first iron steamship was built in 1830.
12. The first steamboat plied the Hudson in 1807.
13. The first passage of the Atlantic by steam in 1839.
14. The first lucifer match was made in 1829.
15. The first steel pen was made in 1830.
16. The first newspaper published in America in 1704.
17. The electric telegraph first used in the U. S. in 1844.

Numbers composed of dollars and cents must be written as in addition.

When one of the given numbers contains *cents*, and the other does not, fill the vacant places with two ciphers.

	(18.)	(19.)	(20.)	(21.)
From	$325.17	$279.00	$105.08	$7.00
Take	84.36	183.42	67.00	.84

Subtract

22. 20762 from 53120.
23. $73.16 from $138.
24. $247 from $382.28.
25. $54.26 from $68.37.
26. 2714 from 5945.
27. $30.52 from $81.76.

From

28. $430.09 take $272.46.
29. 15200 rods take 6472 rods.
30. 120764 tons take 75028 tons.
31. 3886 tons take 1763 tons.
32. 8569 feet take 6245 feet.
33. 9625 days take 7301 days.

Find the difference between

34. 1204 and 5379.
35. 1370 and 1421.
36. 8673 and 3560.
37. $57.46 and $18.00 + $24.25.
38. $50.20 + $4.01 and $76.31.
39. $98.76 and $30.46 + $43.04.

40. 2762 + 10341 and 44701 + 1200 = ?

41. 3000 + 42301 and 720 + 1684 + 7342 = ?

42. What is the sum of 4062 and 12356, increased by the difference between 15000 and 975?

43. From the sum of 23462 and 9030, subtract the difference between 34000 and 7640.

44. From the difference between 19876 and 6032, subtract the difference between 12000 and 673.

45. A man has $10000. How much must he add to this, to be able to pay for a farm worth $13640?

46. California contains 158933 square miles, and Texas 237321 square miles. How much larger is Texas than California?

47. A man having $15740.80, gave $5085 for a store, and $7640.75 for goods. How much money had he left?

48. A merchant bought a quantity of goods for $1248.65, and sold them for $1540. What did he gain?

49. A man bought a western farm for $22700; he expended $4375 for improvements, and $2862 for stock: he sold the whole for $32000. Did he gain or lose, and how much?

50. A man willed $125000 to his wife and two children. To his son he gave $44675, to his daughter $26380, and the remainder to his wife. What was his wife's share?

A convenient method of subtracting the *sum* of two or more numbers from another:

51. From 948 subtract (325 + 106 + 82).

$$\begin{array}{r} 948 \\ \left\{\begin{array}{r} 325 \\ 106 \\ 82 \end{array}\right. \\ \hline 435 \end{array}$$

EXPLANATION.—Write the numbers as in addition, placing the minuend at the top.

Then add and subtract, thus: 2, 8, 13, and 5 more make 18; write 5 in the remainder, and carry 1 to the next column. 1, 9, 11, and 3 more make 14; write 3 in the remainder, and carry 1 to the next column. 1, 2, 5, and 4 more make 9; write 4 in the remainder.

If the work is correct, the sum of the *remainder* and the *parts* of the subtrahend will equal the minuend.

In like manner, solve and prove the following:

	(52.)	(53.)	(54.)	(55.)
Min.	7362	6704	2400	$350.00
Sub.	2043	2362	841	145.73
	1525	501	365	80.50
	367	96	42	10.26

56. Mr. Jones having $12470, paid $4070 for a house, $927 for furniture, and $1000 for improvements. How much money had he left?

57. If a man's income is $10000 a year, and he pays $1500 for rent, $1250 for provisions, $375 for taxes, and $1463 for other expenses, how much has he left?

58. A merchant of Nashville goes to New Orleans with $21600. He invests $7638.50 in groceries, $3210.65 in crockery, $1245.18 in woodenware, and the remainder in hardware How much does he invest in hardware?

59. A man owns property valued at $75860, of which $45640 is invested in real estate, $25175.75 in personal property, and the remainder he has in bank. How much has he in bank?

60. A man deposited in bank $8752; he drew out at one time $4234, at another $1700, at another $962, and at another $49. How much had he remaining in bank?

61. A man buys goods as follows: groceries for $9.68; dry goods for $7.49; clothing, $19.95; nails for $2.58; and a plow for $9.75, and pays for them with a 50-dollar bill. How much change does he receive?

62. A gentleman possessing $47840, at his death gave to each of his two sons $18755, and the remainder to his daughter. How much did the daughter receive?

63. Three persons bought a hotel valued at $42075. The first agreed to pay $8375.50, the second agreed to pay twice as much, and the third the remainder. How much was the third to pay?

64. A man invests in trade at one time $680; at another time $820; at a third time $1580; and on a fourth occasion $420. How much must he add to the sum of these that the amount may be $5000?

65. A man bought a western farm for $22700; he expended $4375 for improvements, and $2862 for stock: he sold the whole for $32000. Did he gain or lose, and how much?

MULTIPLICATION.

INDUCTIVE EXERCISES.

46. 1. At 6 cents each, what is the cost of 4 oranges?

2. How many cents are 6 cents + 6 cents + 6 cents + 6 cents?

3. What is the *sum* of 6 cents taken *four* times?

4. Add by 6's to 24; by 4's to 24; by 3's to 24; 8's to 24.

5. How many are four 6's, or 4 times 6? Six 4's, or 6 times 4?

6. Do the results differ? Why not?

7. James bought 5 pencils, at 7 cents each; how many times 7 cents did he pay? What is the sum of 7 cents repeated 5 times? 5 times 7 = ?

8. Add by 7's to 35; by 5's to 35.

9. What is the difference between five 7's and seven 5's?

10. What is the *sum* of 8 repeated 3 times? Of 3 repeated 8 times? How many are 8+8+8? 3 times 8 = ?

11. What sum is produced by taking $8 five times?

12. What is produced by taking 9 as many times as there are units in 3? In 4? In 5?

13. At $6 a ton, what is the cost of 5 tons of coal?

ANALYSIS.—Since 1 ton of coal costs $6, 5 tons cost 5 times $6, or $30 Hence, 5 tons of coal cost $30.

14. What is the unit of $6? Of 5? Of the number produced by taking $6 five times?

15. Should we repeat $6 five times, or 5 six times? Why?

16. What will the unit of the number produced always be like? *Ans.* Like the unit of the number repeated.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	**4**	6	8	10	12	14	16	18	20	22	24
3	6	**9**	12	15	18	21	24	27	30	33	36
4	8	12	**16**	20	24	28	32	36	40	44	48
5	10	15	20	**25**	30	35	40	45	50	55	60
6	12	18	24	30	**36**	42	48	54	60	66	72
7	14	21	28	35	42	**49**	56	63	70	77	84
8	16	24	32	40	48	56	**64**	72	80	88	96
9	18	27	36	45	54	63	72	**81**	90	99	108
10	20	30	40	50	60	70	80	90	**100**	110	120
11	22	33	44	55	66	77	88	99	110	**121**	132
12	24	36	48	60	72	84	96	108	120	132	**144**

DEFINITIONS.

47. ***Multiplication*** is the process of taking one of two numbers as many times as there are units in the other. Or,

It is a short method of *adding equal* numbers.

1. The **multiplicand** is the number taken or multiplied.

2. The **multiplier** is the number by which to multiply.
It shows *how many times* the multiplicand is taken.

3. The **product** is the result found by multiplication.

4. The multiplicand and multiplier are called **factors**, because they produce the product.

48. The ***Sign of multiplication*** is ×. It is read *times*, or *multiplied by*.

When placed between two numbers, it shows that they are to be multiplied together. Thus, 9 × 7 is read, 9 *multiplied by* 7, or 7 *times* 9.

49. Brief and rapid *oral* exercises should frequently be given the pupil upon the tables, and in the fundamental operations of numbers.

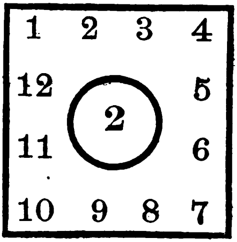
Pupils may be exercised in multiplication upon Tables 1 and 2, as follows:

1 First, by columns, *at sight*, give the product of each number and the one below it. Then taking two columns, give the product of each pair of numbers.

2. Then practice on each line from left to right, and from right to left. Then, taking two lines, give the successive products at sight.

3. Next, multiply the numbers in each line and column by 2, 3, 4, 5, etc., to 12, in every case naming only results.

The following *device*, or the *Arithmetical Chart*, which is constructed on the same plan, is peculiarly adapted to facilitate the memorizing of the tables.



Explanation.—1. Draw on the slate or board a small square, and around the *inside* write the numbers from 1 to 12 inclusive, in regular order, as shown in the diagram.

2. In the centre write one of the numbers, as 2, for a multiplier. Commencing with 1, read rapidly the *product* of the central figure and each figure in the margin, first, to the *right*, then to the *left*, naming only results; thus, 2, 4, 6, etc. Or read from the *open book*.

3. Next practice, in the same manner, with the numbers in the top and the bottom lines, and in the right-hand and left-hand columns.

4. Then use successively the figures in the margin as multipliers, and the central figure as the multiplicand.

5. Now, to vary the exercise and test the pupil, the teacher may point in rapid succession to the numbers in the margin, without regard to the order of arrangement, and the pupil, as promptly, give the *product*.

6. By erasing the central number, and inserting another, a new table and set of products are provided.

7. In like manner, continue this exercise until all the numbers in the margin have been used as *central* numbers.

8. A few minutes, at each recitation, devoted to this drill exercise, on the board, or chart, will soon make the pupils familiar with the tables.

50. An ***abstract number*** is a number in which the kind of unit is not named.

Thus, 4, 7, 9, 15, 40, are abstract numbers.

51. A ***concrete number*** is a number in which the kind of unit is named.

Thus, 3 *men*, 7 *hours*, 9 *tons* are concrete numbers.

52. The ***parenthesis,*** ( ), and ***vinculum,*** $\overline{\quad}$, *connect terms,* and show that the numbers included by them are to be treated as *one number.* The signs + and − *separate* terms.

Thus, 24 × (12−7) signifies that 24 is to be multiplied by the difference between 12 and 7.

INDUCTIVE EXERCISES.

53. 1. What is the product of 8 cents multiplied by 5?

2. What kind of number is the *multiplicand?*
3. What kind of number is the *multiplier?*
4. What kind of number is the *product?*
5. Name two numbers that are *like* and *abstract.*
6. Name two numbers that are *like* and *concrete.*
7. What is the product of 12 feet multiplied by 7?
8. What is the product of 9 multiplied by 6?
9. What kind of numbers are the *factors?* The *product?*
10. What two parts in multiplication are always *like* numbers?
11. Can you multiply 8 miles by 7 days? Why not?

PRINCIPLES.—I. *The multiplier is always regarded as an abstract number.*

II. *The multiplicand and product are like numbers, and may be either concrete or abstract.*

In examples containing concrete numbers, the concrete number is the *true multiplicand,* but when it is the smaller, it is often, for convenience, used *abstractly* as the multiplier.

54. The pupil may read the following rapidly, from the open book, thus: 64, 74; 36, 26; 15, etc.*

8 × 8 + 10 =
9 × 4 − 10 =
12 × 0 + 15 =
10 × 12 − 25 =
[†] 0 × 11 + 12 =

10 × 10 − 14 =
7 × 12 + 16 =
8 × 5 − 7 =
0 × 12 × 8 =
1 × 12 + 8 =

15 × 2 + 15 =
11 × 11 − 9 =
12 × 0 + 25 =
10 × 12 − 16 =
12 × 11 − 12 =

8 × 4 =
3 × 9 =
0 × 5 =

9 × 5 =
7 × 11 =
12 × 6 =

9 × 12 =
11 × 10 =
10 × 12 =

10 × 8 =
9 × 7 =
9 × 11 =

The pupil may make *applied examples* for the above. Thus, for 8 × 4: "What will 4 pounds of sugar cost, at 8 cents a pound?" Another pupil may solve it thus: "4 pounds of sugar will cost 4 times 8 cents, or 32 cents." For 3 × 9: "If a bag holds 3 bushels of wheat, how many bushels will 9 such bags hold?" etc.

ORAL EXERCISES.

Find the result:

1. Of 5 times 6 *units?* Of 6 × 5? Of 8 × 6?
2. Of 5 times 6 *tens?* Of 60 × 5? Of 80 × 6?
3. Of 5 times 6 *hundreds?* Of 600 × 5? Of 800 × 6?
4. Of 5 times 6 thousands? Of 6000 × 5? Of 8000 × 6?
5. Of 7 × 5? 70 × 5? 700 × 5? 7000 × 5?

It will be observed that the product of 6 by 5 is the same, whatever order of units 6 may represent.

Find the product of

6. 8 by 5.
7. 9 by 6.
8. 7 by 8.
9. 12 by 4.
10. 60 by 4.
11. 80 by 6.
12. 40 by 8.
13. 50 by 9.
14. 400 by 6.
15. 300 by 9.
16. 800 by 7.
17. 900 by 8.
18. 3000 by 7.
19. 5000 by 6.
20. 2000 by 9.
21. 7000 by 8.

Table No. 2 will furnish additional exercises of this kind.

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* For suggestions on the right use of signs, see notes on page 55.

[†] Multiplying any number by 0 produces 0; and multiplying 0 by any number produces 0. Thus, 9 × 0, or 0 × 9 = 0.

22. When we say, multiply 8 by 7, what do we mean?

23. In the statement, 9 times $12 are $108, name the multiplicand; the multiplier; the product; the factors; the abstract number; the concrete numbers; the like numbers.

24. Name the two factors of 25; of 27; of 35; of 63.

25. How many are 2 times 5 times 8 cents? Of what are 2, 5, and 8 factors?

26. Name three factors of 24; of 36; of 40.

27. At $8 a ton, what is the cost of 4 tons of coal?

Ans. $8 + $8 + $8 + $8, or 4 times $8 = $32.

28. What is the cost of 6 hats, at $5 a piece? At $7?

29. At 9 cents each, what will 6 melons cost? 8 melons?

30 How many are 7 times $5, minus $6? Plus $10?

31. How many are 10 times 6 bushels, plus 12 bushels?

32. How many are 6 times 12 miles, less 20 miles?

33. James gave 5 cents apiece for 6 oranges. How much change should he receive for 50 cents?

34. How much more than $35 will 5 barrels of flour cost, at $8 a barrel?

35. George had 10 cents, and James had 5 times as many. How many had both?

36. A man paid $4 a yard for 12 yards of cloth, and had $10 left? How much money had he at first?

37. What will be the cost of 6 cows at $26 each?

Analysis.—Six cows will cost 6 times $26. 6 times 6 units are 36 units, or 3 tens and 6 units, and 6 times 2 tens are 12 tens, which, plus 3 tens and 6 units, are 15 tens and 6 units, or 156. Hence, 6 cows will cost $156.

38. If the cars run 25 miles an hour, how far will they run in 7 hours? In 6 hours?

39. If a house rent for $21 a month, what is the rent for 6 months? 8 months? 10 months?

40. When eggs are 18 cents a dozen, what is the cost of 5 dozen? Of 6 dozen? Of 10 dozen?

41. How much less than $64 will 5 barrels of flour cost, at $9 a barrel? How much more is $6\times12+9$ than $7\times9-3$?

42. What is the difference in cost of 8 cords of wood, at $3 a cord, and 6 tons of coal, at $5 a ton?

Find the required *term* in the following equations:

43. $4\times10-4\times5=?$
44. $8\times5+6+10=?$
45. $50-5\times0+7=?$
46. $8\times4-6\times0=?$
47. $4\times10+3+?=50.$
48. $0\times7+8+?=40.$
49. $12+4+8\times?=32.$
50. $9\times8+10-?=75.$
51. $28-20+4\times?=56.$
52. $61+7-?\times10=48.$

WRITTEN EXERCISES.

55. When the multiplier consists of one figure.

1. Multiply 374 by 4.

First.

$$\left.\begin{array}{r}374\\374\\374\\\underline{374}\end{array}\right\}\text{ Equal parts.}$$

$$1496 \quad \text{Sum.}$$

Explanation.—By this method, the result is obtained by *addition*. Since the units, tens, and hundreds in 374 are each to be taken 4 times, the result is obtained by writing 374 four times in a column, and finding the *sum*.

Second.

$$\begin{array}{lr}\text{Multiplicand,} & 374\\ \text{Multiplier,} & \underline{4}\\ \text{Product,} & 1496\end{array}$$

Explanation.—The preceding method is much shortened by *multiplication*. Write the multiplicand 374 but once; and since its parts, 3 hundreds, 7 tens, 4 units are each to be taken 4 times, write the multiplier 4 under the units, and multiply; thus, 4 times 4 units are 16 units, or 1 ten 6 units. Write the 6 units in units' place, and add the 1 ten to the product of the tens.

Next, 4 times 7 tens are 28 tens, and the 1 ten added make 29 tens, or 2 hundreds 9 tens. Write the 9 tens in tens' place, and add the 2 hundreds to the product of the hundreds.

Lastly, 4 times 3 hundreds are 12 hundreds, and the 2 hundreds added make 14 hundreds, or 1 thousand 4 hundreds, which write in the hundreds' and thousands' places.

Hence, the *product* 1496 is the same as the *sum* of its equal parts.

Solve by both methods,

2. 3 times 84.
3. 4 times 135.
4. 5 times 234.
5. 6 times 352.
6. 4 times $204.
7. 5 times $425.

8. Multiply 4621 by 4; by 5; by 6; by 7.
9. Multiply 3062 by 6; by 7; by 8; by 9.
10. Multiply 5604 by 3; by 6; by 8; by 5; by 7.
11. Multiply 28314 by 4; by 6; by 8; by 9; by 5.
12. What will be the cost of 231 tons of coal at $9 a ton?

Although $9 is the *true multiplicand*, for convenience we may use it as the multiplier, and 231 as the multiplicand; but the *product* is *dollars*, because the true multiplicand is dollars.

This is obvious, since 231 tons at $1 a ton would cost $231, and at $9 a ton, 9 times $231.

13. What cost 2140 barrels of flour, at $8 a barrel?
14. What cost 1238 cords of wood, at $5 a cord?
15. What cost 752 pounds of nails, at 7 cents a pound?

SOLUTION.—7 cents × 752 = 5264 cents = $52.64.

When *either* factor contains cents, the *product* is *cents*, and may be changed to dollars and cents by putting the point (.) *two* places from the *right*, and prefixing the sign ($).

	(16.)	(17.)	(18.)	(19.)
Multiply	$43.72	$136.04	87 cents.	$2.06
By	8	7	9	6
Product,	$349.76	$952.28	$7.83	$12.36

20. At 6 cents a pound, what cost 675 pounds of rice?
21. At $4.37 a yard, what is the cost of 7 yards of cloth?
22. What cost 326 tons of coal, at $6 a ton?
23. What cost 379 pounds of nails, at 6 cents a pound?
24. At $6.75, what will 7 tons of coal cost?

56. When the multiplier consists of two or more figures.

1. Multiply 736 by 45.

Multiplicand.	736	
Multiplier.	45	
1st partial prod.	3680	= 736 × 5
2d partial prod.	2944	= 736 × 40
Entire product.	33120	= 736 × 45

Explanation.— Write the multiplier under the multiplicand, like orders of units in the same column.

Since the multiplier, 45, consists of 5 units 4 tens, the multiplicand is to be taken 5 times plus 40 times, or 45 times.

5 times 736 is 3680, the *first* partial product; 4 tens or 40 times 736 is 29440, the second partial product.

The *sum* of the partial products is the entire product, 33120.

In like manner, multiply

2. 475 by 27; by 38.
3. 678 by 36; by 47.
4. $36.42 by 28; by 46.
5. $70.45 by 65; by 82.

Rule.—1. *Write the multiplier under the multiplicand, placing units of the same order in the same column.*

2. *Beginning at the right, multiply each order of the multiplicand by each order of the multiplier, successively, and place the right-hand figure of each partial product under the order of the multiplier used.*

3. *The sum of the partial products is the required product.*

When there are ciphers in the multiplier, multiply by the significant figures only, since the product of any number by 0 is 0.

6. Multiply 6432 by 75; by 67; by 136; $36.45 by 34; by 37; by 126.

7. Multiply 23072 by 128; by 243; by 307; $142.50 by 63; by 234; by 325.

8. What will be the cost of building a line of telegraph 274 miles long, at $967 a mile? Of a brick wall 684 rods long, at $132 a rod?

9. If 1049 pounds of seed cotton be raised from an acre of land, how many pounds will 386 acres produce?

10. If a cotton mill manufactures 628 yards of cloth in a day, how many yards can it make in 297 days?

What is the product

11. Of 2572 bushels by 94?
12. Of $403.06 by 127?
13. Of 86072 pounds by 208?
14. Of 316 times $487.46?
15. Of 507 times 30975 days?
16. Of 325 times 6408 cents?
17. Of 370607 by 4071?
18. Of 600326 by 2645?
19. Of 730096 by 5006?
20. Of 2407068 by 3406?
21. Of 408091 by 2407?
22. Of 73069 by 46035?

Find the cost

23. Of 386 railway coaches, at $7034.75 each.
24. Of 67 hogsheads of sugar, at $37.75 a hogshead?
25. Of 2347 acres of land, at $136 an acre?

57. The process of finding the product of several factors, by multiplying the first by the second, and the result by the third, and so on, is called ***Continued Multiplication.***

Thus, 126 × 72 = 9072, or, 126 × 3 × 4 × 6 = 9072.

PRINCIPLE.—*The product of any number of factors in continued multiplication will be the same, in whatever order they are used.*

Multiply

1. $73.04 by 48, or 8 × 6.
2. 50076 by 72, or 6 × 4 × 3.
3. 46502 by 84, or 7 × 4 × 3.
4. $206.14 by 96, or 4 × 4 × 6.
5. $780.91 by 108.
6. 140086 by 120.
7. 380509 by 144.
8. $557.52 by 240.

9. 127 × 48 × 12 = ?
10. 325 × 25 × 127 = ?
11. 769 × 204 × 0 = ?
12. 2070 × 68 × 102 = ?
13. 376 × 206 × 244 = ?
14. 932 × 95 × 462 = ?
15. 2008 × 0 × 187 = ?
16. 7306 × 306 × 241 = ?

17. A merchant bought 29 pieces of cloth, each piece containing 47 yards, at $8 a yard. What was the cost of the whole?

18. What will be the cost of 45 sets of Cyclopædias, each set containing 16 volumes, at $7 a volume?

19. How many yards of sheeting in 57 bales, each bale containing 26 pieces, and each piece 44 yards?

20. What is the cost of 128 barrels of beef, each containing 216 pounds, worth 13 cents a pound?

21. Three schooners ship 239 cords of wood each, and a fourth ships 248 cords. What is the value of the whole at $4.25 a cord?

58. When there are ciphers at the right of one, or of both factors.

1. Multiply 236 by 100.

$$\begin{array}{r} 236 \\ \underline{100} \\ 23600 \end{array}$$

Explanation.—Since removing a figure *one place* to the left increases its representative value *ten times* (**17**-II), annexing a cipher to a number *multiplies* it by 10; annexing two ciphers multiplies it by 100, etc.

2. Multiply 840 by 300.

$$\begin{array}{r} 840 \\ \underline{300} \\ 252000 \end{array}$$

Explanation.—840 = 84 × 10, and 300 = 3 × 100. First multiply together the two factors 84 and 3, and then multiply their product 252 by 10 × 100, or 1000, by annexing three ciphers, which gives 252000, the required product.

Rule.—*To the product of the significant figures annex as many ciphers as there are on the right of both factors.*

1. For an occasional exercise, require the pupil to give *at sight* the products of each number in columns of Drill Table No. 1 multiplied by 10, 20, 30, to 90; then by 100, 200, 300, etc., and then by 1000, 2000, 3000, etc. See pages 16 and 24.

2. These combinations should be made from *dictation* as well as at sight.

What is the product

3. Of 436 by 10? by 100? by 1000? by 10000?

4. Of 2340 by 60? by 500? by 3200? by 25000?

5. At $160 an acre, what will 500 acres of land cost?

6. At $9 a barrel, what is the cost of 1200 barrels of flour?

7. A merchant bought 240 barrels of flour for $1920, and sold it at $10.50 a barrel. What did he gain?

8. What is the difference in the cost of 48 horses, at $184.50 each, and 130 sheep at $4.80 a head?

9. If a man buys 40 acres of land at $35 an acre, and 56 acres at $29 an acre, and sells the whole at $32 an acre, what does he gain or lose?

10. If my income is $3000 a year, and my expenses $40 a week, what do I save in a year?

11. From 207300 — 236 × 48 take 976 × 98 + 10050.

12. Multiply 675 — (77 + 56) by 3 × 155 — (214 — 28).

13. Bought 150 barrels of flour for $1050, and finding 25 barrels worthless, sold the remainder at $8.50 a barrel. What was gained or lost?

Find the amount of each of the following bills:

14. M. H. DECKER,

Bought of WM. C. DRYER.

136 bushels of oats, at $.37 a bushel.
56 barrels of potatoes, at $2.50 a barrel.
17 tons of hay, at $16.75 a ton.
42 cords of wood, at $3.125 a cord.

15. GEO. MCDOUGAL,

Bought of JOHN CLARK & CO.

128 tons of coal, at $5.25 a ton.
1600 pounds of bar lead, at 6¢ a pound.
750 pounds of printing paper, at 12¢ a pound.

16. THOS. MCMILLAN.

Bought of SAMUEL STONE & CO.

25 barrels of mess pork, at $16.25 a barrel.
14 tubs of butter, of 64 pounds each, at 28¢ a pound.
36 barrels of winter apples, at $4.18 a barrel.
16 barrels of flour, at $7.84 a barrel.

17. A flour merchant bought 1500 barrels of flour, at $7 a barrel; he sold 800 barrels at $10 a barrel, and the remainder at $6 a barrel. What was his gain?

18. A farmer exchanged 584 bushels of wheat at $2 a bushel, for 78 barrels of flour at $9 a barrel, and received the balance in money. How much money did he receive?

19. Two persons start from the same point and travel in opposite directions; one travels at the rate of 32 miles a day, the other at the rate of 39 miles a day. How far apart will they be in 14 days?

20. A planter sold 209 bales of cotton at $76 a bale, and from the proceeds he bought 107 acres of land, at $60 an acre, 18 mules at $75 each, and 4 pairs of horses at $218 a pair. How much money had he left?

21. A man bought 45 acres of land at $38 an acre, and 76 acres at $47 an acre, and sold the whole at $45 an acre. Did he gain or lose, and how much?

22. A man owing $15760, gave in payment 5 lots of land worth $730 each, 6 horses valued at $226.50 each, an interest he owned in a coal mine, worth $2000, and $1589.80 in money How much did he still owe?

Complete the following equations:

23. $(142 + 405) \times (1000 - 850) - 5000 = ?$
24. $97 \times 1000 - (75 \times \overline{500 - 420}) + 1500 = ?$
25. $\$73.46 - (\$.94 + \$3.02) + \$47 \times 35 = ?$
26. $\$246.08 \times 104 + (\$2000 - \$240.50) \times 10 = ?$

DIVISION.

INDUCTIVE EXERCISES.

59. 1. How many 4's are 12? Are 16? Are 24?

2. How many men are 12 men less 6 men, less 6 men?

3. How many feet are 16 feet — 4 feet — 4 feet — 4 feet — 4 feet?

4. How many times can 6 men be taken from 12 men? 4 feet from 16 feet?

5. How many 6's in 12? 4's in 16? 5's in 20?

6. How many times can 6 yards of cloth be cut from a piece containing 24 yards?

7. How many 6's in 24? 4's? 8's? 2's? 3's? 12's?

8. How many times can 5 gallons be taken from a cask containing 30 gallons? How many 5's in 30?

9. Can we say 5 gallons are contained in 30 gallons 6 times? Why?

10. Into how many *equal parts* are 30 gallons separated?

11. What is the size of each part? How many 6's in 30?

12. Distribute 20 cents equally among 4 boys? How many cents will each receive?

13. Can we say, 4 *boys* are contained in 20 *cents* 5 times? Why not?

14. Can we subtract 4 boys from 20 *cents*, 5 times? Why not?

15. When we find how many times 6 cents is contained in 60 cents, what kind of a number is the result?

16. When we find one of the 6 equal parts of 60 cents, what kind of a number is the result?

DEFINITIONS.

60. ***Division*** is the process of finding *how many times* one number is contained in another of the same kind. Or, it is finding *one* of the *equal parts* of a number.

1. The **dividend** is the number to be divided.
2. The **divisor** is the number by which to divide.
3. The **quotient** is the result of the division.
4. The **remainder** is the part of the dividend remaining when the division is not *exact*, and must always be *less* than the divisor.

61. The ***Sign of division*** is ÷, or :. It is read *divided by*.

Thus, 36÷9, or 36 : 9, is read, 36 *divided by* 9.

Division is also indicated by writing the dividend over the divisor, with a line between them; or, by writing the divisor at the left of the dividend, with a curved line between them.

Thus, $\frac{24}{6}$, or 6)24, is read, 24 *divided by* 6.

Division may also be regarded as a short method of performing several *subtractions* of a number.

Thus, 15−5−5−5 = 0. Since 5 can be taken from 15 *three* times, there are *three* 5's in 15, or 5 is contained in 15, 3 times.

62. Since one number is contained in another as many times as it is a *factor* of the other, division may be regarded as the *reverse* of multiplication.

In multiplication, *both factors* are given to find the *product;* in division, *one factor* and the *product* (answering to the *dividend*) are given to find the *other factor*, which answers to the *quotient.*

Thus, 6 × 4 = 24, the factor 6 being taken 4 times; hence, there are *four* 6's in 24, or 6 is contained in 24, 4 times.

The *divisor* and *quotient* are the *factors* of the dividend.

63. The object of division is *twofold:*

First. To find *how many times* one number is contained in another of the *same kind.*

Second. To *separate* a given number into as many *equal parts* as there are units in another.

Thus, if 30 cents are divided equally among a number of boys, giving each boy 5 cents, how many boys are there?

Here the whole number and *one* of the equal parts are given, to find the *number* of equal parts.

Again, if 30 cents are divided equally among 6 boys, how many cents does each boy receive?

Here the whole number and the *number* of equal parts are given, to find the *size* or *value* of *one* of the equal parts.

64. The ***equal parts*** of a number or thing are *named* according to their *size* or *number.* Thus,

1. If a number or thing is separated into *two* equal parts, each part is called ***one half.*** *Written,* $\frac{1}{2}$.

One half ($\frac{1}{2}$) of 16 is $16 \div 2 = 8$; $\frac{1}{2}$ of 12 is $12 \div 2 = 6$.

2. If a number is separated into *three* equal parts, each part is called ***one third.*** *Written,* $\frac{1}{3}$.

One third ($\frac{1}{3}$) of 21 is $21 \div 3 = 7$; $\frac{1}{3}$ of 24 is $24 \div 3 = 8$.

3. If a number is separated into *four* equal parts, each part is called ***one fourth.*** *Written,* $\frac{1}{4}$.

One fourth ($\frac{1}{4}$) of 28 is $28 \div 4 = 7$; $\frac{1}{4}$ of 40 is $40 \div 4 = 10$.

4. In like manner, we obtain the parts named *fifths, sixths, sevenths, eighths, twentieths,* etc.

5. How is *one* of 2 equal parts, or *one half* of a number found? *One* of 3 equal parts, or *one third?* *One* of 8 equal parts, or *one eighth?* One of 12 equal parts, or *one twelfth?*

6. The equal parts into which a number or thing is divided are called *fractional parts,* or *fractions.*

PRINCIPLES.—To find *how many times* one number is contained in another:

I. *The divisor and dividend must be like numbers, and the quotient an abstract number.*

To find one of the *equal parts* of a number:

II. *The dividend and quotient must be like numbers, and the divisor an abstract number.*

III. *The dividend is the product of the divisor and the quotient.*

DRILL EXERCISES.*

65. 1. The product of two factors is 48 miles, and one of the factors is 8 miles; what is the other? If one factor is 6 miles, what is the other? Why?

2. If the dividend is 84 feet and the divisor 12 feet, what is the quotient? If the divisor is 7?

3. How many 2's in 8? In 80? In 800? In 8000?

4. How many 4's in 8? In 80? In 800? In 8000?

5. How many 6's in 12? In 120? In 1200? In 12000?

6. How many 8's in 16? In 160? In 1600? In 16000?

7. How many times 5 feet are 10 feet? 100 feet? 1000 ft.?

8. How many times $8 are $16? $160? $1600?

9. How many times 10 men are 60 men? 7 days are 42 days? 8 miles are 64 miles?

10. If an acre of land is divided into 3 equal parts, what is each part called? If into 4 equal parts? 5? 6? 7? 8? 9? 10? 12? 15? 20?

11. What is *one third* ($\frac{1}{3}$) of 24 cents? Of 36 men?

12. What is *one fourth* ($\frac{1}{4}$) of 32 rods? *One sixth* ($\frac{1}{6}$) of $60?

13. What is *one eighth* of 72 miles? *One ninth* of 54 acres? *One twelfth* ($\frac{1}{12}$) of 84 bushels?

14. How do we obtain $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, etc., of any number?

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* The following dictation and oral exercises and examples are intended simply as *models* or *forms*, each to be largely increased, at the option of the teacher.

15. If 36 marbles are equally divided among 4 boys, what part of 36 marbles does each boy receive? If among 9 boys?

16. If $40 are paid to 8 men in equal parts, what part of $40 is paid to each man? If paid to 5 men?

17. What is $\frac{1}{6}$ of 42 gallons? $\frac{1}{8}$ of 64 feet? $\frac{1}{7}$ of 56 rods?

18. What is $\frac{1}{10}$ of 100 men? $\frac{1}{9}$ of 108 acres? $\frac{1}{12}$ of $120?

19. How many is 7 times $\frac{1}{5}$ of 20? 9 times $\frac{1}{6}$ of 36 bushels?

20. How many is 8 times $\frac{1}{10}$ of 60 days? 10 times $\frac{1}{12}$ of $72?

21. What is $\frac{1}{3}$ of 4 times 6? $\frac{1}{4}$ of 6 times 8? $\frac{1}{3}$ of 2 times 12?

22. What is $\frac{1}{6}$ of 9 times $4? $\frac{1}{7}$ of 3 times 7 pints?

23. What is $\frac{1}{5}$ of 10 times 6 acres? $\frac{1}{9}$ of 8 times 9 pounds?

24. What part of 4 is 1? Is 2? Is 3?

Analysis.—*One* is $\frac{1}{4}$ of 4; 2 is 2 times $\frac{1}{4}$, or $\frac{2}{4}$ of 4; 3 is $\frac{3}{4}$ of 4.

25. How do we obtain one fourth of a number? Two fourths? Three fourths? Two thirds?

26. What part of $5 is $1? Is $2? Is $3? Is $4? Of 6 pounds is 1 pound? 2 pounds? 3 pounds? 5 pounds?

27. What part of 7 is 3? Of 8 is 5? Of 9 is 2?

28. Seven are how many times 3?

Analysis.—Three is in 7, 2 times and 1 remainder, which is $\frac{1}{3}$ of 3. Hence, 7 is 2 times 3 and $\frac{1}{3}$ of 3.

29. 13 are how many times 4? *Ans.* 3 times 4 and $\frac{1}{4}$ of 4.

30. 35 are how many times 8? *Ans.* 4 times 8 and $\frac{3}{8}$ of 8.

31. 42 are how many times 4? 5? 6? 7? 8? 9? 10?

32. 63 are how many times 5? 6? 7? 8? 9? 10? 12?

For additional exercises, use Drill Tables Nos. 1 and 2.

For example, in Drill Table No. 1, divide 10 by each number in a column, naming only the quotient and remainder. In the same way, divide 11, 12, etc., to 20.

Taking *two* columns, divide the larger number by the smaller, naming the quotient and remainder. Taking *three* columns, divide the product of the first and second numbers by the third.

Many similar exercises for drill and dictation may be arranged from these tables by the apt teacher.

66. The pupil should early learn by practice to divide any number not greater than 100 by any number not greater than 12, and give the result *at sight*. The following methods will afford a pleasant exercise, and help in so desirable an acquisition for business purposes:

1. First, to divide any number, as 35, by the numbers less than 13.

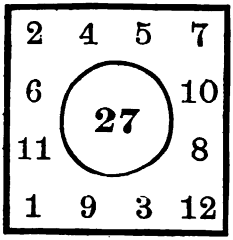
Write on the slate or board the following forms of expression:

$$\frac{35}{4}=\quad\frac{35}{6}=\quad\frac{35}{8}=\quad\frac{35}{5}=\quad\frac{35}{10}=\quad\frac{35}{7}=\quad\frac{35}{11}=\quad\frac{35}{9}=$$

The answers should be given *orally*, and not *written;* and the remainders may be read, either as so many "remainder," or in *fractional* form; thus, 35 is 8 times 4 and 3 remainder; 5 times 6 and 5 remainder, etc.; or thus, 35 is 8 times 4 and $\frac{3}{4}$ of 4; 5 times 6 and $\frac{5}{6}$ of 6, etc.

In like manner, practice upon all dividends, from 12 to 100 inclusive.

2. Another and better method, since there will be less writing, is to use the same *device*, or the Chart, used to memorize the Multiplication Table.



1. Draw on the slate or board a small square, and around the inside write the numbers from 1 to 12 inclusive, without regard to order.

2. In the centre, write for a dividend any number greater than 12 and less than 100, as 27. Commencing with any number in the margin, as 2, use the consecutive numbers as divisors, reading to the *right* around the square; thus, 2 in 27, 13 times and 1 remainder, 4 in 27, 6 times and 3 remainder, etc.; or thus, 27 is 13 times 2 and one half ($\frac{1}{2}$) of 2; 6 times 4 and three fourths ($\frac{3}{4}$) of 4, etc. In the same manner, commence and read to the *left*.

3. Then the teacher may point in rapid succession to the different figures in the margin, and the pupil promptly give the result or quotient.

4. Then erase the number in the center, and insert another for a new dividend, which will give a new set of quotients, and so on, until all the numbers from 13 to 100 have been used as dividends.

ORAL EXERCISES.

67. 1. How many yards of cloth, at $4 a yard, can be bought for $36?*

Analysis.—As many yards as $4 are contained times in $36, which are 9 times. Hence, 9 yards can be bought for $36.

2. At $6 a ton, how many tons of coal can be bought for $24? For $30? For $54? For $72?

3. If 7 barrels of flour cost $63, what will 1 barrel cost?

Analysis.—Since 7 barrels cost $63, 1 barrel will cost 1 *seventh* of $63, or $9.

4. If 6 barrels of flour cost $72, what will 1 barrel cost?

5. If a man travel 48 miles in 4 hours, how far does he travel in 1 hour?

6. What will be the cost of 1 ton of coal, if 8 tons cost $64?

7. If a farm of 120 acres is divided into 12 equal lots, how many acres does each lot contain?

8. At $9 a week, in what time will a man earn $36? $54? $72? $81? $108?

9. If 4 barrels of flour cost $36, what will 7 barrels cost?

Analysis.—One barrel will cost 1 fourth of $36, or $9. and 7 barrels will cost 7 times $9. or $63.

10. What cost 12 yards of cloth, if 6 yards cost $24?

11. If 8 yards of silk cost $32, what will 12 yards cost?

12. What will 15 sheep cost, if 5 sheep cost $35?

13. How many cords of wood, at $4 a cord, will pay for 6 barrels of flour, at $8 a barrel?

Analysis.—Six barrels of flour will cost 6 times $8, or $48; and $4, the price of 1 cord of wood, are contained in $48, 12 times.

14. How many days' labor, at $4 a day, will pay for 3 tons of coal, at $6 a ton, and 2 tons of hay, at $15 a ton?

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* The teacher should make plain the two forms of division, as shown in examples 1 and 3, and require the proper solution for each example.

15. How many pounds of meat, at 12 cents a pound, will cost as much as 9 pounds of cheese at 8 cents a pound?

16. If a stage run 45 miles in 5 hours, how far will it run in 7 hours? In 9 hours? In 12 hours?

17. What cost 9 quarts of milk, if 4 quarts cost 24 cents?

18. How many bags will be required to hold 108 bushels of wheat, if 4 bags hold 9 bushels?

19. If 5 barrels of flour are worth $60, how many cords of wood at $4 a cord will pay for 3 barrels?

20. If 12 yards of cloth cost $40, for how much must it be sold a yard to gain $20?

21. If 8 yards of cloth cost $35, for how much a yard must it be sold to gain $13?

22. A man received $50 for 5 barrels of pears, and paid all but $14 for 4 chairs. What did each chair cost?

23. What is the value of 5 tons of hay, at the rate of $48 for 4 tons?

24. If 6 bushels of wheat are worth $12, how many bushels of wheat must be given for 9 tons of hay, worth $10 a ton?

25. If 15 days' work will pay for 10 cords of wood, at $3 a cord, what is the price of 1 day's labor?

26. If a man receive 16 pounds of sugar in exchange for 20 pounds of cheese at 8 cents a pound, what is the price of the sugar per pound?

27. A newsboy sold 24 papers at 4 cents each, and thereby gained 48 cents. At what rate did he buy the papers?

28. If a woman pay 60 cents for some lemons, at the rate of 10 cents for 6, and sell them at the rate of 9 for 20 cents, how many cents will she gain?

29. If a man earn $5 while a boy earns $2, how many dollars will the man earn while the boy is earning $18?

30. A tailor bought 11 yards of one kind of cloth for $55, and 9 yards of another kind for $36. What was the difference in the price per yard?

68. Two or more operations, to be performed in the same example, are often *indicated* by the use of signs. Thus,

1. To the product of 6 and 10 add 4, and divide the sum by 8.

Written, $\frac{10\times6+4}{8}=8$, for $10\times6+4=64$, and $64\div8=8$.

Copy, read, and give the value of the following expressions:

(2.) $8\times9-8$.

(3.) $6+12\times7$.

(4.) $42+20-12$.

(5.) $42+72\div9$.

(6.) $\frac{4\times6+5\times6}{9}$.

(7.) $\frac{12\times5-3\times4}{6+2}$.

(8.) $\frac{8\times12-27\div3}{9}$.

The operations of multiplication and division, indicated by signs, must be performed before those of addition and subtraction, unless otherwise indicated.

The distinction between *terms* and *factors* must always be observed. *Terms* may be added or subtracted, but *factors* never. To illustrate, take examples 2 and 3. Each expression is composed of two *terms*, one a single number, the other the *product* of two numbers. In Ex. 2, the *term* 8 is not to be subtracted from the *factor* 9, but from the *product* of 8 and 9, or 72, which is the other *term*. In Ex. 3, 6 is not to be added to 12, but to the *product* of 12 and 7, or 84.

If the same are written thus, $8\times\overline{9-8}$, and $(6+12)\times7$, the addition and subtraction must be performed before multiplication, and the expressions will contain but *one* term each, composed of two factors, and equivalent to 8×1 or 8, and 18×7 or 126.

This law or principle becomes more obvious if we substitute the letters a, b, and c for the figures, and express algebraically thus: $a\times b-c$, and $a+b\times c$, or $ab-c$, and $a+bc$. We can no more subtract 8 from 9, or add 6 to 12 in the above examples, than we can subtract c from b, or add a to b in the algebraic expressions.

The law is the same when the sign of division is used.

In Ex. 8, the dividend is composed of two *terms*, 8×12 or 96, and $27\div3$ or 9, and is equivalent to $96-9$. If expressed thus, $(8\times12-27)\div3$, then 27 would be subtracted from the product of 8 and 12, and the remainder divided by 3.

By observing these few hints, and also, that all the numbers included by a parenthesis or a vinculum must be resolved into one number before they can be used with any other, much of the difficulty experienced by the pupil in the use of signs will be obviated.

Complete the following equations:

9. $8 \times 0 + 6 \times 4 \div 8 = ?$
10. $10 \times 12 - 5 \times 6 \div 6 = ?$
11. $9 \times 11 - 54 \div 6 + 20 = ?$
12. $\overline{10 + 4} \div 7 + 20 = ?$
13. $(8 \times 9 - 12) \div 5 = ?$
14. $\overline{40 - 8} \div 4 \times 7 = ?$
15. $(25 + 10) \div 7 + 9 = ?$
16. $72 \div 12 + 50 - 16 = ?$
17. $\overline{48 + 12 - 10} \div ? = 5.$
18. $\overline{108 \div 9 - 4} \times ? = 72.$
19. $(\overline{14 + 7} \div 3) \times ? = 84.$
20. $(144 \div 12 - 3) \times 11 = ?$
21. $24 + 20 - 4 = 80 \div ?$
22. $\overline{56 \div 7} \times 8 = 70 - ?$

69. When the divisor is not greater than 12.

1. Divide 952 by 7.

Divisor. Dividend. Quotient.
7) 952 (136
7
25
21
42
42

PROOF.
136
7
952 (Prin. III).

EXPLANATION.—Write the divisor at the left of the dividend, with a line between them.

Beginning with the left-hand or highest order of the dividend, proceed thus:

7 is contained in 9 hundreds, 1 hundred times, with a remainder. Write the 1 hundred at the right of the dividend for the first figure of the quotient.

Multiply the divisor 7 by the 1 hundred of the quotient, and write the product 7 hundreds under the hundreds of the dividend; subtract, and to the remainder, 2 hundred, annex the 5 tens of the dividend, making 25 tens.

7 is contained in 25 tens, 3 tens times, with a remainder. Writing the 3 tens in the quotient, multiply and subtract as before, and to the remainder annex the 2 units of the dividend, making 42 units.

7 is contained in 42 units, 6 times, which write in the quotient. Multiplying and subtracting, as before, nothing remains.

The solution of the preceding example may be abbreviated, as follows:

7) 952
136 Quotient.

Explanation.—7 is contained in 9, *once*, and 2 remainder. Writing 1, the quotient figure, under the part of the dividend used, prefix the remainder 2 to the next lower order of the dividend, 5, making 25. 7 is contained in 25, 3 times and 4 remainder; prefix the remainder 4 to the next lower order 2, making 42. 7 is contained in 42, 6 times, and no remainder.

Observe, that *the quotient figure is always of the same order as the lowest order of the dividend* used.

The *principles* in the two preceding operations are the same, the difference being, that in the first *all the work is written;* while in the second only the *quotient* is written, the other steps being performed *mentally.* The first is usually termed ***long division,*** the second, ***short division.*** The pupil should be taught always to use the latter, when the divisor does not exceed 12.

In like manner, by *short division* find the quotient, and prove the following:

(2.) 6) 34806 tons.

(3.) 8) 206752 days.

(4.) 7) 44058 rods.

5. Divide 83762 by 7; 79884 by 6; 3263 by 8.

8) 3263
$407\frac{7}{8}$ Quotient.

Explanation.—Since 8 is not contained in 3 thousands, unite the 3 thousands and 2 hundreds, making 32 hundreds. 8 is contained in 32 hundreds, 4 hundreds times, which write in the hundreds' place in the quotient.

Next, 8 is not contained in 6 tens, so write a cipher in tens' place in the quotient, and unite the 6 tens and 3 units. 8 is contained in 63 units, 7 times and 7 units remainder, which write over the divisor and annex as a part of the quotient. Hence the quotient is $407\frac{7}{8}$.

Proof.—$407 \times 8 + 7 = 3263$ (Prin. III).

6. Divide 403076 by 8; 120050 by 7; by 9.

7. Divide 3761201 by 9; 897063 by 6; by 7.

8. Divide 72947 by 7; by 8; by 5; by 6; by 3.

9. Divide 213064 by 4; by 9; by 8; by 5; by 6.

10. How many tons of coal, at $7 a ton, can be bought for $87605?

11. If 36314 bushels of grain be put into 6 bins of equal size, how many bushels must each bin contain?

12. A gentleman divided his estate, worth $42641, equally among his wife and 5 children. How much did each receive?

13. If 75000 bushels of grain are put into 8 bins of equal size, how many bushels does each bin contain?

14. How many times are 8 cents contained in $48.56?

8 cts.) 4856 cts.
607 *times.*

Or, $.08) $48.56
607 *times.*

Explanation.—1. *Eight cents* may be written $.08 (**35**).

2. When the divisor and dividend are like numbers, the quotient is an abstract number (Prin. I). Hence, 8 cents are contained in $48.56, 607 *times*.

3. When the divisor and dividend are both concrete numbers, they must be of the *same name*. Hence, if one is *dollars* and the other *cents*, or dollars and cents, before dividing change, so that both may be *cents*.

15. Find *one* of 8 equal parts of $48.56.

Explanation.—When the divisor is an abstract number, the dividend and quotient are like numbers (Prin. II).

8) $48.56
$6.07

Solve and prove,

(16.)	(17.)	(18.)	(19.)
8) $217.62	7) $6.44	$7) $644	$.08) $48
27.20\frac{2}{8}$	$.92	92 *times.*	600 *times.*

20. How many barrels of flour, at $8 a barrel, can be bought for $12736? For $7068?

21. If 9 acres of land cost $976.50, what is the cost of 1 acre?

22. At 4 cents apiece, how many oranges will $3.72 buy?

23. At 8 cents a yard, how many yards of ribbon can be bought for $7.28?

24. How many pounds of sugar, at 9 cents a pound, can be bought for $14.67? For $27?

Find
25. 1 *fifth* of $863.25.
26. 1 *sixth* of 34807 tons.
27. 1 *eighth* of 20673 days.
28. 1 *ninth* of $7384.50.
29. 1 *tenth* of 9760 feet?
30. 1 *fourth* of 10376 men.

What is
31. $\frac{1}{11}$ of 360723 pounds?
32. $\frac{1}{8}$ of 94000 bushels?
33. $\frac{1}{7}$ of 500322 miles?
34. $\frac{1}{4}$ of 32876 men?
35. $\frac{1}{3}$ of 60349 acres?
36. $\frac{1}{12}$ of 760344 rods?

70. When the divisor is greater than 12.

1. Divide 5437 by 26.

```
Divisor. Dividend. Quotient.
   26 ) 5437 ( 209 3/26
        52
        ---
         237
         234
         ---
           3 Remainder.
```

EXPLANATION.—26 is contained in 54 hundreds, 2 hundred times, with a remainder. Write the 2 hundreds in the quotient, and multiply the divisor 26 by this quotient figure, and subtract the product 52 hundreds from 54 hundreds, the *first* partial dividend, and there remain 2 hundreds. To this annex the 3 tens of the dividend, making 23 tens for the *second* partial dividend.

26 is not contained in 23, so write a cipher in the quotient and bring down the 7 units of the dividend, making 237 units for the *third* partial dividend.

26 is contained in 237 units, 9 times, with a remainder. Write the 9 units in the quotient, and multiplying and subtracting as before, there remain 3 units, which write over the divisor, and annex as a part of the quotient. Hence, the quotient is $209\frac{3}{26}$.

2. Find how many times 204 is contained in 1041835.

Divisor.	Dividend.	Quotient.	Proof.	
204)	1041835 (	$5107\frac{7}{204}$	5107	Quotient.
	1020		204	Divisor.
	218		20428	
	204		10214	
	1435		1041828	
	1428		7	Remainder.
	7	Remainder.	1041835	Dividend.

RULE.—1. *Write the divisor at the left of the dividend.*

2. *Find how many times the divisor is contained in the least number of the left-hand orders of the dividend that will contain it, and write the result for the first figure of the quotient.*

3. *Multiply the divisor by this quotient figure, subtract the product from the partial dividend used, and to the remainder annex the next lower order of the dividend, for a new partial dividend, and divide as before.*

4. *Proceed in the same manner until all the orders of the dividend have been used.*

5. *If any partial dividend does not contain the divisor, write a cipher in the quotient, and annex the next lower order of the dividend, and proceed as before.*

6. *If the division is not exact, write the last remainder, with the divisor underneath, as a part of the quotient.*

To test or prove the work, see Prin. III.

1. If the product of the divisor and quotient be *greater* than the *partial* dividend, the quotient is *too large*, and must be *diminished.*

2. If any remainder is *equal to* or *greater* than the divisor, the quotient is *too small* and must be *increased.*

3. Divide $46.92 by $23.

2300) 4692 ($2\frac{92}{2300}$ *times.*
4600
92

4. Divide $46 by 23 cents.

23) 4600 (200 *times.*
46
00 (Exp. 3, p. 58.)

In like manner, solve and prove the following:

5. $325.72 ÷ 34.
6. $938.07 ÷ 63.
7. $3176.46 ÷ 126.
8. $49.56 ÷ 14 cents.
9. $87.36 ÷ 21 cents.
10. $288.96 ÷ $.43.
11. $810.98 ÷ $.46.
12. $594 ÷ 18 cents.
13. $1385 ÷ $105.
14. $32.48 ÷ $7.

15. Divide 35280 by 45; by 49; by 48; by 55.
16. Divide 1276704 by 36; by 33; by 39; by 42.
17. Divide 95762 by 114; by 152; by 171; by 204.
18. A planter raised 43240 pounds of cotton on 94 acres. What was the average number of pounds to the acre?
19. How many acres of land can be bought for $8370, at $18 an acre? At $25?
20. How many barrels of pork, at $22 a barrel, can be bought for $35354?

Divide
21. 33490 by 85.
22. 740070 by 135.
23. 1554768 by 216.
24. 5497800 by 175.
25. 3931476 by 556.
26. 5120401 by 587.

Divide
27. 863256 by 736.
28. 1646301 by 381.
29. 5226412 by 2567.
30. 11214887 by 3076.
31. 75862500 by 10115.
32. 313194105 by 7153.

33. If 867 shares of railroad stock are valued at $84099, what is the value of each share?
34. A plantation of 736 acres was sold for $55936. What was the price of an acre?

35. The annual receipts of a company are $57069[illegible]. What is the average per day, if there are 313 working days?

36. A drover received $26460 for 196 head of cattle. How much was their average value per head?

37. If 43776 pounds of cotton be packed in 96 bales, what is the average weight of the bales?

Find the second member of the following equations:

38. $(6300 \div 15) - (372 \div 3) = ?$
39. $(12460 \div 10) \times (475 - 377) = ?$
40. $(867 \times 97) \div (200 \times 5 \times 3) = ?$
41. $(6070 - 1200) + (4680 \div 15) = ?$

71. When the divisor has ciphers on the right.

1. Divide 47325 by 100.

Explanation.—Removing any order of units one place to the *right*, by cutting off or taking away the right-hand figure of a number, *divides* the number by 10 (**17**-III). For like reasons, cutting off *two* figures *divides* by 100; *three* figures *divides* by 1000, etc. The remaining figures are the *quotient*, and the figures cut off, the *remainder*.

1|00) 473|25
473....25 Rem.
$473\frac{25}{100}$ Quotient.

2. Divide 38736 by 800.

Explanation.—Resolve the divisor into the factors 8 and 100. First divide by 100, by cutting off the two right-hand figures of the dividend, and the quotient is 387, and a remainder of 36. Dividing 387 by 8, the quotient is 48, and a second remainder of 3, which prefixed to the first remainder, 36, gives the true remainder, 336. Hence the quotient is $48\frac{336}{800}$.

8|00) 387|36
48....336 Rem.
$48\frac{336}{800}$ Quotient.

In like manner, divide

3. 124260 by 100.
4. 304631 by 1000.
5. 71032000 by 10000.
6. 10800 by 900.
7. 47620 by 420.
8. 378000 by 1200.

RULE.—1. *Cut off the ciphers from the right of the divisor, and as many figures from the right of the dividend, and divide the remaining part of the dividend by the remaining part of the divisor, for the quotient.*

2. *Prefix the remainder, if any, to the figures cut off, and the result will be the true remainder.*

9. How many bales, each weighing 510 pounds, can be made of 76500 pounds of cotton?

10. How many horses, at $260 each, can be bought for $19500?

11. If it require $34400 to pay a regiment of 800 men, how much does each man receive?

12. At $3400 each, how many lots can be bought for $68000?

GENERAL PRINCIPLES OF DIVISION.

72. The *relative* values of the dividend and divisor determine the value of the quotient.

There are six cases of changing the terms by multiplication and division, illustrated by comparing the equation $48 \div 8 = 6$ with the following, thus:

I. $\frac{48 \times 2}{8}$ or $\frac{48}{8 \div 2} = 12$ — *Multiplying the dividend, or dividing the divisor,* MULTIPLIES *the quotient by the same number.*

II. $\frac{48 \div 2}{8}$ or $\frac{48}{8 \times 2} = 3$ — *Dividing the dividend, or multiplying the divisor,* DIVIDES *the quotient by the same number.*

III. $\frac{48 \times 2}{8 \times 2}$ or $\frac{48 \div 2}{8 \div 2} = 6$ — *Multiplying or dividing both the dividend and divisor by the same number,* DOES NOT CHANGE *the quotient.* Hence,

A change in the DIVIDEND *by multiplication or division produces a* LIKE *change in the quotient; but such a change in the* DIVISOR *produces an* OPPOSITE *change in the quotient.*

15. How many pounds of meat, at 12 cents a pound, will cost as much as 9 pounds of cheese at 8 cents a pound?

16. If a stage run 45 miles in 5 hours, how far will it run in 7 hours? In 9 hours? In 12 hours?

17. What cost 9 quarts of milk, if 4 quarts cost 24 cents?

18. How many bags will be required to hold 108 bushels of wheat, if 4 bags hold 9 bushels?

19. If 5 barrels of flour are worth $60, how many cords of wood at $4 a cord will pay for 3 barrels?

20. If 12 yards of cloth cost $40, for how much must it be sold a yard to gain $20?

21. If 8 yards of cloth cost $35, for how much a yard must it be sold to gain $13?

22. A man received $50 for 5 barrels of pears, and paid all but $14 for 4 chairs. What did each chair cost?

23. What is the value of 5 tons of hay, at the rate of $48 for 4 tons?

24. If 6 bushels of wheat are worth $12, how many bushels of wheat must be given for 9 tons of hay, worth $10 a ton?

25. If 15 days' work will pay for 10 cords of wood, at $3 a cord, what is the price of 1 day's labor?

26. If a man receive 16 pounds of sugar in exchange for 20 pounds of cheese at 8 cents a pound, what is the price of the sugar per pound?

27. A newsboy sold 24 papers at 4 cents each, and thereby gained 48 cents. At what rate did he buy the papers?

28. If a woman pay 60 cents for some lemons, at the rate of 10 cents for 6, and sell them at the rate of 9 for 20 cents, how many cents will she gain?

29. If a man earn $5 while a boy earns $2, how many dollars will the man earn while the boy is earning $18?

30. A tailor bought 11 yards of one kind of cloth for $55, and 9 yards of another kind for $36. What was the difference in the price per yard?

68. Two or more operations, to be performed in the same example, are often *indicated* by the use of signs. Thus,

1. To the product of 6 and 10 add 4, and divide the sum by 8.

Written, $\frac{10\times6+4}{8}=8$, for $10\times6+4=64$, and $64\div8=8$.

Copy, read, and give the value of the following expressions:

(2.) $8\times9-8$. (3.) $6+12\times7$. (4.) $42+20-12$. (5.) $42+72\div9$.

(6.) $\frac{4\times6+5\times6}{9}$. (7.) $\frac{12\times5-3\times4}{6+2}$. (8.) $\frac{8\times12-27\div3}{9}$.

The operations of multiplication and division, indicated by signs, must be performed before those of addition and subtraction, unless otherwise indicated.

The distinction between *terms* and *factors* must always be observed. *Terms* may be added or subtracted, but *factors* never. To illustrate, take examples 2 and 3. Each expression is composed of two *terms*, one a single number, the other the *product* of two numbers. In Ex. 2, the *term* 8 is not to be subtracted from the *factor* 9, but from the *product* of 8 and 9, or 72, which is the other *term*. In Ex. 3, 6 is not to be added to 12, but to the *product* of 12 and 7, or 84.

If the same are written thus, $8\times\overline{9-8}$, and $(6+12)\times7$, the addition and subtraction must be performed before multiplication, and the expressions will contain but *one* term each, composed of two factors, and equivalent to 8×1 or 8, and 18×7 or 126.

This law or principle becomes more obvious if we substitute the letters a, b, and c for the figures, and express algebraically thus: $a\times b-c$, and $a+b\times c$, or $ab-c$, and $a+bc$. We can no more subtract 8 from 9, or add 6 to 12 in the above examples, than we can subtract c from b, or add a to b in the algebraic expressions.

The law is the same when the sign of division is used.

In Ex. 8, the dividend is composed of two *terms*, 8×12 or 96, and $27\div3$ or 9, and is equivalent to $96-9$. If expressed thus, $(8\times12-27)\div3$, then 27 would be subtracted from the product of 8 and 12, and the remainder divided by 3.

By observing these few hints, and also, that all the numbers included by a parenthesis or a vinculum must be resolved into one number before they can be used with any other, much of the difficulty experienced by the pupil in the use of signs will be obviated.

Complete the following equations:

9. $8 \times 0 + 6 \times 4 \div 8 = ?$
10. $10 \times 12 - 5 \times 6 \div 6 = ?$
11. $9 \times 11 - 54 \div 6 + 20 = ?$
12. $\overline{10 + 4} \div 7 + 20 = ?$
13. $(8 \times 9 - 12) \div 5 = ?$
14. $\overline{40 - 8} \div 4 \times 7 = ?$
15. $(25 + 10) \div 7 + 9 = ?$
16. $72 \div 12 + 50 - 16 = ?$
17. $\overline{48 + 12 - 10} \div ? = 5.$
18. $\overline{108 \div 9 - 4} \times ? = 72.$
19. $(\overline{14 + 7} \div 3) \times ? = 84.$
20. $(144 \div 12 - 3) \times 11 = ?$
21. $24 + 20 - 4 = 80 \div ?$
22. $\overline{56 \div 7} \times 8 = 70 - ?$

69. When the divisor is not greater than 12.

1. Divide 952 by 7.

Divisor. Dividend. Quotient.

$$\begin{array}{rl} 7\ ) & 952\ (\ 136 \\ & \underline{7} \\ & 25 \\ & \underline{21} \\ & 42 \\ & \underline{42} \end{array}$$

Proof.

$$\begin{array}{r} 136 \\ \underline{7} \\ 952 \end{array}\ \text{(Prin. III).}$$

Explanation.—Write the divisor at the left of the dividend, with a line between them.

Beginning with the left-hand or highest order of the dividend, proceed thus:

7 is contained in 9 hundreds, 1 hundred times, with a remainder. Write the 1 hundred at the right of the dividend for the first figure of the quotient.

Multiply the divisor 7 by the 1 hundred of the quotient, and write the product 7 hundreds under the hundreds of the dividend; subtract, and to the remainder, 2 hundred, annex the 5 tens of the dividend, making 25 tens.

7 is contained in 25 tens, 3 tens times, with a remainder. Writing the 3 tens in the quotient, multiply and subtract as before, and to the remainder annex the 2 units of the dividend, making 42 units.

7 is contained in 42 units, 6 times, which write in the quotient. Multiplying and subtracting, as before, nothing remains.

The solution of the preceding example may be abbreviated, as follows:

$$7\,\overline{)\,952}$$
136 Quotient.

Explanation.—7 is contained in 9, *once*, and 2 remainder. Writing 1, the quotient figure, under the part of the dividend used, prefix the remainder 2 to the next lower order of the dividend, 5, making 25. 7 is contained in 25, 3 times and 4 remainder; prefix the remainder 4 to the next lower order 2, making 42. 7 is contained in 42, 6 times, and no remainder.

Observe, that *the quotient figure is always of the same order as the lowest order of the dividend* used.

The *principles* in the two preceding operations are the same, the difference being, that in the first *all the work is written;* while in the second only the *quotient* is written, the other steps being performed *mentally.* The first is usually termed ***long division,*** the second, ***short division.*** The pupil should be taught always to use the latter, when the divisor does not exceed 12.

In like manner, by *short division* find the quotient, and prove the following:

(2.) 6) 34806 tons.

(3.) 8) 206752 days.

(4.) 7) 44058 rods.

5. Divide 83762 by 7; 79884 by 6; 3263 by 8.

$$8\,\overline{)\,3263}$$
$407\frac{7}{8}$ Quotient.

Explanation.—Since 8 is not contained in 3 thousands, unite the 3 thousands and 2 hundreds, making 32 hundreds. 8 is contained in 32 hundreds, 4 hundreds times, which write in the hundreds' place in the quotient.

Next, 8 is not contained in 6 tens, so write a cipher in tens' place in the quotient, and unite the 6 tens and 3 units. 8 is contained in 63 units, 7 times and 7 units remainder, which write over the divisor and annex as a part of the quotient. Hence the quotient is $407\frac{7}{8}$.

Proof.—$407 \times 8 + 7 = 3263$ (Prin. III).

6. Divide 403076 by 8; 120050 by 7; by 9.
7. Divide 3761201 by 9; 897063 by 6; by 7.
8. Divide 72947 by 7; by 8; by 5; by 6; by 3.
9. Divide 213064 by 4; by 9; by 8; by 5; by 6.
10. How many tons of coal, at $7 a ton, can be bought for $87605?
11. If 36314 bushels of grain be put into 6 bins of equal size, how many bushels must each bin contain?
12. A gentleman divided his estate, worth $42641, equally among his wife and 5 children. How much did each receive?
13. If 75000 bushels of grain are put into 8 bins of equal size, how many bushels does each bin contain?

14. How many times are 8 cents contained in $48.56?

8 cts.) 4856 cts.	Or,	$.08) $48.56
607 *times.*		607 *times.*

Explanation.—1. *Eight cents* may be written $.08 (**35**).

2. When the divisor and dividend are like numbers, the quotient is an abstract number (Prin. I). Hence, 8 cents are contained in $48.56, 607 *times*.

3. When the divisor and dividend are both concrete numbers, they must be of the *same name*. Hence, if one is *dollars* and the other *cents*, or dollars and cents, before dividing change, so that both may be *cents*.

15. Find *one* of 8 equal parts of $48.56.

Explanation.—When the divisor is an abstract number, the dividend and quotient are like numbers (Prin. II).

8) $48.56
$6.07

Solve and prove,

(16.)	(17.)	(18.)	(19.)
8) $217.62	7) $6.44	$7) $644	$.08) $48
27.20\frac{2}{8}$	$.92	92 *times.*	600 *times.*

20. How many barrels of flour, at \$8 a barrel, can be bought for \$12736? For \$7068?

21. If 9 acres of land cost \$976.50, what is the cost of 1 acre?

22. At 4 cents apiece, how many oranges will \$3.72 buy?

23. At 8 cents a yard, how many yards of ribbon can be bought for \$7.28?

24. How many pounds of sugar, at 9 cents a pound, can be bought for \$14.67? For \$27?

Find

25. 1 *fifth* of \$863.25.
26. 1 *sixth* of 34807 tons.
27. 1 *eighth* of 20673 days.
28. 1 *ninth* of \$7384.50.
29. 1 *tenth* of 9760 feet?
30. 1 *fourth* of 10376 men.

What is

31. $\frac{1}{11}$ of 360723 pounds?
32. $\frac{1}{8}$ of 94000 bushels?
33. $\frac{1}{7}$ of 500322 miles?
34. $\frac{1}{4}$ of 32876 men?
35. $\frac{1}{5}$ of 60349 acres?
36. $\frac{1}{12}$ of 760344 rods?

70. When the divisor is greater than 12.

1. Divide 5437 by 26.

Divisor. Dividend. Quotient.

$$\begin{array}{rrl} 26\;) & 5437 & (\;209\frac{3}{26} \\ & \underline{52} & \\ & 237 & \\ & \underline{234} & \\ & 3 & \textbf{Remainder.} \end{array}$$

Explanation.—26 is contained in 54 hundreds, 2 hundred times, with a remainder. Write the 2 hundreds in the quotient, and multiply the divisor 26 by this quotient figure, and subtract the product 52 hundreds from 54 hundreds, the *first* partial dividend, and there remain 2 hundreds. To this annex the 3 tens of the dividend, making 23 tens for the *second* partial dividend.

26 is not contained in 23, so write a cipher in the quotient and bring down the 7 units of the dividend, making 237 units for the *third* partial dividend.

26 is contained in 237 units, 9 times, with a remainder. Write the 9 units in the quotient, and multiplying and subtracting as before, there remain 3 units, which write over the divisor, and annex as a part of the quotient. Hence, the quotient is $209\frac{3}{26}$.

2. Find how many times 204 is contained in 1041835.

Divisor.	Dividend.	Quotient.	Proof.	
204)	1041835 (	$5107\frac{7}{204}$	5107	Quotient.
	1020		204	Divisor.
	218		20428	
	204		10214	
	1435		1041828	
	1428		7	Remainder.
	7	Remainder.	1041835	Dividend.

RULE.—1. *Write the divisor at the left of the dividend.*

2. *Find how many times the divisor is contained in the least number of the left-hand orders of the dividend that will contain it, and write the result for the first figure of the quotient.*

3. *Multiply the divisor by this quotient figure, subtract the product from the partial dividend used, and to the remainder annex the next lower order of the dividend, for a new partial dividend, and divide as before.*

4. *Proceed in the same manner until all the orders of the dividend have been used.*

5. *If any partial dividend does not contain the divisor, write a cipher in the quotient, and annex the next lower order of the dividend, and proceed as before.*

6. *If the division is not exact, write the last remainder, with the divisor underneath, as a part of the quotient.*

To test or prove the work, see Prin. III.

1. If the product of the divisor and quotient be *greater* than the *partial* dividend, the quotient is *too large*, and must be *diminished.*

2. If any remainder is *equal to* or *greater* than the divisor, the quotient is *too small* and must be *increased.*

3. Divide $46.92 by $23.

2300) 4692 ($2\frac{92}{2300}$ *times.*
4600
92

4. Divide $46 by 23 cents.

23) 4600 (200 *times.*
46
00 (Exp. 3, p. 58.)

In like manner, solve and prove the following:

5. $325.72 ÷ 34.
6. $938.07 ÷ 63.
7. $3176.46 ÷ 126.
8. $49.56 ÷ 14 cents.
9. $87.36 ÷ 21 cents.
10. $288.96 ÷ $.43.
11. $810.98 ÷ $.46.
12. $594 ÷ 18 cents.
13. $1385 ÷ $105.
14. $32.48 ÷ $7.

15. Divide 35280 by 45; by 49; by 48; by 55.
16. Divide 1276704 by 36; by 33; by 39; by 42.
17. Divide 95762 by 114; by 152; by 171; by 204.
18. A planter raised 43240 pounds of cotton on 94 acres. What was the average number of pounds to the acre?
19. How many acres of land can be bought for $8370, at $18 an acre? At $25?
20. How many barrels of pork, at $22 a barrel, can be bought for $35354?

Divide
21. 33490 by 85.
22. 740070 by 135.
23. 1554768 by 216.
24. 5497800 by 175.
25. 3931476 by 556.
26. 5120401 by 587.

Divide
27. 863256 by 736.
28. 1646301 by 381.
29. 5226412 by 2567.
30. 11214887 by 3076.
31. 75862500 by 10115.
32. 313194105 by 7153.

33. If 867 shares of railroad stock are valued at $84099, what is the value of each share?
34. A plantation of 736 acres was sold for $55936. What was the price of an acre?

35. The annual receipts of a company are \$5706[illegible]. What is the average per day, if there are 313 working days?

36. A drover received \$26460 for 196 head of cattle. How much was their average value per head?

37. If 43776 pounds of cotton be packed in 96 bales, what is the average weight of the bales?

Find the second member of the following equations:

38. $(6300 \div 15) - (372 \div 3) = ?$
39. $(12460 \div 10) \times (475 - 377) = ?$
40. $(867 \times 97) \div (200 \times 5 \times 3) = ?$
41. $(6070 - 1200) + (4680 \div 15) = ?$

71. When the divisor has ciphers on the right.

1. Divide 47325 by 100.

$$1|00\,)\,\underline{473|25\qquad}$$
$$473\ldots\ldots25 \text{ Rem.}$$
$$473\tfrac{25}{100} \text{ Quotient.}$$

Explanation.—Removing any order of units one place to the *right*, by cutting off or taking away the right-hand figure of a number, *divides* the number by 10 (**17**-III). For like reasons, cutting off *two* figures *divides* by 100; *three* figures *divides* by 1000, etc. The remaining figures are the *quotient*, and the figures cut off, the *remainder*.

2. Divide 38736 by 800.

$$8|00\,)\,\underline{387|36\qquad}$$
$$48\ldots\ldots336 \text{ Rem.}$$
$$48\tfrac{336}{800} \text{ Quotient.}$$

Explanation.—Resolve the divisor into the factors 8 and 100. First divide by 100, by cutting off the two right-hand figures of the dividend, and the quotient is 387, and a remainder of 36. Dividing 387 by 8, the quotient is 48, and a second remainder of 3, which prefixed to the first remainder, 36, gives the true remainder, 336. Hence the quotient is $48\frac{336}{800}$.

In like manner, divide

3. 124260 by 100.
4. 304631 by 1000.
5. 71032000 by 10000.
6. 10800 by 900.
7. 47620 by 420.
8. 378000 by 1200.

RULE.—1. *Cut off the ciphers from the right of the divisor, and as many figures from the right of the dividend, and divide the remaining part of the dividend by the remaining part of the divisor, for the quotient.*

2. *Prefix the remainder, if any, to the figures cut off, and the result will be the true remainder.*

9. How many bales, each weighing 510 pounds, can be made of 76500 pounds of cotton?

10. How many horses, at $260 each, can be bought for $19500?

11. If it require $34400 to pay a regiment of 800 men, how much does each man receive?

12. At $3400 each, how many lots can be bought for $68000?

GENERAL PRINCIPLES OF DIVISION.

72. The *relative* values of the dividend and divisor determine the value of the quotient.

There are six cases of changing the terms by multiplication and division, illustrated by comparing the equation $48 \div 8 = 6$ with the following, thus:

I. $\frac{48 \times 2}{8}$ or $\frac{48}{8 \div 2} = 12$ *Multiplying the dividend, or dividing the divisor,* MULTIPLIES *the quotient by the same number.*

II. $\frac{48 \div 2}{8}$ or $\frac{48}{8 \times 2} = 3$ *Dividing the dividend, or multiplying the divisor,* DIVIDES *the quotient by the same number.*

III. $\frac{48 \times 2}{8 \times 2}$ or $\frac{48 \div 2}{8 \div 2} = 6$ *Multiplying or dividing both the dividend and divisor by the same number,* DOES NOT CHANGE *the quotient.* Hence,

A change in the DIVIDEND *by multiplication or division produces a* LIKE *change in the quotient; but such a change in the* DIVISOR *produces an* OPPOSITE *change in the quotient.*

CANCELLATION.

73. ***Cancellation*** is the process of rejecting or cancelling equal factors from both dividend and divisor.

PRINCIPLES.—I. *Rejecting a factor from any number divides the number by that factor.*

II. *Rejecting equal factors from both dividend and divisor does not change the quotient.*

WRITTEN EXERCISES.

74. 1. Divide 56 × 24 by 48 × 7.

$$\frac{\overset{\cancel{7}}{\cancel{56}} \times \overset{4}{\cancel{24}}}{\underset{\cancel{6}}{\cancel{48}} \times \cancel{7}} = 4$$

EXPLANATION.—Since 8 will divide both 56 and 48, reject 8 as a factor of 56, retaining the factor 7, and also of 48, retaining the factor 6.

Again, since 6 will divide both 24 in the dividend and 6 in the divisor, reject 6 as a factor from both, retaining the factor 4 in the dividend. Finally, rejecting the factor 7, common both to the dividend and to the divisor, there remains only the factor 4 in the dividend, which is the required quotient.

2. Divide the product of 25, 18, 4, and 3, by the product of 7, 6, 5, and 3.

$$\frac{\overset{5}{\cancel{25}} \times \overset{3}{\cancel{18}} \times 4 \times \cancel{3}}{7 \times \cancel{6} \times \cancel{5} \times \cancel{3}} = \frac{5 \times 3 \times 4}{7} = \frac{60}{7} = 8\tfrac{4}{7}. \qquad \text{Or,}$$

$$\begin{array}{r|l} 7 & \cancel{25}^{\,5} \\ \cancel{6} & \cancel{18}^{\,3} \\ \cancel{5} & 4 \\ \cancel{3} & \cancel{3} \\ \hline 7 & 60 \\ \cline{2-2} & 8\tfrac{4}{7} \end{array}$$

1. The *vertical* line is much used, and by many is thought to be more convenient, wherever *cancellation* is applied. It serves the same purpose as the *curved* line in division, since it separates the *dividend* or its factors on the *right* from the *divisor* or its factors on the *left*.

2. The *horizontal* line may be used by those who prefer it.

3. Divide 13 × 7 × 5 × 3 by 3 × 5 × 7.

4. Divide 42 × 18 × 6 × 4 by 36 × 21 × 6.

RULE.—*Cancel the factors common to both dividend and divisor, and divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor; the result will be the required quotient.*

5. What is the quotient of 21 × 8 × 60 × 8 × 6 divided by 7 × 12 × 3 × 8 × 3?

6. Multiply 8 times 32 by 6 times 27, and divide the product by 9 times 96.

7. How many tons of hay, at $18, must be given for 45 cords of wood, at $4 a cord?

8. How many flour barrels, at $.80 each, will pay for 112 bushels of corn, at $.70 a bushel?

9. How many tubs of butter, each containing 56 pounds, at 30 cents a pound, must be given for 7 barrels of sugar, each containing 195 pounds, at 10 cents a pound?

10. How many bushels of apples, at 75 cents a bushel, will pay for 95 pounds of coffee, at 30 cents a pound?

11. A farmer exchanged 45 bushels of potatoes, worth 40 cents a bushel, for 18 pounds of tea. What was the tea worth a pound?

12. From 6 pieces of cloth, each piece containing 24 yards, worth $3 a yard, how many suits can be made, worth $18 a suit?

13. When hay is worth $18 a ton, how many tons must be given for 12 barrels of flour, at $7 a barrel?

14. If 6 acres of land cost $180, what will 150 acres cost?

15. How many bales of cloth, each containing 40 pieces, and each piece containing 36 yards, worth $3 a yard, must be given for 120 government bonds, worth $108 each?

16. $(240 \times 56 \times 18) \div (60 \times 28 \times 9) = ?$

17. $(72 \times 48 \times 28 \times 5) \div (84 \times 15 \times 7 \times 6) = ?$

18. $(66 \times 18 \times 27 \times 25) \div (84 \times 45 \times 7 \times 30) = ?$

19. $(80 \times 60 \times 50 \times 16 \times 14) \div (70 \times 50 \times 24 \times 20) = ?$

20. How many bushels of oats, at $.60 a bushel, will pay for 12 tons of coal, at $7.20 a ton?

21. How many days' work, at $1.25 a day, will pay for 75 bushels of corn, at $.80 a bushel?

22. A laborer gave 12 days' work for 48 bushels of potatoes, worth 50 cents a bushel. What were his daily earnings?

REVIEW.

WRITTEN EXERCISES.

75. 1. How many times is 246 contained in 3690?

2. How many times can 246 be subtracted from 3690?

3. How many times 246 will produce 3690?

4. The factors of a number are 84 + 23 and 427 — 259. What is the number?

5. The product of two numbers is 14304, and one of the numbers is 2400÷25. What is the other?

6. If 2, 2, and 7 are three factors of 364, what is the other factor?

7. What is the difference between 106 × 127, and 15341÷29?

8. The product of three numbers is 107100; one of the numbers is 42, and another 34. What is the third number?

9. The quotient of one number divided by another is 56, the divisor is 375, and the remainder 124. What is the dividend?

10. A coal dealer bought 250 tons of coal, receiving 2240 pounds for a ton; he sold it at 2000 pounds for a ton. How many tons did he gain?

11. A farmer sold 300 bushels of wheat, at $2 a bushel, and corn and oats to the amount of $750; with the proceeds he bought 120 head of sheep at $3 a head, one pair of oxen for $90, and 25 acres of land for the remainder. How much did the land cost him an acre?

12. How many pounds of sugar worth 13 cents a pound, can be bought for 126 pounds of butter worth 28 cts. a pound?

13. If a man earns $60 a month, and spends $45 in the same time, in what time will he save $900?

14. A grocer bought two kinds of syrup; one for 54 cents a gallon, and the other for 62 cents. What was the *average* cost a gallon?

Solution.—(54 cents + 62 cents) ÷ 2 = 58 cents.

The *average* of *two* numbers is *one half* their sum, the average of *three* numbers is *one third* their sum, etc.

15. A merchant bought three kinds of tea, some at 60 cents, some at 78 cents, and some at 90 cents a pound. What was the average cost a pound?

16. A keeper of a toll bridge received $104 toll on Monday, $97 on Tuesday, $128 on Wednesday, and $99 on Thursday. What was the average daily receipts?

17. Sold three city lots for $1500, $2976, and $1895, respectively. What was the average price of each?

18. If a young man receive a salary of $25 a week, and he pays $8.75 for his board, and $4.65 for other expenses, in how many weeks can he pay a debt of $487.20?

19. A man having $4578, paid out all but $1642 in 8 weeks. What was the average amount paid out each week?

20. Bought 140 acres of land for $7560, and sold 86 acres of it at $75 an acre, and the remainder at cost. How much was gained?

21. The sales of a merchant are $75000 a year. What is the average per week? Per month?

22. How many pounds of sugar, at 20 cents a pound, must be given for 5 pieces of cotton cloth, containing 44 yards each, at 42 cents a yard?

23. Sold a quantity of wood for $2492, that cost $1424, thus gaining $3 a cord. How many cords were there, and what was the cost per cord?

24. Bought 150 acres of land for $7650, and sold it for $63 an acre. How much was the whole gain?

25. If 100 barrels of flour cost $600, what will 350 barrels cost at the same rate?

26. How many pounds of butter, at 40 cents a pound, are worth as much as 1600 bushels of oats, at 75 cents a bushel?

27. If a man gain $638.75 by selling 365 barrels of flour at $9.25 a barrel, at what price did he buy it?

28. A grocer bought 12 barrels of sugar at $16 per barrel, and 17 barrels at $13 per barrel; how much would he gain by selling the whole at $18 per barrel?

29. A farmer sold 16 cords of wood at $3 a cord, and 40 bushels of wheat at $2 a bushel. He received 15 yards of cloth at $4 a yard, and the remainder in money; how much money did he receive?

30. If a mechanic receive $1500 a year for his labor, and his expenses are $968, in what time can he save enough to buy 28 acres of land at $133 an acre?

31. A planter sold 40 bales of cotton at $68 a bale. With the proceeds he bought 8 horses at $180 each, and with the remainder bought cows at $32 a head. How many cows did he purchase?

32. Bought 288 barrels of flour for $1482, and sold the same for $2058. What was the gain on each barrel?

Find the second member in the following equations:

33. $6300 \div 15 - 372 \div 3 = ?$
34. $(12460 \div 10) \times (475 - 377) = ?$
35. $\overline{450 + 12} \times 5 \div \frac{22}{6} + 125 = ?$
36. $(6070 - 1200) + (4680 \div 15) = ?$
37. $384 + 4621 - 872 + \overline{3176 \div 8} \times 12 = ?$
38. $25 \times 370 - 62 \times 8 + (335 + 206) \times 45 = ?$
39. $16 \times 3125 - 127 \times 0 + (972 - 122) \div 50 = ?$
40. $\overline{(125 \times 30) \div (25 \times 25)} \times (324 - 216) + \overline{264 - 450 \div 5} = ?$

76. The pupil may be required to illustrate the following problems by original examples:

I. ADDITION.

Given	*to find*
1. The parts,	the whole, or the sum total.
2. The less of two numbers and their difference, or the subtrahend and remainder,	the greater number, or the minuend.

II. SUBTRACTION.

Given	*to find*
1. The sum of two numbers and one of them,	the other.
2. The greater and the less of two numbers, or the minuend and subtrahend,	the difference or remainder.

III. MULTIPLICATION.

Given	*to find*
1. Two numbers,	their product.
2. Any number of factors,	their continued product.
3. The divisor and quotient,	the dividend.

IV. DIVISION.

Given	*to find*
1. The dividend and divisor,	the quotient.
2. The dividend and quotient,	the divisor.
3. The product and one of two factors,	the other factor.
4. The continued product of several factors, and the product of all but one factor,	that one factor.

FACTORS AND DIVISORS.

INDUCTIVE EXERCISES.

77. 1. What two numbers, besides the number itself and 1, will give a product of 8? 16? 25? 42? 64?

2. What numbers, other than the given number and 1, will exactly divide 9? 15? 36? 48? 55?

3. Of what sets of two numbers is 24 the product?

4. Of what sets of three numbers is 36 the product?

5. What are the smallest numbers, other than 1, that will exactly divide 18? 21? 49? 55?

6. What is the largest number, other than the given number itself, that will exactly divide 22? 24? 30? 40?

7. Name the numbers between 12 and 30, that are the product of *two* factors greater than 1. Between 30 and 50.

8. Name the numbers between 5 and 20, that have no other factors than the numbers themselves and 1.

DEFINITIONS.

78. The ***integral factors*** of a number are the integers whose continued product equals that number.

Thus, 4 and 5 are factors of 20; 2, 4, and 8, of 64.

79. A ***composite number*** is a number that has other integral factors besides itself and 1.

Thus, 24 is a *composite* number, since it is the product of 4 and 6, or 3 and 8, or 2, 3, and 4.

80. A ***prime number*** is a number that has no integral factors besides itself and 1.

Thus, 2, 3, 5, 7, 11, etc., are *prime* numbers.

81. A ***prime factor*** is a prime number used as a factor.

Numbers are said to be *prime to each other* when they have no common integral factors or divisors.

Thus, 9 and 4, 12 and 25 are prime to each other.

82. An ***exact divisor*** of a number is any integral factor of that number.

Thus, 3 and 4 are exact factors or divisors of 12.

1. An exact divisor of a number is also called a *measure* of that number.

2. When any integral factor is common to *two* or *more* numbers, it is called a *common divisor*, *factor*, or *measure* of those numbers.

83. An ***even number*** is one that is exactly divisible by 2.

All numbers terminating with 0, 2, 4, 6, or 8 are *even*.

84. An ***odd number*** is one that is not exactly divisible by 2.

All numbers terminating with 1, 3, 5, 7, or 9 are *odd*.

Any number is exactly divisible

1. By 2, when it is an *even* number.
2. By 3, when the sum of its digits is divisible by 3.
3. By 4, when its units and tens together are divisible by 4.
4. By 5, when it ends with a 0 or 5.
5. By 6, when it is an *even* number and divisible by 3.
6. By 8, when its three right-hand figures are ciphers, or express a number divisible by 8.
7. By 9, when the sum of its digits is divisible by 9.
8. By 10, when it ends with one or more ciphers.

Find by inspection some of the exact divisors of the following numbers:

1. 1536.
2. 1683.
3. 3348.
4. 6105.
5. 12936.
6. 43560.
7. 32472.
8. 71460.
9. 197200.

FACTORING.

INDUCTIVE EXERCISES.

85. 1. What are the *prime* numbers from 8 to 24?

2. What are the composite numbers from 8 to 24?

3. Name all the prime factors of 36.

4. Name all the composite factors of 36.

5. What prime factors are common to 21 and 42?

6. What composite factors are common to 36 and 72?

7. What factors are common to 16 and 24? To their *sum?* To their *difference?*

8. Form a composite number by using 2 twice as a factor; 3 twice as a factor; 2 three times as a factor.

9. What is *one* of *two* equal factors of 9? of 16? of 25?

10. *One* of *three* equal factors of 8? of 27? of 64?

When a composite number is composed of equal factors, it is called a ***power*** of that factor; and one of the equal factors is called a ***root*** of the number.

The power of a number is named according to the *number* of equal factors used. Thus,

3 × 3 is the *second power* or ***square*** of 3.

3 × 3 × 3 is the *third power* or ***cube*** of 3.

3 × 3 × 3 × 3 is the *fourth power* of 3.

So, also, 3 is called the *second root* or ***square root*** of 9; the *third root* or ***cube root*** of 27.

11. Required the *second* power of 3; of 4; of 5; of 6, of 8; of 9; the *third* power of 2; of 3; of 4; of 5; of 6.

12. What is *one* of *two equal* factors of 9? of 25? of 36?

13. *One* of *three equal* factors of 8? 27? 64? 125?

14. What is the second root of 9? of 16? of 25? of 64?

15. What is the third root of 8? of 27? of 64? of 125?

Instead of repeating the factor, it is generally written but once, and a small figure, called the ***exponent***, is placed at the right, and a little above the numbers, to show how many times it is used as a factor. Thus,

$3^4 = 3 \times 3 \times 3 \times 3$, and denotes that 3 is used as a factor 4 times.

$6^3 = 6 \times 6 \times 6$, and denotes that 6 is used as a factor 3 times.

PRINCIPLES.—I. *Every composite number is the product of all its prime factors.*

II. *A factor common to two or more numbers is a factor of their* SUM*; also, of the* DIFFERENCE *of any two of them.*

86. ***Factoring*** is the process of separating a composite number into its factors.

WRITTEN EXERCISES.

87. To resolve a number into its prime factors.

1. What are the prime factors of 252 ?

ANALYSIS.—Since the given number is even, divide by the least prime factor 2, and the quotient also by 2.

Next divide by the prime numbers 3 and 3 successively, obtaining 7 for the last quotient, which being prime, the division can be carried no further. Hence the divisors 2, 2, 3, 3, and the last quotient 7 are all the prime factors or divisors of 252, and may be written 7, 3^2, 2^2.

$$\begin{array}{r|r} 2 & 252 \\ \hline 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

PROOF.—$7 \times 3 \times 3 \times 2 \times 2 = 252$.

In like manner, find the prime factors or divisors

2. Of 225.
3. Of 154.
4. Of 200.
5. Of 625.
6. Of 945.
7. Of 1155.
8. Of 2934.
9. Of 6300.

The principles and the operation make the *rule* obvious.

COMMON DIVISORS.

INDUCTIVE EXERCISES.

88. 1. What numbers are exact divisors of 21? 36?

2. What are the prime divisors of 25? 35? 49?

3. What are the composite divisors of 72? 80? 108?

4. What prime divisor is common to 28, 42, and 56?

5. What is a common measure of 22, 33, and 55?

6. What factors or divisors are common to 12, 24, and 36?

7. What two numbers will exactly divide or measure 24 miles? 35 acres? 48 pounds?

8. What two numbers will exactly measure 10 and 20? Their *sum?* Their *difference?* Their *product?*

9. What is the greatest exact divisor of the sum and difference of 16 and 24? Of 18 and 45?

10. What is the G.C.D. of 28, 35, and 42?

11. Name three numbers of which 9 is the G.C.D.

12. What is the G.C.D. of 22, 44, and 66?

13. What divisor is common to 6 and 9? To 6 and 9, and to 54, their product? To 8 and 12, and to 96, their product?

14. What divisor is common to 18 and 27, and to 45, their sum? To 18 and 27, and to 9, their difference?

PRINCIPLES.—I. *Every prime factor of a number is an exact divisor of that number.*

II. *The G.C.D. of two or more numbers is the product of all their common prime factors.*

89. A ***common divisor*** of two or more numbers is a *common factor* of each of them.

90. The ***greatest common divisor*** of two or more numbers is the *greatest common factor,* and is the product of *all the* common *prime* factors.

WRITTEN EXERCISES.

91. To find the *G.C.D.* of two or more numbers.

1. What is the G.C.D. of 18, 30, and 42?

1ST METHOD.

2	18	30	42
3	9	15	21
	3	5	7

EXPLANATION.—By factoring, we find 2 and 3 to be the prime factors common to all the given numbers. The product of these prime divisors, 2 × 3 = 6, is the G.C.D. of 18, 30, and 42 (PRIN. II).

In like manner, find the G.C.D.

2. Of 42 and 112.
3. Of 96 and 544.
4. Of 56, 72, and 92.
5. Of 21, 42, and 77.
6. Of 36, 48, and 216.
7. Of 105, 216, and 405.

8. Find the G.C.D. of 247 and 323.

		323
247	1	247
228	3	76
19	4	76
		0

EXPLANATION.—Draw two vertical lines, and place the greater number on the right, and the less on the left, one line lower down. Divide 323 by 247, and write the quotient 1 between the vertical lines, the product 247 under the greater number, and the remainder 76 below.

Next, divide 247 by this remainder 76, writing the quotient 3 between the vertical lines, the product 228 on the left, and the remainder 19 below. Again, dividing the last divisor, 76, by 19, there is no remainder. Hence, 19, the last divisor, is the G.C.D. of 247 and 323.

In like manner, find the G.C.D. of

9. 154 and 210.
10. 316 and 664.
11. 126 and 189.
12. 300 and 375.
13. 225 and 270.
14. 468 and 1184

RULES FOR FINDING THE G. C. D.

1st. By prime factors:

Resolve the given numbers into their prime factors; the product of all the common prime factors is the G.C.D.

2d. By continued division:

1. *Divide the greater number by the less, and the less number by the remainder, if any, and so continue to divide the last divisor by the last remainder, till nothing remains. The last divisor will be the G. C. D.*

2. *If more than two numbers are given, find the G. C. D. of two of them, then of this divisor and a third number, and so on*

If the last divisor is 1, the numbers are *prime to each other*, and have no common divisor greater than 1.

Find the greatest common divisor

15. Of 432 and 648.
16. Of 135 and 315.
17. Of 182 and 364.
18. Of 756 and 1575.
19. Of 1008 and 1036.
20. Of 216, 360, and 432.
21. Of 84, 126, and 462.
22. Of 141, 799, and 940.
23. Of 3281 and 10778.
24. Of 10353 and 14877.

25. What is the greatest number that will divide 620, 1116, and 1488? 396, 5184, and 6914?

26. A speculator has 3 fields, the first containing 18, the second 24, and the third 40 acres, which he wishes to divide into the largest possible lots having the same number of acres in each. How many acres in each lot?

27. The Erie R. R. has 3 switches, or side tracks, of the following lengths: 3013, 2231, and 2047 feet. What is the length of the longest rail that will exactly lay the track on each switch?

28. A farmer wishes to put 336 bushels of corn and 812 bushels of wheat into the least number of bins possible, that shall contain the same number of bushels, without mixing the two kinds of grain. How many bushels must each bin hold?

29. Three persons have respectively $630, $1134, and $1386, with which they agree to purchase horses, at the highest price per head that will allow each man to invest all his money. How many horses can each man buy?

MULTIPLES.

INDUCTIVE EXERCISES.

92. 1. What numbers between 5 and 30 are exactly divisible by 4? By 5? By 6? By 8?

2. Name some numbers of which 5 is a factor. Of which 6 is a factor.

3. What prime factors are common to 6 and 5 times 6?

4. Name some numbers that are exactly divisible by 3 and 5. 4 and 6. 3 and 7.

5. A number which contains another number one or more times is called a *multiple* of that number.

6. By what three prime numbers can 42 be divided?

7. What numbers less than 40 are exactly divisible by 7?

8. What numbers from 4 to 36 have both 2 and 3 as factors? Have 2 and 5 as factors?

9. What is the least number of which 3, 4, and 5 are factors?

10. Name two numbers of which 6 and 9 are factors. Name the least number of which they are common factors.

11. Name three multiples of 3? 4? 5? 6? 8?

PRINCIPLES.—I. *Every multiple of a number contains all the prime factors of that number.*

II. *The* L. C. M. *of two or more numbers contains all the prime factors of each of those numbers.*

III. *If two or more numbers are prime to each other, their product is their* L. C. M.

93. A ***multiple*** of a number is any number of times that number.

1. A multiple is necessarily *composite;* a divisor may be either prime or composite.

2. A number is a *divisor* of all its multiples, and a *multiple* of all its divisors.

94. A ***common multiple*** of two or more numbers is a number exactly divisible by each of them.

95. The ***least common multiple*** (L.C.M.) of two or more numbers is the least number exactly divisible by each of them.

Two or more numbers can have but *one* least common multiple.

WRITTEN EXERCISES.

96. To find the *L.C.M.* of two or more numbers.

1. Find the L.C.M. of 14, 21, and 45.

FIRST METHOD.

$$\begin{array}{l} 14 = 2 \times 7 \\ 21 = \quad\;\; 7 \times 3 \\ 45 = \underline{\qquad\qquad 3 \times 3 \times 5} \\ \qquad\; 2 \times 7 \times 3 \times 3 \times 5 \end{array}$$

EXPLANATION.—Resolving the given numbers into their prime factors, we find the L.C.M. must contain the factor 2 *once*, 7 *once*, 3 *twice*, and 5 *once*, and no other factors (PRIN. II). Hence, 2 × 7 × 3 × 3 × 5 = 630, the L.C.M.

SECOND METHOD.

$$\begin{array}{r|rrr} 3 & 14 & 21 & 45 \\ \hline 7 & 14 & 7 & 15 \\ \hline 3 & 2 & 1 & 15 \\ \hline & 2 & 1 & 5 \end{array}$$

EXPLANATION.—Divide the given numbers 21 and 45 by the prime factor 3, and write the quotients and undivided number 14 in the second line. In the same manner, divide successively by the prime factors 7 and 3. The last quotient, 5, being a prime number, is not divisible. Hence, 3 × 7 × 3 × 2 × 5 = 630, L.C.M. (PRIN. III).

Find the L.C.M.

2. Of 14, 16, and 18.
3. Of 27, 36, and 44.
4. Of 16, 48, and 108.
5. Of 14, 42, and 63.
6. Of 12. 16, 18, and 20.
7. Of 10, 45, 75, and 90.

RULE.—*Resolve each of the numbers into their prime factors; the product of the highest power of each prime factor is the L.C.M.* Or,

1. *Write the numbers in a horizontal line, omitting such of the smaller numbers as are factors of the larger, and divide by any prime factor that will exactly divide one or more of the given numbers, and write the quotients and undivided numbers, if any, in a line beneath.*

2. *In like manner, divide the quotients and undivided numbers, until the results are prime to each other. The product of the divisors and numbers in the last line will be the L.C.M.*

Find the L.C.M.

8. Of 144 and 180.
9. Of 324 and 360.
10. Of 225 and 375.
11. Of 16, 40, and 96.
12. Of 84, 100, and 224.
13. Of 16, 20, 48, and 72.

14. Find the L.C.M. of 25, 60, 100, and 125.

15. Find the L.C.M. of 7, 15, 21, 25, and 35.

16. Find the L.C.M. of the first five odd numbers.

17. What is the smallest sum of money which can be exactly expended for books at $5, or $3, or $4, or $6 each?

18. What is the shortest piece of rope that can be cut exactly into pieces either 15, 18, or 20 feet long?

19. What is the product of the L.C.M. of 12, 16, 24, and 32, multiplied by their G.C.D.?

20. Divide the L.C.M. of 7, 42, 6, 9, 10, and 630, by the G.C.D. of 110, 140, and 680.

21. What is the smallest sum of money which can be exactly expended for sheep at $8, or cows at $28, or oxen at $54, or horses at $162 each?

22. What is the smallest quantity of grain that will fill an exact number of bins, whether they hold 36, 48, 80, or 144 bushels?

23. What is the least number of acres in a farm that can be exactly divided into lots of 10 acres, 14 acres, 16 acres, and 20 acres each?

FRACTIONS.

INDUCTIVE EXERCISES.

97. 1. If any unit, as an apple or a yard, is divided into 2 *equal parts*, what is each part named? **(64.)**

2. If into 3 equal parts? If into 5 equal parts?

3. What are the largest equal parts that can be made of a whole? The next largest? The next?

4. What is 1 of 4 equal parts of a unit called? 2 of 3 equal parts? 3 of 5 equal parts? 5 of 7 equal parts?

5. What is meant by 1 *fourth* of anything? 3 fourths? 2 fifths? 3 eighths? 5 ninths?

6. What are 3 of the 7 equal parts of a week called?

7. What are 5 of the 8 equal parts of a yard called?

8. Which is the greater, a third or a fourth? Why?

9. Which is the smaller, fifths or thirds? Why?

DEFINITIONS.

98. A ***fraction*** is one or more of the equal parts of a unit, or of anything regarded as a whole.*

Thus, 1 *half*, 2 *thirds*, and 4 *fifths* are fractions.

1. A **fractional unit** is *one* of the equal parts into which the integer or thing is divided.

Thus, in the expression *three fourths* of a mile, the *fractional unit* is *one fourth* of a mile.

Fractions are classified into *common* and *decimal*.

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* *Properly* speaking, the value of a fraction is less than 1. Hence, when the value is greater than 1, it is called an *improper* fraction (**100**, 2).

99. A ***common fraction*** is expressed by two numbers, called the *numerator* and the *denominator*, the former written over the latter, with a line between them. Thus,

One third	is written	$\frac{1}{3}$.	Nine tenths	is written	$\frac{9}{10}$.
Three fourths	"	$\frac{3}{4}$.	Seven twentieths	"	$\frac{7}{20}$.
Five sixths	"	$\frac{5}{6}$.	Twelve thirty-fifths	"	$\frac{12}{35}$.
Seven eighths	"	$\frac{7}{8}$.	Thirty forty-ninths	"	$\frac{30}{49}$

1. The **denominator** of a fraction, written below the line, shows the *number* of *equal parts* into which the unit is divided, and also *names* the parts.

Thus, in the fraction $\frac{7}{8}$, 8 is the *denominator*, and shows that the unit is divided into *eight* equal parts, named *eighths*.

2. The **numerator** of a fraction, written above the line, shows the number of equal parts *taken* to form the fraction.

Thus, in $\frac{7}{8}$, 7 is the *numerator*, and shows that 7 of the eight equal parts are *taken*, or *expressed* by the fraction.

3. The **terms** of a fraction are its numerator and denominator.

Thus, 6 and 7 are the *terms* of the fraction $\frac{6}{7}$.

4. The **reciprocal of a fraction** is 1 divided by that fraction, or it is the fraction *inverted*.

Thus, the *reciprocal* of $\frac{3}{4}$ is $1 \div \frac{3}{4} = \frac{4}{3}$; of $\frac{7}{12}$, it is $\frac{12}{7}$.

5. The **value** of a fraction is the quotient of its numerator divided by its denominator. Thus, $\frac{12}{3} = 4$.

Name the fractional unit, the terms, the reciprocal, and the value of each of the following fractions:

$$\frac{4}{5} \quad \frac{6}{7} \quad \frac{8}{9} \quad \frac{6}{5} \quad \frac{7}{8} \quad \frac{9}{7} \quad \frac{11}{9} \quad \frac{7}{10} \quad \frac{18}{24} \quad \frac{12}{20}$$

100. ***Common fractions*** are classified as *proper* or *improper*, according to their *value*.

1. A **proper fraction** is a fraction whose numerator is less than its denominator. Hence, its *value* is *less* than 1.

Thus, $\frac{4}{5}$, $\frac{5}{8}$, and $\frac{11}{14}$ are *proper* fractions.

2. An **improper fraction** is a fraction whose numerator equals or exceeds its denominator. Hence, its *value* is *equal to* or *greater than* 1.

Thus, $\frac{6}{6}$, $\frac{10}{4}$, and $\frac{21}{9}$ are *improper* fractions.

101. A ***mixed number*** is a number composed of an integer and a fraction united.

Thus, $12\frac{3}{5}$ is a *mixed* number, equivalent to $12+\frac{3}{5}$.

102. Since fractions indicate division, all changes in the *terms* of a fraction will affect the *value* of the fraction according to the "General Principles of Division" (**72**), as shown in the following illustrations:

$$\text{I.}\begin{cases}\dfrac{4\times 2}{6}=\dfrac{8}{6}\\[2ex]\dfrac{4}{6\div 2}=\dfrac{4}{3}\end{cases}$$

1. The *value* of each fractional unit remains the same, but the *number* of units is *twice* as many.

2. The *value* of each fractional unit is *twice* as large, but the *number* of units remains the same. Hence,

In both cases, the value of the fraction is multiplied (**72**, I).

$$\text{II.}\begin{cases}\dfrac{4\div 2}{6}=\dfrac{2}{6}\\[2ex]\dfrac{4}{6\times 2}=\dfrac{4}{12}\end{cases}$$

1. The *value* of each fractional unit remains the same, but the *number* of units is *one half* as many.

2. The *value* of each fractional unit is *one half* as large, but the *number* of units remains the same. Hence

In both cases, the value of the fraction is divided (**72**, II).

$$\text{III.}\begin{cases}\dfrac{4\times 2}{6\times 2}=\dfrac{8}{12}\\[2ex]\dfrac{4\div 2}{6\div 2}=\dfrac{2}{3}\end{cases}$$

1. The *value* of each fractional unit is only *one half* as large, but the *number* of units is *twice* as many.

2. The *value* of each fractional unit is *twice* as large, but the *number* of units is *one half* as many. Hence,

In either case, the value of the fraction is not changed (**72**, III).

103. GENERAL PRINCIPLES OF FRACTIONS.

I.	***Multiplying the numerator***, or ***Dividing the denominator***,	*MULTIPLIES the fraction.*
II.	*Dividing the numerator*, or *Multiplying the denominator*,	*DIVIDES the fraction.*
III.	*Multiplying or dividing both numerator and denominator by the same number*,	*DOES NOT CHANGE the value of the fraction.*

The above may be reduced to one *general principle*, **viz.**:

A change in the NUMERATOR, by multiplication or division, produces a LIKE change in the value of the fraction; but such a change in the DENOMINATOR produces an OPPOSITE change in the value of a fraction.

REDUCTION.

104. ***Reduction of fractions*** is the process of changing their *form*, without altering their value.

1. A fraction is changed to **higher terms**, when its numerator and denominator are expressed in *larger* numbers; to **lower terms**, when expressed in *smaller* numbers; and to its **lowest terms**, when the numerator and denominator are *prime to each other*.

2. All higher terms of a fraction are *multiples* of its *lowest terms*.

105. To change fractions to higher or lower terms.

INDUCTIVE EXERCISES.

1. One half is equal to how many fourths?

ANALYSIS.—Since 1 is equal to 4 *fourths*, $\frac{1}{2}$ is equal to 1 half of 4 fourths, or 2 fourths.

2. One third of a mile is how many *sixths* of a mile?

3. One half of a dollar is how many *fourths* of a dollar?

4. Name some equivalent fractions for *halves*. *Thirds*.

5. Express $\frac{2}{3}$ in terms 3 times as great. 4 times as great.

6. The number of twelfths in a unit is how many times the number of *halves?* Of *thirds?* *Fourths?*

7. How is the fraction $\frac{4}{5}$ changed to *tenths* expressing the same value? To 15ths? To 20ths? To 40ths?

8. The denominators 4, 6, 8, and 10 are *multiples* of what number?

9. Name the first three multiples of 3, the denominator of $\frac{2}{3}$. The first three of 4 in $\frac{3}{4}$.

10. Change $\frac{3}{4}$, $\frac{7}{6}$, $\frac{5}{8}$, and $\frac{11}{12}$ to 24ths, to 48ths, and 96ths.

11. Multiply both terms of $\frac{3}{4}$ by 3, and show that the *value* of the fraction is not changed.

Analysis.—If both terms of $\frac{3}{4}$ are multiplied by 3, the resulting fraction is $\frac{9}{12}$, which is equivalent to $\frac{3}{4}$, since the *fractional unit* is $\frac{1}{3}$ as large, while the number of fractional units taken is 3 times as many (**102**, III).

12. Name three equivalent fractions for $\frac{4}{5}$; for $\frac{1}{6}$; for $\frac{3}{8}$.

13. How are 5ths changed to 20ths? 8ths to 40ths?

14. Express $\frac{3}{7}$ in 28ths; in 35ths; in 56ths; in 49ths.

15. How are fractions changed to *higher* terms?

16. 8 *twelfths* are how many *thirds?*

Analysis.—Since 1 third is equal to 4 twelfths, 8 twelfths are equal to as many thirds as 4 twelfths are contained times in 8 twelfths, which is 2 times. Hence there are $\frac{2}{3}$ in $\frac{8}{12}$.

17. How many *fourths* of a rod are 9 *twelfths* of a rod?

18. Divide both terms of $\frac{15}{20}$ by 5, and show that the *value* of the fraction is not changed.

Analysis.—If both terms of $\frac{15}{20}$ are divided by 5, the resulting fraction is $\frac{3}{4}$, which is equivalent to $\frac{15}{20}$, since the *fractional unit* is 5 times as large, while the number taken is $\frac{1}{5}$ as many (**102**, III).

19. Change $\frac{15}{30}$ to a fraction having lower terms; $\frac{18}{27}$; $\frac{16}{32}$.

20. How many *thirds* in $\frac{10}{15}$? *Fifths in* $\frac{24}{40}$?

21. Express the value of $\frac{24}{36}$ in terms $\frac{1}{12}$ as great; $\frac{1}{6}$ as great; $\frac{1}{4}$ as great; $\frac{1}{3}$ as great; $\frac{1}{2}$ as great.

22. What common divisors have the terms of $\frac{14}{28}$? $\frac{25}{30}$? $\frac{42}{56}$?

23. Is $\frac{12}{18}$ in its lowest terms? Why not? Name two factors of its terms. Why is $\frac{2}{3}$ equal to $\frac{12}{18}$?

24. In what lower terms can $\frac{12}{24}$ be expressed?

25. Change $\frac{9}{36}$ to its lowest terms; $\frac{16}{24}$, $\frac{24}{40}$; $\frac{21}{35}$.

26. How is a fraction reduced to *lower* terms?

27. How is a fraction reduced to its *lowest* terms?

WRITTEN EXERCISES.

106. 1. Reduce $\frac{5}{8}$ to sixteenths.

$$16 \div 8 = 2$$

$$\frac{5 \times 2}{8 \times 2} = \frac{10}{16}$$

EXPLANATION.—Since the given denominator 8 is contained in the required denominator 2 times, multiply both terms of the fraction by 2. Hence, $\frac{5}{8} = \frac{10}{16}$.*

Reduce	Change
2. $\frac{4}{5}$ to thirty-fifths.	5. $\frac{11}{15}$ to seventy-fifths.
3. $\frac{5}{7}$ to sixty-thirds.	6. $\frac{16}{18}$ to 108ths.
4. $\frac{7}{12}$ to eighty-fourths.	7. $\frac{21}{32}$ to 256ths.

8. Reduce $\frac{36}{48}$ to its lowest terms.

$$\frac{36 \div 4}{48 \div 4} = \frac{9}{12}; \quad \frac{9 \div 3}{12 \div 3} = \frac{3}{4};$$

$$\text{or,} \quad \frac{36 \div 12}{48 \div 12} = \frac{3}{4}.$$

EXPLANATION. — Dividing both terms of the given fraction $\frac{36}{48}$ by 4, the result is $\frac{9}{12}$ (**102**, III).

Again, dividing both terms of $\frac{9}{12}$ by 3, the result is $\frac{3}{4}$. Since the terms of $\frac{3}{4}$ are *prime to each other*, the lowest terms of $\frac{36}{48}$ are $\frac{3}{4}$.

The same result may be obtained more directly by dividing the terms of the given fraction by their greatest common divisor 12.

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* *Rules* are omitted when the oral exercises and explanations are so full and explicit that the pupil can readily construct a rule for himself.

Reduce to their lowest terms,

9. $\frac{96}{100}$.
10. $\frac{63}{81}$.
11. $\frac{72}{108}$.
12. $\frac{168}{252}$.
13. $\frac{32}{256}$.
14. $\frac{324}{486}$.
15. $\frac{420}{2310}$.
16. $\frac{595}{665}$.
17. $\frac{128}{162}$.
18. $\frac{292}{657}$.
19. $\frac{5400}{7200}$.
20. $\frac{639}{1737}$.
21. $\frac{1344}{1536}$.
22. $\frac{5643}{5940}$.
23. $\frac{4680}{10800}$.
24. $\frac{5222}{7042}$.

25. Express in its simplest form the quotient of 441 divided by 462. Of 189 divided by 273.

26. Express in the simplest form 1344 divided by 1536.

27. Which is the greater fraction, and why, $\frac{75}{135}$, or $\frac{168}{252}$?

28. Change 168÷252 to the form of a fraction in its lowest terms. 81÷63. 160÷400. 324÷612.

107. To change an integer or a mixed number to the form of an improper fraction.

ORAL EXERCISES.

1. How many *thirds* in 4?

ANALYSIS.—Since in any number there are 3 times as many *thirds* as whole ones, in 4 whole ones there are 3 times 4 *thirds* = $\frac{12}{3}$. Hence, etc.

2. In 4 bushels, how many *eighths* of a bushel?

3. In $1 how many fifths of a dollar? Sixths? Eighths? Tenths? Twelfths?

4. In $5\frac{3}{4}$ yards, how many fourths of a yard?

ANALYSIS.—Since in any number there are 4 times as many *fourths* as there are whole ones, in 5 yards there are 4 times 5 *fourths* or $\frac{20}{4}$ of a yard, plus $\frac{3}{4}$ of a yard, equals $\frac{23}{4}$ yards. Hence, etc.

5. How many sixths in $$2\frac{5}{6}$? In $5\frac{1}{6}$ rods? In $7\frac{2}{3}$ acres?

6. In $6\frac{3}{4}$ cords of wood, how many *fourths* of a cord?

7. Among how many boys can you distribute $5\frac{3}{4}$ quarts of chestnuts, if you give them $\frac{1}{4}$ of a quart each?

8. How is an integer or a mixed number changed to the form of an improper fraction?

WRITTEN EXERCISES.

108. 1. Change 81 to a fraction having 24 for its denominator.

SOLUTION. $81 \times 24 = 1944$. Hence, $81 = \frac{1944}{24}$.

2. Change $49\frac{7}{12}$ to *twelfths*.

SOLUTION. $49\frac{7}{12} = 49 + \frac{7}{12}$; $49 \times \frac{12}{12} = \frac{588}{12}$; $\frac{588}{12} + \frac{7}{12} = \frac{595}{12}$.

3. Change 44 bushels to *fourths* of a bushel.

4. Change $15\frac{2}{3}$ years to *thirds* of a year.

5. Change $29\frac{4}{7}$ weeks to *sevenths* of a week.

6. In 78 pounds, how many sixteenths of a pound?

Reduce to improper fractions,

7.	$75\frac{9}{13}$.	10.	$86\frac{5}{17}$.	13.	$1227\frac{13}{153}$.
8.	$224\frac{5}{18}$.	11.	$450\frac{13}{20}$.	14.	$2475\frac{24}{30}$.
9.	$307\frac{18}{25}$.	12.	$261\frac{7}{80}$.	15.	$1307\frac{9}{75}$.

109. To change an improper fraction to the form of an integer, or a mixed number.

ORAL EXERCISES.

1. In $\frac{18}{4}$, how many units or ones?

ANALYSIS.—Since 4 *fourths* equal 1, 18 *fourths* are as many times 1 as 4 fourths is contained times in 18 fourths, which is $4\frac{2}{4}$ times. Hence, $\frac{18}{4} = 18 \div 4 = 4\frac{2}{4}$.

2. How many times 1 are $\frac{28}{7}$? $\frac{46}{8}$? $\frac{75}{12}$? $\frac{60}{9}$? $\frac{90}{15}$?

3. How many yards are $\frac{27}{5}$ of a yard? $\frac{45}{5}$? $\frac{62}{5}$?

4. How many dollars are $\$\frac{21}{4}$? $\$\frac{36}{8}$? $\$\frac{75}{12}$? $\$\frac{84}{20}$?

5. In $\frac{90}{12}$ of a foot, how many feet? In $\frac{108}{9}$ of an acre, how many acres? In $\frac{180}{20}$ of a ton, how many tons?

6. If a man buy $\frac{30}{8}$ of a yard of cloth, how many yards does he buy? If $\frac{75}{12}$? If $\frac{108}{16}$?

7. How is an improper fraction changed to the form of an integer, or a mixed number?

WRITTEN EXERCISES.

110. 1. Change $\frac{147}{9}$ to the form of a mixed number.

SOLUTION.—$\frac{147}{9} = 147 \div 9 = 16\frac{1}{3}$. Hence, $\frac{147}{9} = 16\frac{1}{3}$.

2. In $\frac{161}{16}$ of a pound, how many pounds?
3. In $\frac{401}{25}$ of a dollar, how many dollars?
4. How many rods in $\frac{525}{40}$ of a rod?

Reduce to integers or mixed numbers,

5. $\frac{651}{7}$.
6. $\frac{241}{12}$.
7. $\frac{476}{17}$.
8. $\frac{1345}{22}$.
9. $\frac{1345}{24}$.
10. $\frac{4407}{113}$.
11. $\frac{4483}{35}$.
12. $\frac{5312}{59}$.
13. $\frac{2184}{42}$.
14. $\frac{5812}{112}$.
15. $\frac{13001}{33}$.
16. $\frac{32460}{48}$.

111. To reduce two or more fractions to equivalent fractions having a common denominator.

ORAL EXERCISES.

1. How many *sixths* in 1? In $\frac{1}{2}$? In $\frac{1}{3}$?
2. How many *eighths* in 1? In $\frac{1}{2}$? In $\frac{1}{4}$? In $\frac{3}{4}$?
3. How can $\frac{2}{3}$ and $\frac{3}{4}$ be changed to 12ths? To 24ths?
4. Change $\frac{3}{4}$ and $\frac{5}{8}$ to fractions of the same denominator?
5. If the *denominator* is multiplied by any number, how is the *value* of the fraction preserved? Why? (102, I, 1.)
6. Change $\frac{2}{3}$ and $\frac{4}{5}$ to 15ths; to 30ths; to 45ths.
7. Change $\frac{12}{15}$ and $\frac{18}{30}$ to 5ths; to 10ths.
8. If the *denominator* be divided by any number, how is the *value* of the fraction preserved? Why? (102, I, 2.)
9. What is a *multiple* of a number? A common multiple of two or more numbers? (93, 94.)
10. Name a multiple of 2; of 4; of 6; of 8.
11. What is the common multiple of 3 and 4? Of 4 and 5?
12. What is the L.C.M. of 2, 5, and 10? Of 2, 3, and 4?
13. What is the L.C.M. of the denominators $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$? Of $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{5}{6}$? Of $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{4}{5}$?

112. A ***common denominator*** is a denominator common to two or more fractions.

A common denominator of two or more fractions having different denominators must be a *common multiple* of the given denominators.

113. The ***least common denominator*** of two or more fractions is the L.C.M. of their denominators.

When the fractions are in their lowest terms, and the denominators are *prime to each other*, their *continued product* is the L.C.D.

WRITTEN EXERCISES.

114. 1. Change $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{4}{5}$ to equivalent fractions having a common denominator.

$$2 \times 3 \times 5 = 30$$
$$\tfrac{1}{2} \text{ of } \tfrac{30}{30} = \tfrac{15}{30}$$
$$\tfrac{2}{3} \text{ of } \tfrac{30}{30} = \tfrac{20}{30}$$
$$\tfrac{4}{5} \text{ of } \tfrac{30}{30} = \tfrac{24}{30}$$

EXPLANATION.—Since the denominators are *prime to each other*, and in their lowest terms, their product 30 is their L.C.M., and hence the L.C.D. of the fractions $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{4}{5}$.

To find the numerators, take such part of the common denominator as the given fraction is of 1. Thus, $\frac{1}{2}$ of $30 = 15$; $\frac{2}{3}$ of $30 = 20$; and $\frac{4}{5}$ of $30 = 24$. Hence, etc.

Change to fractions having a common denominator,

2. $\frac{3}{7}$ and $\frac{5}{8}$.
3. $\frac{7}{12}$ and $\frac{4}{5}$.
4. $\frac{3}{7}$, $\frac{5}{8}$, and $\frac{2}{3}$.
5. $\frac{7}{16}$, $\frac{2}{3}$, and $\frac{1}{4}$.
6. $\frac{9}{10}$, $\frac{5}{7}$, and $\frac{1}{2}$.
7. $\frac{1}{6}$, $\frac{2}{9}$, and $\frac{2}{15}$.

8. Change $\frac{2}{3}$, $\frac{4}{7}$, and $\frac{5}{14}$ to equivalent fractions having the least common denominator.

$$\begin{array}{r|rrr} 7 & 3 & 7 & 14 \\ \hline & 3 & 1 & 2 \end{array} \qquad \tfrac{2}{3} = \tfrac{28}{42}$$
$$\tfrac{4}{7} = \tfrac{24}{42}$$
$$2 \times 3 \times 7 = 42 \qquad \tfrac{5}{14} = \tfrac{15}{42}$$

EXPLANATION.—First find the L.C.M. of the given denominators, which is 42. This must be the least common denominator of the given fractions. (**113.**) Reduce mixed numbers to improper fractions and fractions to their lowest terms, before finding the L.C.M.

Change to fractions having the least common denominator:

9. $\frac{7}{8}$, $\frac{11}{16}$, and $\frac{17}{24}$.
10. $\frac{4}{13}$, $\frac{15}{26}$, and $\frac{7}{39}$.
11. $\frac{20}{21}$, $\frac{9}{56}$, and $\frac{61}{84}$.
12. $\frac{3}{4}$, $2\frac{5}{7}$, $\frac{3}{5}$, and $1\frac{7}{10}$.
13. $6\frac{1}{4}$, $\frac{7}{20}$, 7, and $1\frac{1}{2}$.
14. $\frac{15}{40}$, $\frac{25}{120}$, $\frac{14}{64}$, and $2\frac{4}{5}$.

ADDITION AND SUBTRACTION.

INDUCTIVE EXERCISES.

115. 1. What is the sum of $\frac{3}{8}$ and $\frac{5}{8}$? Of $\frac{4}{9}$ and $\frac{7}{9}$?

2. How many times 1 is the sum of $\frac{6}{7}$, $\frac{3}{7}$, and $\frac{5}{7}$?

3. What is the difference between $\frac{4}{7}$ and $\frac{6}{7}$? $\frac{5}{9}$ and $\frac{8}{9}$?

4. How much is $\frac{9}{14}$ less $\frac{5}{14}$? $\frac{13}{20}$ less $\frac{7}{20}$? $\frac{16}{25}$ less $\frac{9}{25}$?

5. A gentleman who owned a sail-boat sold $\frac{5}{16}$ of it. What part did he still own?

6. How many times 1 are $\frac{4}{8}+\frac{5}{8}+\frac{7}{8}$? $\frac{4}{9}+\frac{5}{9}+\frac{8}{9}$?

7. A boy paid $\$\frac{1}{5}$ for a slate, $\$\frac{2}{5}$ for a reader, and $\$\frac{4}{5}$ for an arithmetic. How much did he pay for all?

How are fractions having a common denominator *added* or *subtracted*?

8. What is the fractional unit of $\frac{2}{3}$? Of $\frac{5}{6}$? Can $\frac{2}{3}$ and $\frac{5}{6}$ in their present form be added? Why not? Can one be subtracted from the other? Why not?

9. What change must be made in $\frac{3}{4}$ and $\frac{5}{6}$ before their sum or their difference can be found?

10. Mary paid $\$\frac{3}{4}$ for some ribbon, and $\$\frac{5}{6}$ for a pair of gloves. How much did she pay for both?

ANALYSIS.—She paid the sum of $\$\frac{3}{4}$ and $\$\frac{5}{6}$. $\frac{3}{4}$ is equal to $\frac{9}{12}$, and $\frac{5}{6}$ is equal to $\frac{10}{12}$; $\frac{9}{12}$ and $\frac{10}{12}$ are $\frac{19}{12}$, or $1\frac{7}{12}$. Hence, she paid $\$1\frac{7}{12}$.

11. A man having $\frac{2}{3}$ of a ton of coal, bought $\frac{2}{5}$ of a ton more. How much had he then?

12. A boy having $\$\frac{3}{4}$, gave $\$\frac{1}{3}$ for a necktie. What had he left?

ANALYSIS.—He had left the difference between $\$\frac{3}{4}$ and $\$\frac{1}{3}$. $\frac{3}{4}$ is equal to $\frac{9}{12}$, and $\frac{1}{3}$ equals $\frac{4}{12}$; $\frac{9}{12}$ less $\frac{4}{12}$ are $\frac{5}{12}$. Hence, he had $\$\frac{5}{12}$ left.

13. A man owning $\frac{2}{3}$ of an acre of ground, sold $\frac{1}{6}$ of an acre. What part remained?

How are fractions having different denominators *added* or *subtracted*?

14. A farmer sold 3¼ tons of hay to one man, and 5⅜ to another. How much did he sell to both?

Analysis.—The sum of 3¼ tons and 5⅜ tons. 5 and 3 are 8; and ¼ and ⅜ are ⅝, which added to 8 makes 8⅝ tons.

15. A man bought 5½ cords of wood at one time, and $7\frac{9}{16}$ at another. How much did he buy in all?

16. From a piece of cloth containing 12½ yards, 5⅛ yards were cut. How many yards remained?

Analysis.—The difference between 12½ yards and 5⅛ yards. ⅛ from ½ leaves ⅜, and 5 from 12 leaves 7. Hence, 7⅜ yards remained.

17. If a ton of coal costs $7⅘, and a cord of wood $4⅓, what is the difference in their cost?

How are mixed numbers *added* or *subtracted*?

18. What is the sum of ½, ⅓, and ⅚? Of ⅔, ¼, and ⅙?

19. Find the sum of ⅝, ½, and $\frac{7}{12}$. Of ⅘, ¾, and $\frac{7}{10}$.

20. Subtract ½ from ¾; ⅔ from ⅚; ¾ from ⅞; ⅖ from ⅚.

21. If a grocer buys tea at $⅝ a pound, and sells it at $¾, does he gain or lose, and how much?

22. How much greater than 2 is ⅔ + ½ + 1⅙?

23. What is the difference between 4 and 2⅜? 5¼ and 7¼?

24. Write on the slate or board,

$$\frac{1}{2} \qquad \frac{2}{3} \qquad \frac{4}{6} \qquad \frac{5}{3} \qquad \frac{1}{6} \qquad \frac{4}{2} \qquad \frac{7}{6} \qquad \frac{8}{2} \qquad \frac{5}{4} \qquad \frac{10}{8}$$

1. Give *sum* of each fraction and the one at its right.

2. Give the *difference*.

3. Give the *sum* of each three fractions successively.

4. Give the difference between the *sum* of two fractions and the one at the right.

Principle.—*The sum, or difference of two or more fractions can be found only when they have a common denominator, and when they express fractional units of the same kind.*

WRITTEN EXERCISES.

116. 1. From the sum of $\frac{4}{10}$ and $\frac{7}{8}$ take the difference between $1\frac{1}{4}$ and $\frac{3}{4}$.

2. A man owning $\frac{19}{27}$ of a cotton-mill, sold $\frac{1}{3}$ of the whole to one man and $\frac{2}{9}$ to another. What part had he left?

3. From the sum of $14\frac{1}{2}$ and $9\frac{1}{3}$ take the difference between $25\frac{3}{4}$ and $16\frac{1}{5}$.

4. A clerk earned \$$75\frac{3}{4}$ one month, and paid for board \$$28\frac{3}{8}$, and other expenses \$$27\frac{7}{16}$. How much had he left?

Find the value of

5. $\frac{1}{7}+\frac{3}{4}+\frac{5}{8}-\frac{9}{14}$.
6. $4\frac{5}{6}+2\frac{3}{8}-5\frac{1}{4}$.
7. $18-5\frac{1}{4}+3\frac{1}{6}-\frac{7}{10}$.
8. $8\frac{4}{7}+\overline{12\frac{3}{5}-11\frac{1}{7}}+\frac{12}{35}$.
9. $\frac{11}{16}+1\frac{1}{8}+5\frac{1}{2}-2\frac{5}{32}$.
10. $28\frac{1}{4}+36\frac{5}{9}-(44-13\frac{1}{6})$.
11. $86\frac{9}{40}-(50\frac{1}{8}+15\frac{1}{5}-6\frac{1}{2})$.
12. $8\frac{1}{2}+\frac{3}{4}+5\frac{1}{6}-7\frac{8}{18}$.

13. A lady having \$50, paid \$$12\frac{3}{8}$ for a bonnet, \$$17\frac{5}{8}$ for a shawl, and \$$3\frac{7}{10}$ for a veil. How much money had she left?

Find the *sum*

14. Of $\frac{5}{12}$, $\frac{13}{15}$, and $\frac{7}{20}$.
15. Of $\frac{3}{5}$, $\frac{7}{8}$, and $\frac{1}{2}$.
16. Of 42, $31\frac{5}{12}$, and $9\frac{7}{18}$.
17. Of $204\frac{7}{15}$, $50\frac{10}{21}$, and $7\frac{8}{35}$.

Find the *difference* between

18. $\frac{5}{8}$ and $\frac{3}{11}$.
19. $1\frac{3}{4}$ and $\frac{5}{6}$.
20. $\frac{14}{15}$ and $2\frac{4}{5}$.
21. 16 and $3\frac{2}{11}$.
22. $63\frac{7}{9}$ and $71\frac{1}{8}$.
23. 106 and $95\frac{15}{36}$.

24. Bought a quantity of coal for \$$136\frac{9}{16}$, and of lumber for \$$350\frac{2}{3}$. I sold the coal for \$$184\frac{1}{2}$, and the lumber for \$$416\frac{3}{4}$. What was my whole gain?

25. A merchant sold $46\frac{3}{4}$ yards of cloth for \$$127\frac{7}{16}$, $64\frac{5}{8}$ yards for \$$226\frac{5}{6}$, and $76\frac{1}{2}$ yards for \$$312\frac{2}{3}$. How many yards of cloth did he sell, and how much did he receive for the whole?

26. A man having \$$25\frac{1}{8}$, paid \$$6\frac{1}{2}$ for coal, \$$2\frac{1}{5}$ for dry goods, and \$$\frac{3}{4}$ for a pound of tea. How much had he left?

27. A man bought a ton of hay for $\$15\frac{3}{8}$, a barrel of flour for $\$9\frac{5}{12}$, and a barrel of apples for $\$3\frac{7}{16}$. What change should be given to him for 3 ten-dollar bills?

Complete the following equations:

28. $\frac{4}{5}+\frac{6}{7}-\overline{\frac{3}{8}+\frac{5}{7}} = ?$
29. $8\frac{5}{6}+2\frac{2}{3}-5\frac{4}{11} = ?$
30. $48-(16\frac{4}{7}-3\frac{1}{2}) = ?$
31. $41\frac{1}{3}+\overline{56-24\frac{7}{30}}-4\frac{11}{15} = ?$
32. $120-51\frac{2}{3}+90\frac{10}{27}-\frac{2}{3} = ?$
33. $342-(21\frac{3}{15}+\frac{4}{5}-9) = ?$

MULTIPLICATION.

117. In all examples of multiplication of fractions, either *one* or *both* of the factors will be *fractional.*

INDUCTIVE EXERCISES.

118. 1. What part of a bushel is 3 times $\frac{1}{4}$ of a bushel?

2. What part of a cord is 5 times $\frac{1}{8}$ of a cord?

3. At $\$\frac{3}{8}$ a pound, what will 4 pounds of sugar cost?

ANALYSIS.—4 pounds will cost 4 times $\$\frac{3}{8}$, or $\$\frac{3\times 4}{8} = \$\frac{12}{8} = \$1\frac{1}{2}$; or $\$\frac{3}{8\div 4} = \$\frac{3}{2} = \$1\frac{1}{2}$. (**102**, I.)

4. At $\$\frac{5}{6}$ a bushel, what is the cost of 3 bushels of oats?

5. Of 5 bushels? Of 6 bushels? Of 8 bushels?

6. If a horse eats $\frac{5}{8}$ of a bushel of oats in a day, how much will 2 horses eat? 4 horses? 6 horses? 8 horses?

How many ways to multiply a fraction by an integer, and what are they?

Show that multiplying the numerator of $\frac{3}{20}$ by 4, or dividing the denominator by 4, multiplies the fraction by 4. (**102**, I.)

7. Multiply $\frac{4}{9}$ by 7; $\frac{4}{18}$ by 6; $\frac{9}{20}$ by 5; $\frac{7}{40}$ by 8; $\frac{3}{8}$ by 9.

A fraction is *multiplied* by a number equal to its *denominator* by *cancelling* the denominator. Thus, $\frac{7}{8}\times 8 = 7$.

Cancelling a *factor* of the denominator *multiplies* the fraction by that factor. Thus, $\frac{5}{12}\times 4 = \frac{5}{3}$.

8. Multiply $\frac{5}{9}$ by 9; $\frac{4}{10}$ by 5; $\frac{6}{7}$ by 7; $\frac{9}{48}$ by 8; $\frac{11}{24}$ by 12.

9. At $3¾ a box, what will 4 boxes of raisins cost?

Analysis. $3¾ = $¹⁵⁄₄, and 4 times $¹⁵⁄₄ = $15. Or, 4 times $¾ = $3, and 4 times $3 = $12; $12 + $3 = $15.

10. Multiply 6⅘ by 4; by 5; 10⅝ by 4; 9⁴⁄₇ by 6.

11. At $8⅚ a barrel, what is the cost of 3 barrels of flour?

12. What is ½ of 12 pounds? ¼ of 24 men? ⅕ of $40?

13. Multiplying by ½, ⅓, ¼, ⅕, ⅙, etc., is the same as dividing by what integers?

When a *fractional part* of an integer, or of a fraction, is to be taken, the word *of*, and not *times*, should be used.

14. At $7 a ton, what will ¾ of a ton of coal cost?

Analysis.—It will cost ¾ of $7, or 3 times ¼ of $7. ¼ of $7 is $1¾, and 3 times $1¾ are $5¼.

15. If an acre of land produce 40 bushels of corn, how much will ¾ of an acre produce? ⅝? ⅘? ⅗? ⁷⁄₁₀?

16. What is ⅗ of $5? Of $7? ⅝ of $16? ⅘ of $20?

17. Multiply 6 by ¾; 8 by ⅘; ⅘ by 8; 15 by ⅗; 24 by ⅜.

Show that a fraction of an integer equals the product of the integer by the fraction.

How is an integer multiplied by a fraction?

18. At $12 a ton, what will 5⅜ tons of cheese cost?

Analysis.—It will cost 5⅜ times $12. 5 times $12 are $60, and ⅜ of $12 are $4½, which added to $60 make $64½.

19. If a man earn $4 a day, how much will he earn in 6⅜ days? In 5⅙ days? In 8¾ days? In 10⅝ days?

20. What is ½ of 6 tenths? ⅓ of 9 twelfths? ¼ of 8 ninths?

21. Which is greater, ⅓ of ½, or ½ of ⅓? ¼ of ⅘, or ⅕ of ¾?

22. If I own ⁶⁄₇ of a farm, and sell ⅓ of it, what part of the farm do I sell? What part do I retain?

23. A boy having $¾, gave ⅚ of it for a knife. What part of a dollar did he pay for the knife?

Analysis.—He paid ⅚ of $¾, or 5 times ⅙ of $¾. ⅙ of $¾ is $³⁄₂₄, and 5 times $³⁄₂₄ are ¹⁵⁄₂₄, or $⅝.

24. If a pound of coffee cost $\$\frac{3}{8}$, what cost $\frac{2}{3}$ of a pound?

25. At $\$\frac{4}{5}$ a gallon, what will $\frac{3}{4}$ of a gallon of syrup cost?

Fractions with the word *of* between them are sometimes called *compound fractions*. The word *of* is equivalent to the sign (×) of multiplication. Thus, $\frac{5}{6}$ *of* $\frac{2}{3} = \frac{5}{6} \times \frac{2}{3}$; $\frac{4}{5}$ *of* $9 = \frac{4}{5} \times 9$, etc.

26. What is $\frac{2}{3}$ of $\frac{5}{7}$? $\frac{4}{5}$ of $\frac{3}{4}$? $\frac{5}{6}$ of $\frac{3}{8}$? $\frac{3}{7}$ of $\frac{5}{8}$?

27. What is $\frac{4}{7} \times \frac{3}{5}$? $\frac{9}{10} \times \frac{2}{5}$? $\frac{3}{8} \times \frac{5}{12}$? $\frac{7}{9} \times \frac{3}{14}$?

28. A man owning $\frac{4}{5}$ of a mill, sold $\frac{5}{8}$ of his share to his brother. What part of the mill did each then own?

29. At $\$8\frac{4}{5}$ a barrel, what will $\frac{3}{4}$ of a barrel of flour cost?

ANALYSIS.—It will cost $\frac{3}{4}$ of $\$8\frac{4}{5}$, or 3 times $\frac{1}{4}$ of $\$8\frac{4}{5}$. $\frac{1}{4}$ of $\$8\frac{4}{5}$ is $\$2\frac{1}{5}$, and 3 times $\$2\frac{1}{5}$ are $\$6\frac{3}{5}$.

30. At $\$9\frac{1}{4}$ a case, what will $\frac{3}{5}$ of a case of slates cost?

31. At $\$5\frac{1}{2}$ a yard, what will $\frac{4}{5}$ of a yard of cloth cost?

32. At $\$\frac{2}{3}$ a yard, what will $6\frac{1}{4}$ yards of flannel cost?

ANALYSIS.—It will cost $6\frac{1}{4}$ times $\$\frac{2}{3}$. 6 times $\$\frac{2}{3}$ are $4, and $\frac{1}{4}$ of $\$\frac{2}{3}$ is $\$\frac{1}{6}$, which added to $4 makes $\$4\frac{1}{6}$.

33. If a man hoe $\frac{7}{8}$ of an acre of corn in a day, how many acres can he hoe in $5\frac{1}{3}$ days? In $6\frac{3}{4}$ days? In $8\frac{4}{5}$ days?

34. What will $2\frac{1}{4}$ yards of silk cost, at $\$3\frac{1}{3}$ a yard?

ANALYSIS. $2\frac{1}{4} = \frac{9}{4}$, and $3\frac{1}{3} = \frac{10}{3}$; and $\frac{10}{3} \times \frac{9}{4} = \frac{15}{2} = \$7\frac{1}{2}$.

Reduce mixed numbers to improper fractions, and then proceed as in multiplying one fraction by another.

Find the value of

35. $\frac{3}{7} \times \frac{3}{4}$.
36. $1\frac{3}{4} \times \frac{5}{8}$.
37. $\frac{7}{10} \times 6\frac{1}{7}$.
38. $4\frac{1}{3} \times 1\frac{2}{3}$.

39. $\frac{3}{7}$ of 10 weeks.
40. $\frac{7}{8}$ of 12 rods.
41. $\frac{5}{9}$ of 27 yards.
42. $\frac{5}{6}$ of 40 acres.

43. $\frac{3}{4}$ of $\frac{8}{9}$ of a mile.
44. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $1.
45. $\frac{3}{5}$ of $4\frac{7}{10}$ leagues.
46. $\frac{3}{4}$ of $6\frac{2}{3}$ pecks.

What is the product

47. Of $2\frac{1}{4}$ times 12 cents?
48. Of $5\frac{2}{3}$ times $7?
49. Of $4\frac{1}{3}$ times 9 feet?

Find the result

50. Of $16 - \frac{3}{10} \times 12\frac{2}{3}$.
51. Of $4\frac{1}{8} + \frac{3}{4}$ of $1\frac{1}{2} - \frac{7}{16}$.
52. Of $\frac{13}{33}$ of $\frac{11}{9} + 2\frac{1}{3} \times 0$.

119. Principles.—I. *A fraction is multiplied by multiplying its numerator, or by dividing its denominator by any integer.*

II. *The product of two or more factors, whether integral or fractional, is the same, in whatever order they are used.*

WRITTEN EXERCISES.

120. All operations in multiplication of fractions may be reduced to *one uniform* and simple process, and often much abbreviated by *cancellation.*

Since the analyses and principles of the Inductive Exercises are equally applicable to similar written exercises, further explanations are unnecessary.

1. Multiply $\frac{7}{75}$ by 15.

$\frac{7}{75}\times 15 = \frac{7\times\cancel{15}}{\cancel{75}}$

$= \frac{7}{5} = 1\frac{2}{5}$. Prin. I.

Or, $\frac{7}{75}\times 15 = \frac{7}{75\div 15}$

$= \frac{7}{5} = 1\frac{2}{5}$.

Or,

$$\begin{array}{r|l} {}^{5}\,\cancel{75} & 7 \\ 1 & \cancel{15} \\ \hline 5 & 7 \\ \cline{2-2} & 1\frac{2}{5} \end{array}$$

2. Multiply 15 by $\frac{7}{75}$.

$15\times\frac{7}{75} = \frac{\cancel{15}\times 7}{\cancel{75}}$

$= \frac{7}{5} = 1\frac{2}{5}$.

Or,

$15\times\frac{7}{75} = \overline{15\div 75}\times 7$

$= 1\frac{2}{5}$.

Or,

$$\begin{array}{r|l} 1 & \cancel{15} \\ {}^{5}\,\cancel{75} & 7 \\ \hline 5 & 7 \\ \cline{2-2} & 1\frac{2}{5} \end{array}$$

All factors common to the numerators and the denominators should be cancelled before multiplying.

It is obvious, that in multiplying a fraction by an integer, or an integer by a fraction, the result is the same. (Prin. II.)

3. Multiply $17\frac{2}{9}$ by 6.

$$\begin{array}{rl} 17\frac{2}{9} & \\ 6 & \\ \hline 102 & = 17 \times 6 \\ 1\frac{1}{3} & = \frac{2}{9}\times 6 \\ \hline 103\frac{1}{3} & = 17\frac{2}{9}\times 6. \end{array}$$

Or, $17\frac{2}{9} = \frac{155}{9}$.

$$\begin{array}{r|l} {}^{3}\,\cancel{9} & 155 \\ 1 & \cancel{6}\,{}^{2} \\ \hline 3 & 310 \\ \cline{2-2} & 103\frac{1}{3} \end{array}$$

4. Multiply 27 by $6\frac{4}{9}$.

$$\begin{array}{rl} 27 & \\ 6\frac{4}{9} & \\ \hline 162 & = 27\times 6 \\ 12 & = 27\times\frac{4}{9} \\ \hline 174 & = 27\times 6\frac{4}{9}. \end{array}$$

Or, $6\frac{4}{9} = \frac{58}{9}$.

$$\begin{array}{r|l} 1 & \cancel{27}\,{}^{3} \\ \cancel{9} & 58 \\ \cline{2-2} & 174 \end{array}$$

5. Multiply $\frac{15}{16}$ by $\frac{8}{9}$.

$\frac{15}{16} \times \frac{8}{9} = \frac{120}{144} = \frac{5}{6}$. Or,

Or,

$\frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{16}}} \times \frac{\cancel{8}}{\underset{3}{\cancel{9}}} = \frac{5}{6}$.

$$\begin{array}{r|l} {}^{2}\,\cancel{16} & \cancel{15}\,{}^{5} \\ {}^{3}\,\cancel{9} & \cancel{8} \\ \hline 6 & 5 = \frac{5}{6}. \end{array}$$

6. Multiply $\frac{5}{6}$ of $4\frac{1}{2}$ by 2.

$\frac{5}{\underset{2}{\cancel{6}}} \times \frac{\overset{3}{\cancel{9}}}{\cancel{2}} \times \frac{\cancel{2}}{1} = \frac{15}{2} = 7\frac{1}{2}$.

Or,

$$\begin{array}{r|l} {}^{2}\,\cancel{6} & 5 \\ \cancel{2} & \cancel{9}\,{}^{3} \\ 1 & \cancel{2} \\ \hline 2 & 15 = 7\frac{1}{2}. \end{array}$$

RULE.—1. *Change integers and mixed numbers, if any, to the form of fractions, and write the numerators on the right, and the denominators on the left of a vertical line.* (74, N. 2).

2. *Cancel equal factors, if any. Then the product of the remaining factors on the right will be the numerator, and the product of the remaining factors on the left the denominator of the required product.*

When the multiplier is a mixed number, and the multiplicand large, and an integer, we may multiply by the integer and fractional part separately, and add their products.

In like manner, multiply

7. $\frac{9}{14}$ by 12.
8. $\frac{17}{63}$ by 9.
9. $\frac{7}{75}$ by 15.
10. $\frac{45}{120}$ by 36.
11. $\frac{4}{102}$ by 17.
12. $\frac{101}{132}$ by 22.

13. 139 by $\frac{9}{13}$.
14. 290 by $\frac{5}{6}$.
15. 216 by $\frac{10}{13}$.
16. 525 by $\frac{8}{27}$.
17. 796 by $9\frac{1}{5}$.
18. 2112 by $10\frac{3}{8}$.

19. $\frac{7}{15}$ by $\frac{4}{5}$.
20. $\frac{11}{12}$ by $\frac{9}{14}$.
21. $\frac{9}{16}$ by $\frac{18}{27}$.
22. $\frac{19}{14}$ by $\frac{7}{30}$.
23. $\frac{15}{21}$ by $\frac{17}{25}$.
24. $\frac{18}{49}$ by $\frac{7}{9}$.

Find the result of

25. $\frac{3}{7} \times \frac{3}{4}$.
26. $1\frac{3}{4} \times \frac{5}{8}$.
27. $\frac{7}{10} \times 6\frac{1}{7}$.
28. $4\frac{1}{3} \times 1\frac{2}{3}$.
29. $10 - \frac{4}{5}$ of $3\frac{1}{4}$.
30. $6\frac{7}{8} + 2\frac{1}{2} \times \frac{5}{6}$.
31. $\frac{15}{16} - \frac{1}{3}$ of $2\frac{3}{4}$.
32. $\frac{11}{12} \times \frac{4}{5} + 8\frac{1}{7}$.
33. $2\frac{1}{4} \times \frac{5}{6} - \frac{3}{8}$ of $1\frac{3}{5}$.
34. $16 - \frac{3}{10} \times 12\frac{2}{3}$.
35. $4\frac{1}{8} + \frac{5}{4}$ of $1\frac{1}{2} - \frac{7}{16}$.
36. $\frac{18}{35}$ of $\frac{11}{9} + 2\frac{1}{3} \times 0$.

Find the cost.

37. Of $\frac{7}{8}$ of a yard of flannel, at $\$\frac{3}{8}$ a yard.
38. Of $6\frac{1}{4}$ pounds of rice, at $6\frac{1}{8}$ cents a pound.
39. Of $8\frac{3}{4}$ bushels of apples, at $37\frac{1}{2}$ cents a bushel.
40. Of $21\frac{4}{5}$ yards of calico, at $\$\frac{1}{8}$ a yard.
41. Of 15 tons of coal, at $\$7\frac{5}{8}$ a ton.
42. Of $9\frac{3}{5}$ pounds of coffee, at $28\frac{1}{2}$ cents a pound.
43. Of 56 pounds of cotton, at $18\frac{3}{4}$ cents a pound.
44. Of $\frac{2}{3}$ of $5\frac{1}{5}$ pounds of wool, at 47 cents a pound.
45. Of $\frac{4}{5}$ of 9 pounds of honey, at $\frac{1}{2}$ of $18\frac{3}{4}$ cents a pound.
46. Of 15 cords of bark, at $\$4\frac{5}{8}$ a cord.
47. Of $24\frac{3}{4}$ pounds of tea, at $\$\frac{7}{8}$ a pound.
48. Of 80 yards of cloth, at $\$4\frac{5}{16}$ a yard.
49. Of $21\frac{3}{5}$ bushels of corn, at $\$\frac{9}{16}$ a bushel.
50. Of $\frac{3}{4}$ of $5\frac{1}{2}$ tons of hay, at $\$15\frac{3}{10}$ a ton.
51. Of $18\frac{2}{3}$ barrels of crude oil, at $\$7\frac{4}{5}$ a barrel.
52. Of $\frac{1}{3}$ of $18\frac{1}{2}$ yards of silk, at $\frac{3}{4}$ of $5 a yard.
53. Of 126 pounds of beef, at $9\frac{1}{4}$ cents a pound.
54. Of $36\frac{7}{10}$ tons of railroad iron, at $\$62\frac{1}{2}$ a ton.
55. Of 35 horses, at $\$205\frac{3}{8}$ each.
56. Of $\frac{4}{5}$ of $156\frac{3}{8}$ acres of land, at $\frac{5}{6}$ of $\$54\frac{9}{10}$ an acre.
57. Of $28\frac{5}{8}$ bushels of sweet potatoes, at $\$1\frac{3}{8}$ a bushel.
58. Of $\frac{5}{8}$ of a yard of satin, at $\$\frac{13}{20}$ a yard.
59. Of $\frac{9}{40}$ of an acre of land, at $125 an acre.
60. Of $7\frac{9}{20}$ tons of middlings, at $\$26\frac{4}{5}$ a ton.

61. Paid $\$365\frac{7}{8}$ for a horse, and sold him for $\frac{4}{5}$ of what he cost. What was the loss?

62. When peaches are worth $\$\frac{7}{8}$ a basket, what are $126\frac{3}{4}$ baskets worth?

63. What is the cost of $\frac{4}{5}$ of $6\frac{1}{3}$ tons of coal, at $\frac{2}{3}$ of $\$7\frac{1}{4}$ per ton?

64. If a family consume $1\frac{1}{4}$ barrels of flour a month, how many barrels will 5 such families consume in $4\frac{9}{10}$ months?

65. Find the value of $(129-76\frac{4}{5})\times\frac{7}{12}$ of $\overline{12\frac{1}{4}-2\frac{2}{3}}+21\frac{1}{2}\times 6\frac{1}{5}$.

DIVISION.

121. In all examples of division of fractions, the divisor and dividend are either the *one* an integral, and the *other* a fractional number, or *both* are fractional.

INDUCTIVE EXERCISES.

122. 1. What is the quotient of 8 ninths divided by 4?

2. Dividing $\frac{8}{9}$ by 4 is taking what part of $\frac{8}{9}$?

3. $\frac{1}{4}$ of $\frac{8}{9}$ is what part of 1? $\frac{1}{3}$ of $\frac{6}{8}$? $\frac{1}{4}$ of $\frac{6}{7}$? $\frac{1}{5}$ of $\frac{5}{13}$?

4. Dividing by 2, 3, 4, 5, etc., is the same as multiplying by what fractions?

5. If 4 slates cost \$$\frac{8}{9}$, what is the cost of 1 slate?

ANALYSIS. $\frac{1}{4}$ of \$$\frac{8}{9}$, or \$$\frac{8 \div 4}{9}$ = \$$\frac{2}{9}$; or, \$$\frac{8}{9 \times 4}$ = \$$\frac{2}{9}$. (**102**, II.)

6. What is the quotient of $\frac{18}{21}$ divided by 2? 3? 4? 6? 9?

7. Divide $\frac{3}{8}$ by 4; $\frac{4}{7}$ by 5; $\frac{7}{9}$ by 6; $\frac{9}{10}$ by 8; $\frac{14}{25}$ by 7.

How many ways to divide a fraction by an integer, and what are they? Show that dividing the numerator of $\frac{6}{7}$ by 3, or multiplying the denominator by 3, divides the fraction. (**102**, II.)

8. Divide $\frac{8}{15}$ by 4; $\frac{10}{21}$ by 5; $\frac{3}{11}$ by 7; $\frac{5}{6}$ by 9; $\frac{2}{3}$ by 12.

9. At \$4 a yard, how many yards of silk can be bought for \$21$\frac{5}{7}$?

ANALYSIS.—As many yards as \$4 is contained times in \$21$\frac{5}{7}$, or $\frac{1}{4}$ of 21$\frac{5}{7}$ = $\frac{1}{4}$ of $\frac{152}{7}$, or $\frac{38}{7}$ = 5$\frac{3}{7}$ times.

Or, 4 is contained in 21$\frac{5}{7}$, 5 times and 1$\frac{5}{7}$ or $\frac{12}{7}$ remainder; $\frac{1}{4}$ of $\frac{12}{7}$ is $\frac{3}{7}$, which added to 5 makes 5$\frac{3}{7}$. Hence, etc.

10. Divide $\frac{5}{8}$ by $\frac{3}{5}$ of 20. $\frac{12}{17}$ by $\frac{2}{3}$ of 15; $\frac{7}{9}$ by $\frac{1}{8}$ of 40.

11. Divide 8$\frac{1}{3}$ by 5; 10$\frac{2}{7}$ by 7; 18$\frac{1}{2}$ by 12; 4$\frac{3}{4}$ by 8.

12. Divide $\frac{3}{4}$ of 21 by $\frac{2}{3}$ of 10; $\frac{7}{8}$ of 29 by $\frac{1}{9}$ of 63.

13. If a man walks 18$\frac{2}{3}$ miles in 4 hours, how far does he walk in 1 hour? In 3 hours? In 5 hours?

14. What is the quotient of 1 divided by $\frac{1}{2}$? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{5}$?

Show that dividing by $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ is the same as multiplying by 2, 3, and 4. (**102**, I, II.)

15. In 2 are how many times $\frac{1}{3}$? $\frac{2}{3}$? $\frac{3}{4}$? $\frac{4}{5}$? $\frac{1}{6}$? $\frac{5}{6}$?

16. Dividing 4 by $\frac{3}{5}$ is the same as multiplying 4 by what fraction?

17. At \$$\frac{4}{5}$ a yard, how many yards of cloth can be bought for \$6? For \$9? For \$10?

Analysis.—As many yards as \$$\frac{4}{5}$ is contained times in \$6. 6 is equal to $\frac{30}{5}$, and 4 fifths is contained in 30 fifths $7\frac{1}{2}$ times.

Or, \$$\frac{4}{5}$ is contained in \$1, $\frac{5}{4}$ times, and in \$6, 6 times $\frac{5}{4}$, or $\frac{30}{4}$, equal to $7\frac{1}{2}$ times. Hence, etc.

18. Divide 7 by $\frac{1}{2}$; 8 by $\frac{2}{3}$; 12 by $\frac{1}{4}$; 9 by $\frac{3}{4}$; 10 by $\frac{2}{5}$.

19. At \$$\frac{2}{3}$ a bushel, how many bushels of turnips can be bought for \$4? \$6? \$7? \$8? \$10?

20. How much butter at \$$\frac{3}{8}$ a pound can be bought for \$2? \$4? \$6? \$9? 8? 12?

How is an integer divided by a fraction?

What is the *reciprocal* of a fraction? (**99**, 4.)

Show that multiplying an integer by the *reciprocal* of a fraction *divides* the integer by that fraction.

21. How many times is $\frac{2}{3}$ contained in $\frac{3}{4}$ of 16?

22. What will 1 ton of coal cost, if $\frac{3}{8}$ of a ton cost \$7?

23. How many times is $\frac{5}{7}$ contained in 6? In 8? In 11?

24. How many turkeys can be bought for \$9, at \$$1\frac{1}{8}$ each?

25. How many garments can be made from 15 yards of cloth, if each garment contains $2\frac{1}{2}$ yards?

26. How many times is 2 eighths of a yard contained in 6 eighths of a yard? $\frac{3}{7}$ of a mile in $\frac{6}{7}$ of a mile?

27. At \$$\frac{3}{16}$ each, how many pineapples can be bought for \$$\frac{6}{16}$? For \$$\frac{9}{16}$? For \$$\frac{11}{16}$? For \$$\frac{15}{16}$?

How is one fraction divided by another, when they have a *common denominator*?

28. At $\$\frac{2}{5}$ a pound, how much tea can be bought for $\$\frac{3}{4}$?

ANALYSIS.—As many pounds as $\$\frac{2}{5}$ is contained times in $\$\frac{3}{4}$. $\$\frac{2}{5}=\$\frac{8}{20}$, and $\$\frac{3}{4}=\$\frac{15}{20}$; $\$\frac{8}{20}$ is contained in $\$\frac{15}{20}$, $1\frac{7}{8}$ times.

Or, multiply the dividend by the *reciprocal* of the divisor $\frac{2}{5}$.

29. How many pounds of honey, at $\$\frac{1}{4}$ a pound, can be bought for $\$\frac{5}{8}$? For $\$\frac{4}{5}$? For $\$\frac{5}{6}$? For $\$\frac{9}{10}$?

30. Divide $\frac{4}{5}$ by $\frac{2}{3}$; $\frac{3}{4}$ by $\frac{1}{6}$; $\frac{7}{8}$ by $\frac{3}{4}$; $\frac{6}{7}$ by $\frac{1}{3}$; $\frac{7}{12}$ by $\frac{1}{4}$.

How is one fraction divided by another when they have *not* a common denominator?

31. In $2\frac{1}{4}$ acres of land, how many building lots of $\frac{3}{8}$ of an acre each?

Reduce mixed numbers to improper fractions, then divide as we divide one fraction by another.

32. At $\$\frac{3}{16}$ a yard, how many yards of cambric can be bought for $\$\frac{3}{4}$? For $\$\frac{7}{8}$? For $\$\frac{1}{2}$? For $\$3\frac{1}{4}$?

33. At $\$\frac{3}{5}$ a bushel, how many bushels of onions can be bought for $\$4\frac{1}{2}$? For $\$5\frac{1}{4}$? For $\$3\frac{4}{5}$?

Divide

34. $\frac{2}{7}$ of a week by 6.
35. $2\frac{4}{5}$ miles by 7.
36. $16\frac{2}{3}$ rods by 10.
37. $\frac{3}{4}$ of $\frac{2}{3}$ of a ton by $\frac{4}{5}$.

Divide

38. $\frac{1}{2}$ of $\frac{5}{8}$ by $\frac{3}{4}$.
39. $\frac{6}{7}$ of 21 by $\frac{3}{5}$.
40. $\frac{1}{2}$ of $4\frac{5}{6}$ gallons by $\frac{7}{8}$.
41. 9 times $\frac{2}{3}$ of a day by $1\frac{1}{2}$.

Find the value of

42. $\frac{15}{16}\div 5$.
43. $\frac{7}{10}\div 8$.
44. $1\frac{11}{12}\div 6$.
45. $4\frac{3}{8}\div 7$.
46. $9\frac{4}{5}\div 5$.
47. $13\frac{4}{7}\div 4-2\frac{2}{3}$.
48. $27\frac{3}{4}\div 6+1\frac{3}{8}$.
49. $\frac{21}{25}\div 7\times 10$.
50. $1\frac{7}{20}\div 9\times\frac{5}{6}$.
51. $5\frac{3}{8}\times 4+13\frac{1}{2}\div 3$.
52. $\frac{14}{15}\div 7\times\frac{3}{5}$ of $1\frac{1}{2}$.
53. $\frac{7}{20}-\frac{1}{5}\times\frac{5}{8}\div 4$.

123. **PRINCIPLES.**—I. *A fraction is divided by an integer, either by dividing its numerator, or by multiplying its denominator by that integer.*

II. *Any integral or fractional number is divided by multiplying it by the* RECIPROCAL *of its divisor.*

WRITTEN EXERCISES.

124. In division, as in multiplication of fractions, all operations may be reduced to *one process*, only differing from multiplication by inverting the terms of the divisor (**99**, 4).

The Inductive Exercises will furnish all instruction necessary for the pupil to understand and explain all operations in the Written Exercises.

1. Divide $\frac{18}{25}$ by 6.

$\frac{18}{25} \div 6 = \frac{18 \div 6}{25} = \frac{3}{25}$.

Or, (**123, I.**)

$\frac{18}{25} \div 6 = \frac{18}{25 \times 6} = \frac{3}{25}$.

$$\begin{array}{r|l} 25 & \cancel{18}\,^{3} \\ \cancel{6} & \\ \hline 25 & 3 = \frac{3}{25}. \end{array}$$

2. Divide 6 by $\frac{18}{25}$.

$6 \div \frac{18}{25} = \frac{6 \times 25}{18} = 8\frac{1}{3}$.

Or,

$6 = \frac{150}{25}$;

$\frac{150}{25} \div \frac{18}{25} = 8\frac{1}{3}$.

$$\begin{array}{r|l} & \cancel{6} \\ ^{3}\,\cancel{18} & 25 \\ \hline 3 & 25 = 8\frac{1}{3}. \end{array}$$

3. Divide $50\frac{2}{5}$ by 8.

$50\frac{2}{5} \div 8 = \frac{252}{5 \times 8} = 6\frac{3}{10}$.

Or,

$$\begin{array}{r} 8\,)\,\underline{50\frac{2}{5}} \\ 6\frac{3}{10}. \end{array}$$

$$\begin{array}{r|l} 5 & \overset{63}{\cancel{252}} \\ ^{2}\,\cancel{8} & \\ \hline 10 & 63 \\ \hline & 6\frac{3}{10}. \end{array}$$

4. Divide $\frac{9}{10}$ by $\frac{3}{8}$.

$\frac{9}{10} \div \frac{3}{8} = \frac{9}{10} \times \frac{8}{3} = 2\frac{2}{5}$.

Or,

$\frac{9}{10} \div \frac{3}{8} = \frac{36}{40} \div \frac{15}{40} = 2\frac{2}{5}$.

$$\begin{array}{r|l} ^{5}\,\cancel{10} & \cancel{9}\,^{3} \\ \cancel{3} & \cancel{8}\,^{4} \\ \hline 5 & 12 \\ \hline & 2\frac{2}{5} \end{array}$$

5. Divide $3\frac{1}{4}$ by $\frac{4}{5}$ of $2\frac{1}{2}$.

$3\frac{1}{4} = \frac{13}{4}$;

$\frac{4}{5}$ of $2\frac{1}{2} = 2$.

$\frac{13}{4} \div \frac{2}{1} = \frac{13}{4} \times \frac{1}{2} = 1\frac{5}{8}$.

$$\begin{array}{r|l} 4 & 13 \\ ^{2}\,\cancel{4} & \cancel{5} \\ \cancel{5} & \cancel{2} \\ \hline 8 & 13 = 1\frac{5}{8}. \end{array}$$

6. Divide $3\frac{1}{2}$ by $4\frac{3}{8}$.

$3\frac{1}{2} = \frac{7}{2}$; $4\frac{3}{8} = \frac{35}{8}$;

$3\frac{1}{2} \div 4\frac{3}{8} = \frac{7}{2} \times \frac{8}{35}$

$= \frac{4}{5}$.

$$\begin{array}{r|l} \cancel{2} & \cancel{7} \\ ^{5}\,\cancel{35} & \cancel{8}\,^{4} \\ \hline 5 & 4 = \frac{4}{5}. \end{array}$$

RULE.—*Change integers and mixed numbers, if any, to the form of fractions; then multiply the dividend by the* RECIPROCAL *of the divisor* (**123, II**).

Apply cancellation when practicable.

In like manner divide

7. $\frac{36}{45}$ by 18.
8. $\frac{27}{12}$ by 10.
9. $\frac{2}{30}$ by 14.
10. 56 by $1\frac{5}{9}$.
11. $\frac{128}{135}$ by 80.
12. $1\frac{14}{31}$ by $\frac{35}{62}$.
13. $73\frac{3}{4}$ by 10.
14. $125\frac{5}{7}$ by 20.
15. $236\frac{1}{8}$ by 16.
16. $\frac{3}{10}$ by $\frac{11}{24}$.
17. $45\frac{3}{4}$ by 8.
18. 92 by $5\frac{1}{6}$.
19. $\frac{1}{4}$ of 30 by 12.
20. $\frac{4}{5}$ of 142 by 15.
21. $\frac{5}{6}$ of 8 by 20.
22. $16\frac{1}{2}$ by $13\frac{1}{5}$.
23. $\frac{9}{10}$ of 4 by $\frac{5}{6}$ of $3\frac{1}{4}$.
24. 44 by $\frac{2}{3}$ of $5\frac{1}{2} \times 7$.

25. If a man dig $29\frac{5}{16}$ rods of ditch in 14 days, how much does he dig in 1 day?

26. If $1\frac{1}{3}$ bushels of oats cost $\$\frac{5}{8}$, what costs 1 bushel?

27. If $\frac{3}{5}$ of a farm cost \$6270, what did the whole cost?

28. If $\frac{3}{7}$ of an acre of land cost \$63, what cost 1 acre?

29. If 54 horses cost $\$4622\frac{2}{5}$, what is the cost of each?

30. If 12 ploughs cost $\$124\frac{3}{4}$, what is the cost of each?

31. At $\$3\frac{3}{8}$ a cord, how many cords of wood can be bought for \$40? For \$150?

32. How many bushels of pears at $\$\frac{5}{8}$ a bushel can be bought for \$18? For \$39?

Divide

33. $\frac{5}{8}$ by $\frac{3}{4}$.
34. $\frac{9}{10}$ by $\frac{3}{5}$.
35. $\frac{3}{5}$ by $\frac{6}{7}$.
36. $\frac{4}{14}$ by $\frac{6}{7}$.
37. $\frac{12}{25}$ by $\frac{9}{10}$.
38. $\frac{7}{36}$ by $\frac{14}{27}$.
39. $\frac{20}{21}$ by $\frac{1}{3}$ of $\frac{4}{5}$.
40. $\frac{24}{65}$ by $\frac{3}{5}$ of $1\frac{5}{8}$.
41. $\frac{1}{2}$ of $\frac{3}{4}$ by $\frac{7}{8}$.
42. $\frac{4}{5}$ of $\frac{5}{8}$ by $\frac{7}{20}$.
43. $\frac{11}{15}$ by $\frac{1}{7}$ of $\frac{3}{5}$.
44. $9\frac{1}{6}$ by $\frac{1}{2}$ of 5.
45. $4\frac{6}{7}$ by $2\frac{3}{4}$.
46. $12\frac{9}{10}$ by $\frac{1}{3}$ of $6\frac{3}{4}$.
47. $\frac{3}{4}$ of $26\frac{2}{3}$ by $2\frac{6}{7}$.

48. How many pounds of tea, at $\$1\frac{1}{3}$ a pound, can be bought for $\$15\frac{1}{3}$?

49. At $\$6\frac{3}{8}$ a ton, how many tons of coal can be bought for \$160? For \$248?

50. At $\$\frac{11}{12}$ a yard, how many yards of cloth can be bought for \$9? For \$24? For \$64?

51. At $\$\frac{3}{8}$ a pound, how many pounds of butter will \$110 buy?

52. What is the value of $\frac{2\frac{4}{9}}{3\frac{2}{3}}$?

$$\frac{2\frac{4}{9}}{3\frac{2}{3}} = \frac{\frac{22}{9}}{\frac{11}{3}} = \frac{22}{9} \div \frac{11}{3};$$

and $$\frac{22}{9} \div \frac{11}{3} = \frac{\overset{2}{\cancel{22}}}{\underset{3}{\cancel{9}}} \times \frac{\cancel{3}}{\cancel{11}} = \frac{2}{3}.$$

Or,

$$\begin{array}{r|l} {}^{3}\,\cancel{9} & \cancel{22}\,{}^{2} \\ \cancel{11} & \cancel{3} \\ \hline 3 & 2 = \frac{2}{3}. \end{array}$$

The above is only an expression of *division*, and simply means, divide $2\frac{4}{9}$ by $3\frac{2}{3}$. Such expressions are sometimes called *Complex Fractions.*

Find the value of

53. $\frac{\frac{7}{8}}{\frac{3}{4}}$, or of $\frac{7}{8} \div \frac{3}{4}$.

54. $\frac{2\frac{1}{4}}{3\frac{3}{8}}$, or of $2\frac{1}{4} \div 3\frac{3}{8}$.

55. $\frac{15}{12\frac{1}{2}}$.

56. $\frac{3}{\frac{3}{5}}$.

57. $\frac{\frac{3}{5} \text{ of } \frac{11}{12}}{\frac{1}{18} \text{ of } 5\frac{1}{2}}$.

58. $\frac{\frac{3}{4} \text{ of } 2\frac{1}{3}}{\frac{9}{10}}$.

59. If a horse eat $\frac{4}{7}$ of a bushel of oats a day, in how many days will he eat $12\frac{3}{5}$ bushels?

60. At $6\frac{1}{8}$ a bushel, how many bushels of clover seed can be bought for $40\frac{3}{4}$?

61. At $\frac{1}{4}$ of $\frac{7}{8}$ a yard, how many yards of muslin can be bought for $\frac{1}{3}$ of $\frac{3}{4}$?

For further exercise upon fractional numbers, use the following, thus: take the first set of numbers, $\frac{3}{4}$ and $\frac{2}{3}$; find, in the simplest form, their *sum, difference, product,* and, lastly, the *quotient* of the first divided by the second. In like manner, treat each set in the double columns.

62. $\frac{3}{4}$, $\frac{2}{3}$.
63. $\frac{7}{8}$, $\frac{4}{5}$.
64. $\frac{5}{6}$, $\frac{4}{7}$.
65. $\frac{11}{24}$, $\frac{2}{9}$.
66. $\frac{2}{3}$, $\frac{27}{28}$.
67. $\frac{7}{16}$, $\frac{5}{6}$.
68. $\frac{14}{15}$, $\frac{7}{10}$.
69. $\frac{4}{15}$, $\frac{5}{24}$.
70. $4\frac{5}{8}$, $\frac{2}{3}$.
71. $3\frac{1}{4}$, $\frac{3}{4}$.
72. $3\frac{1}{2}$, $2\frac{1}{3}$.
73. $4\frac{4}{7}$, $2\frac{3}{4}$.
74. $\frac{5}{9}$, $2\frac{1}{2}$.
75. $5\frac{1}{3}$, 8.
76. $21\frac{3}{5}$, $\frac{9}{16}$.
77. $24\frac{3}{4}$, $\frac{7}{8}$.
78. $28\frac{5}{8}$, $1\frac{2}{5}$.
79. $62\frac{1}{2}$, $36\frac{7}{10}$.
80. $16\frac{1}{2}$, $13\frac{1}{5}$.
81. $3\frac{1}{2}$, $\frac{4}{5}$ of $2\frac{1}{2}$
82. $9\frac{1}{2}$, $\frac{1}{2}$ of 7.
83. $\frac{1}{3}$, $\frac{2}{3}$ of $4\frac{1}{2}$.
84. $\frac{2\frac{1}{4}}{3\frac{3}{5}}$, $\frac{\frac{5}{8}}{2\frac{1}{2}}$.

RELATION OF NUMBERS.

125. To find what fractional part one number is of another.

1. What part of 5 is 3?

Analysis.—Since 1 is 1/5 of 5, 3 is 3 times 1/5 or 3/5 of 5.

2. What part of 9 is 5? Of 12 is 7? Of 24 is 18?

3. Ten yards are what part of 25 yards? 8 pounds, of 20 pounds? 9 eggs, of a dozen? 10 ounces, of a pound?

4. What part of 3 is 5/8?

Analysis.—1 is 1/3 of 3, and 5/8 of 1 is 5/8 of 1/3, or 5/24 of 3; or, 5/8 ÷ 3 = 5/24.

5. What part of 9 is 1/4? Of 8 is 3/10? Of 20 is 2/3?

6. What part of 15 is 1 1/2? Of 18 is 2 1/3? Of 25 is 6 1/4?

7. What part of 4/5 is 2/3?

Analysis.—1 is 5/4 of 4/5, and 2/3 of 1 is 2/3 of 5/4, or 10/12 = 5/6. Or, 4/5 = 12/15, and 2/3 = 10/15, and the relation of 2/3 to 4/5 is the same as that of 10 to 12, 10/12 = 5/6.

8. What part of 3/4 is 1/2? Of 9/16 is 3/8? Of 5/6 is 1/6?

9. What part of 16/21 is 8/14? Of 3 1/2 is 3/4? Of 2 4/5 is 1 2/3?

Rule.—*Divide the number denoting the part by the number denoting the whole.*

126. To find a number when a fractional part of it is given.

1. 15 is 3/4 of what number?

Analysis.—15 is 3/4 of 4 times 1/3 of 15, which is 20.

2. 16 is 4/5 of what number? 2/7 of what number?

3. 31 1/5 is 4/7 of what number? 5/8 of what number?

4. 6/7 is 3/5 of what number?

Analysis.—6/7 is 3/5 of 5 times 1/3 of 6/7, which is 1 3/7.

5. 7/8 is 3/8 of what number? 3/4 of what number?

6. 1 3/8 is 1/5 of what number? 5/8 of what number?

7. 36 is $\frac{9}{8}$ of how many times 4?

Analysis.—36 is $\frac{9}{8}$ of 8 times $\frac{1}{9}$ of 36, which is 32, and 4 is contained in 32, 8 times.

8. 28 is $\frac{7}{16}$ of how many times 8? 12? 9? 16?
9. 35 is $\frac{7}{4}$ of how many times $\frac{1}{7}$ of 28? $\frac{1}{6}$ of 30?
10. $\frac{3}{8}$ of 56 is $\frac{7}{10}$ of what number?

Analysis.—$\frac{3}{8}$ of 56 is 3 times $\frac{1}{8}$ of 56, which is 21; and 21 is $\frac{7}{10}$ of 10 times $\frac{1}{7}$ of 21, which is 30.

11. $\frac{5}{9}$ of 27 is $\frac{5}{7}$ of what number? $\frac{5}{8}$ of what number?
12. $\frac{3}{8}$ of $\frac{2}{3}$ of 64 is $\frac{2}{5}$ of what number? $\frac{4}{5}$ of what number?

13. 54 is $\frac{9}{10}$ of how many times 5? 6? 8? 9? 12?
14. 45 is $\frac{5}{8}$ of how many times 8? 9? 6? 7? 11?

REVIEW.

WRITTEN EXERCISES.

127. 1. Change 21 to a fraction having 25 for its denominator; 34 to 50ths, 75 to 18ths.

2. Reduce $\frac{1}{3}$ of $\frac{3}{4}$, $\frac{3}{5}$, and $1\frac{5}{8}$ to equivalent fractions having the least common denominator.

3. The difference between two numbers is $\frac{17}{28}$, and the smaller $9\frac{4}{7}$; what is the greater?

4. The product of two numbers is $56\frac{2}{3}$, and one of the numbers is $12\frac{1}{3}$; what is the other?

5. What number divided by $\frac{2}{25}$ will give a quotient of $\frac{3}{8}$?

6. Find the value of $3 \times \frac{7}{8} \times \frac{5}{9} \times 4\frac{2}{5} - 3\frac{7}{8} \times \frac{5}{9} \times 4 \times \frac{2}{5}$.

7. Reduce the fractions $\dfrac{4}{\frac{1}{5}}$ and $\dfrac{2\frac{1}{2}}{1\frac{1}{3}}$ to their simplest form.

8. A farmer put $\frac{1}{5}$ of his corn in one bin, $\frac{1}{4}$ in another, and 77 bushels in a third. How much corn had he?

9. If $3\frac{3}{5}$ pounds of sugar cost \$.33, what will be the cost of $65\frac{1}{2}$ pounds?

10. A man owning 3/5 of an iron foundry, sold 1/3 of his share for \$540 3/4. What was the value of the foundry?

11. If 3/8 of a farm is valued at \$1729 1/2, what is the value of the whole?

12. A man having \$150, gave \$25 for a robe, and 3/5 of the remainder for a harness. What part of \$150 had he left?

13. Bought a horse for \$275, and sold him for \$160. For what part of the cost was he sold?

14. A pole stands 1/6 in the mud, 1/4 in the water, and 35 feet above the water. What is the length of the pole?

15. At \$3/8 a pound, how many pounds of coffee can be bought for \$6 1/2?

16. If from 18 3/4 yards of cloth 2 7/8 yards are cut, what part of the whole is taken?

17. If 48 is 3/8 of some number, what is 3/4 of the same number?

18. If 7 1/2 tons of hay cost \$120, how many tons can be bought for \$78?

19. Gave 6 5/8 pounds of butter, at 36 cents a pound, for 3 1/8 gallons of oil. What was the oil worth a gallon?

20. If 3/10 of an acre of land is worth \$79 1/4, what is 1 acre worth?

21. What number taken from 2 1/2 times 12 4/5 will leave 20 3/4?

22. At \$7/8 a yard, how many yards of silk can be bought for \$10 5/8?

23. What is the sum, the difference, and the product of 7/8 and its reciprocal?

24. How many pounds of tea, at \$7/8 a pound, will pay for 10 1/2 baskets of peaches, at \$9/10 a basket?

25. A can do a certain piece of work in 8 days, and B can do the same piece of work in 6 days. In what time can both together do it?

26. A horse and wagon cost \$270; the horse cost 1 1/4 times as much as the wagon. What was the cost of the wagon?

27. If $7\frac{1}{2}$ barrels of pork cost \$60, how many barrels can be bought for \$156, at the same rate?

28. What is the quotient of $1\frac{1}{3}$ divided by its reciprocal?

29. A certain sum of money is divided among 4 persons; A has $\frac{3}{7}$, B $\frac{1}{4}$, C $\frac{3}{14}$, and D has the remainder, which is \$30. What is the whole sum divided?

30. How many times can a bottle, holding $\frac{1}{4}$ of $\frac{2}{3}$ of a gallon, be filled from a demijohn containing $\frac{3}{4}$ of $1\frac{2}{3}$ gallons?

31. If 5 be added to both terms of the fraction $\frac{7}{8}$, will its value be increased or diminished?

32. If 5 be added to both terms of the fraction $\frac{9}{7}$, will its value be increased or diminished?

33. If $235\frac{1}{8}$ acres of land cost \$$4725\frac{3}{8}$, what will 628 acres cost?

34. From $100\frac{5}{8}$ tons of iron were sold $40\frac{3}{4}$ tons at \$$42\frac{1}{8}$ a ton, and the remainder at \$$40\frac{1}{4}$. What was received for all?

35. A gentleman owning $\frac{5}{8}$ of an iron foundry, sold $\frac{4}{5}$ of his share for \$$2570\frac{3}{8}$; how much was the whole foundry worth?

36. Bought 320 sheep at \$$2\frac{1}{4}$ per head; afterward bought 435 at \$$1\frac{7}{8}$ per head; then sold $\frac{3}{5}$ of the whole number at \$$1\frac{3}{4}$ per head, and the remainder at \$$2\frac{1}{8}$. Did I gain or lose, and how much?

What is the value of

37. $(\frac{2}{3}\text{ of }\frac{5}{6}\text{ of }3\frac{3}{4}+8\frac{2}{3}))\div(10\frac{1}{2}-7\frac{5}{18})$?

38. $(\frac{3}{5}\text{ of }\frac{6}{11}\div15)\times(15\div\frac{3}{4}\text{ of }\frac{4}{7})$?

39. $(12\frac{3}{8}\div\frac{11}{12}-15\frac{5}{6}\times\frac{1}{3})\times4$?

40. $(\frac{2}{5}\text{ of }8\frac{1}{3}-3\times\frac{3}{10})\times(\frac{7}{8}\div\frac{1}{4}+5\frac{7}{8})$?

41. $(\overline{\frac{3}{5}\times14}\div\frac{1}{6}\text{ of }15)\times37\frac{1}{2}$?

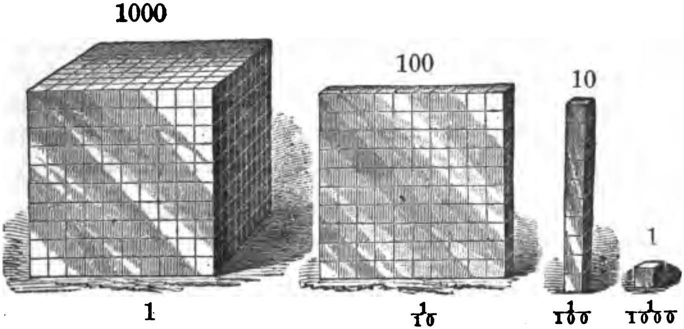
42. $\frac{5}{16}\text{ of }(2\frac{4}{7}\div6\frac{5}{8})\times(9\frac{3}{5}\div4\frac{4}{7})$?

43. $\dfrac{12\frac{1}{2}}{\frac{4}{7}}\div\dfrac{\frac{2}{3}\text{ of }\frac{3}{4}}{\frac{1}{3}}$?

44. $\dfrac{2\frac{2}{3}\text{ of }\frac{4}{7}}{6\frac{2}{7}\text{ of }\frac{3}{11}}\div\dfrac{1\frac{1}{3}}{9}$?

DECIMALS.

128. The numbers 1000, 100, 10, 1, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, etc., increase from right to left, and decrease from left to right, by the same uniform scale of *ten.* Thus, 1000 is ten times 100; 100 is 10 times 10; 10 is ten times 1; 1 is ten times $\frac{1}{10}$, etc. Also, $\frac{1}{1000}$ is one tenth of $\frac{1}{100}$; $\frac{1}{100}$ is one tenth of $\frac{1}{10}$; $\frac{1}{10}$ is one tenth of 1; 1 is one tenth of 10, etc. The value of any decimal figure is always $\frac{1}{10}$ of the value of the same figure in the next place to the left.



1. If a *unit* or 1 is divided into *ten* equal parts, what is *one* part called? 2 parts? 5 parts?

2. If 1 *tenth* of 1 is divided into *ten* equal parts, what is *one* part called? 3 parts? 7 parts? 25 parts?

3. If 1 *hundredth* of 1 is divided into *ten* equal parts, what is *one* part called? 9 parts? 75 parts?

4. What is $\frac{1}{10}$ of $\frac{1}{10}$ of 1? $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$ of 1? $\frac{2}{10}$ of $\frac{1}{100}$?

5. What is the fractional unit of 3 tenths? Of 5 hundredths? Of 25 thousandths?

NOTATION AND NUMERATION.

129. A ***decimal fraction*** or ***decimal*** is one or more of the *decimal* divisions of a unit.*

Thus, $\frac{1}{10}$, $\frac{3}{10}$, $\frac{5}{100}$, $\frac{25}{1000}$, etc., are *decimal* fractions.

The denominators of decimal fractions increase and decrease by the *uniform scale* of 10. Hence, the fractional units are always *tenths, hundredths, thousandths*, etc.

130. The ***decimal sign*** **(.)**, or *decimal point*, is used to distinguish a *decimal* from an *integer*, and is *always* placed before the numerator of the decimal, when expressed alone.

The number of figures after the decimal sign indicates the *denominator*, and determines the *value* of the decimal expression.

131. *Decimals* may be written in *two* ways, viz.: either as *common fractions*, the denominator being expressed, or in *decimal notation*, the denominator being omitted. Thus,

$\frac{5}{10}$	is written,	.5,	and is read,		5 *tenths.*
$\frac{25}{100}$	"	.25,	"	"	25 *hundredths.*
$\frac{125}{1000}$	"	.125,	"	"	125 *thousandths.*
$\frac{2436}{10000}$	"	.2436,	"	"	2436 *ten-thousandths.*

1. The **denominator** of a decimal, when expressed, is always 1 with as many ciphers annexed as there are figures in the given decimal.

Thus, $.4 = \frac{4}{10}$, $.09 = \frac{9}{100}$, $.007 = \frac{7}{1000}$. Hence,

2. The **numerator**, when expressed alone, must have as many decimal places as there are ciphers in the denominator.

Thus, $\frac{8}{10} = .8$, $\frac{16}{100} = .16$, $\frac{12}{1000} = .012$, $\frac{14}{10000} = .0014$.

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* The terms "*fraction*" and "*decimal*" will hereafter be used to distinguish the common from the decimal *form* of expression. Thus, $\frac{3}{4}$ or $\frac{75}{100}$, and .75 are two *forms* of expression for the *same thing*.

132. Express in the form of a *decimal*, and read,

1.	$\frac{7}{10}$.	5.	$\frac{4}{100}$.	9.	$\frac{20}{100}$.	13.	$\frac{3027}{10000}$.
2.	$\frac{12}{100}$.	6.	$\frac{42}{100}$.	10.	$\frac{8}{1000}$.	14.	$\frac{309}{10000}$.
3.	$\frac{9}{10}$.	7.	$\frac{125}{1000}$.	11.	$\frac{30}{1000}$.	15.	$\frac{42}{10000}$.
4.	$\frac{17}{100}$.	8.	$\frac{22}{1000}$.	12.	$\frac{206}{1000}$.	16.	$\frac{145}{100000}$.

Express in the form of a *fraction*, and read,

17.	.6.	21.	.040.	25.	.2162.	29.	.14036.
18.	.02.	22.	.024.	26.	.0056.	30.	.00402.
19.	.28.	23.	.010.	27.	.0320.	31.	.013760.
20.	.15.	24.	.005.	28.	.0008.	32.	.000342.

133. A ***mixed decimal*** consists of an integer and a decimal written together.

In reading a mixed number, the word "*and*" is always read in place of the *point*. Thus, 7.12 is read, 7 *and* 12 hundredths; 26.134 is read, 26 *and* 134 thousandths.

134. The relation of *integers* and *decimals* to each other is clearly shown in the following

TABLE.

NAMES.	Hund.-millions.	Ten-millions.	**Millions.**	Hund.-thousands.	Ten-thousands.	**Thousands.**	Hundreds.	Tens.	**UNITS.**	Tenths.	Hundredths.	**Thousandths.**	Ten-thousandths.	Hund.-thousandths.	**Millionths.**	Ten-millionths.	Hund.-millionths.
ORDERS	9th.	8th.	7th.	6th.	5th.	4th.	3d.	2d.	1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.	9th.
UNITS.	5	5	5	5	5	5	5	5	5.	5	5	5	5	5	5	5	5

INTEGERS. DECIMALS.

The number is read, 555 million 555 thousand 555, and 55 million 555 thousand 555 *hundred-millionths.*

135. As in integers, so in decimals, make the order of *units* the starting-point of notation and numeration, extending the scale to the *left* of units' place in writing *integers*, and to the *right* of units' place in writing *decimals*.

The first order to the *left* of units is *tens*, and the first order to the *right* of units is *tenths;* the second order at the left is *hundreds*, and the second order to the right is *hundredths*, and so on. The *orders* on the left and the *decimals* on the right, equally distant from units' place, correspond in name, and are *reciprocals* of each other. Thus, the 2d order on the left is 1 *ten*, and the second order on the right is 1 *tenth*, and $\frac{1}{10}$, or .1, is the *reciprocal* of 10; the 3d order on the left is 1 *hundred*, and the 3d order on the right is 1 *hundredth*, and $\frac{1}{100}$, or .01, is the *reciprocal* of 100, etc.

The value of any decimal order is always *ten times* the value of the next lower order, or 1 *tenth* the value of the next higher order.

136. In writing decimals, as in writing integers, *vacant orders* must be filled with ciphers.

Since decimals are governed by the same laws of notation as integers, it follows that,

137. Each removal of a decimal figure one place to the right, causes it to represent units of *one tenth* the value it did before, and hence *divides* the number by 10. (**17**, III.)

Thus, .03 is $\frac{1}{10}$ the value of .3; .003 is $\frac{1}{10}$ the value of .03; or, $.3 \div 10 = .03$; $.03 \div 10 = .003$.

138. Each removal of a decimal figure one place to the left, causes it to represent units of *ten times* the value it did before, and hence *multiplies* the number by 10. (**17**, II.)

Thus, .05 is 10 times .005; .5 is 10 times .05; or, $.005 \times 10 = .05$; $.05 \times 10 = .5$; $.5 \times 10 = 5$.

139. Ciphers annexed or rejected at the right of any decimal do not change its value, since they do not change the place of the significant figures of the decimal.

Thus, .6 is the same as .60, or .600, or .6000.

WRITTEN EXERCISES.

140. Express decimally:

1. Seventy-five thousandths. $\frac{75}{1000} = .075$.

2. Fifteen hundredths.
3. Seven thousandths.
4. Fifty-three thousandths.
5. Nine ten-thousandths.
6. 22 ten-thousandths.
7. 245 ten-thousandths.
8. 1042 hundred-thousandths.
9. 14605 millionths.

10. $\frac{128}{1000}$.
11. $7\frac{9}{1000}$.
12. $\frac{3906}{100000}$.
13. $127\frac{12}{100}$.
14. $\frac{572}{1000000}$.
15. $84\frac{308}{100000}$.
16. $48\frac{175}{100000}$.
17. $218\frac{3046}{10000000}$.

18. Four hundred thirty-seven thousand five hundred 49 millionths.

19. Three million forty thousand 12 ten-millionths.

20. Six hundred and 24 hundred-millionths.

21. Four hundred ninety-five million seven hundred five thousand and 43075 ten-millionths.

22. Four million seven hundred thirty-five thousand and 903624 hundred-millionths.

23. Three million forty thousand ten ten-millionths.

24. Twenty-four hundred-millionths.

25. Eight thousand six hundred forty-five hundred-thousandths.

Copy and read the following:

26. .705.
27. .0023.
28. .3607.
29. .00705.
30. .400564.
31. .000256.
32. .0010275.
33. .0000407.
34. 18.0031.
35. 6.306.
36. 49.0703.
37. 10.0064.
38. 22.09042.
39. 1.10106.
40. 14.00370.
41. 70.00063.
42. .00078.
43. .3050040.
44. .0003006.
45. 42.0637.
46. 108.0094.
47. 230.40685.
48. 30.26002015.
49. 8.040103463.

DECIMAL CURRENCY.

141. ***Currency*** is a term applied to all kinds of money in circulation, both *coin* and *paper*.

The *legal currency* of the United States is a *decimal* currency, and is sometimes called *Federal money*.

142. The **unit** of United States money is the *dollar;* and dimes, cents, and mills are respectively *tenths, hundredths,* and *thousandths* of the unit.

The legal **coin** of the United States is as follows: **gold,** the double-eagle, eagle, half-eagle, quarter-eagle, three-dollar, and one-dollar pieces; **silver,** new currency-dollar, half-dollar, quarter-dollar, and ten-cent pieces; **nickel,** the five-cent and three-cent pieces; **bronze,** one-cent.

The *trade-dollar*, used for commercial purposes, weighs 420 grains.

The *new currency-dollar* of 1878 weighs 412½ grains.

TABLE OF UNITS.				
10 Mills (*m.*)	=	1 Cent,	. .	*ct.*
10 Cents	=	1 Dime	. .	*d.*
10 Dimes	=	1 Dollar,	. .	*$.*
10 Dollars	=	1 Eagle,	. .	*E.*

EQUIVALENTS.	
1 *E.* =	10000 *m.*
	1000 *ct.*
	100 *d.*
	10 *$.*

143. In business transactions, *cents* are often written as fractions of a dollar; and the *half-cent* either as a fraction (½) or as 5 mills.

Thus, $7.42 may be written, 7\frac{42}{100}$; 87½ cents, $.87½ or $ 375.

Generally, in business calculations, if in the final result the mills are less than 5 they are not regarded; if 5 or more than 5, they are considered a *cent*.

Thus, $6.424 would be called $6.42, and $6.426 would be called $6.43. This usage will generally be adopted in the answers to examples in this book.

144. To change *cents* to *mills*, multiply by 10; *dollars* to *cents*, multiply by 100; *dollars* to *mills*, multiply by 1000, omitting the sign $, and writing *cts.*, or *m.*, after the result.

Thus, $.36 = 36 *cts.*; $8 = 800 *cts.*, or 8000 *m.*

145. To change *mills* to *cents*, divide by 10; *cents* to *dollars*, divide by 100; *mills* to *dollars*, divide by 1000, and prefix the sign $ to the result.

Thus, 740 *mills* = $.74; 645 *cents* = $6.45; 4200 *mills* = $4.20.

Since the dollar is the *unit*, of which cents are *hundredths*, and mills are *thousandths*, it follows that

146. *All the operations in* DECIMAL CURRENCY *are the same as the corresponding operations in* DECIMALS.

REDUCTION.

INDUCTIVE EXERCISES.

147. 1. How many tenths in 2 units? In 5 units?
2. How many tenths in 20 hundredths? In .40?
3. How many hundredths in 2 units? In 4 units?
4. How many hundredths in 200 thousandths?
5. How many hundredths in 5 tenths? In .6? .7? .8?
6. How many thousandths in .06? In .25? .48? .75?
7. How many hundredths in .150? In .260? In .2500?
8. In 400 thousandths how many hundredths? Tenths?
9. How many tenths of a dollar in $6? Hundredths?
10. Change 4 dollars 50 cents to cents. To mills.
11. How many dollars are 300 cents? 540 cents?
12. How many cents are 2600 mills? Dollars?
13. How are thousandths changed to hundredths? Hundredths to tenths? Tenths to units?

14. How are units changed to tenths? Tenths to hundredths? Hundredths to thousandths?

15. Express decimally 7 cents; 8 cents; 9 cents.

16. Express decimally 5 mills; 6 mills; 6 cents 6 mills; 14 cents 5 mills; 22 cents 7 mills.

17. What part of a dollar is 9 cents? 12 cents? 24 cents? 18 cents? 50 cents? 75 cents?

18. Change .5 to hundredths. To thousandths.

19. Change .4 and .05 to thousandths; .07 and .01.

20. Change .5000 to thousandths. To hundredths. To tenths.

21. What is a common denominator of .3 and .05?

22. Change .2, .04, and .005 to equivalent decimals having a common denominator. .15, .6, and .125.

How are decimals reduced to a common denominator?

23. Change .5, .08, and .023 to equivalent decimals, having a common denominator of 1000. Also, .14, .009, and .6. .7, .007, and .091.

24. How many halves in $\frac{5}{10}$? In $\frac{50}{100}$? In $\frac{500}{1000}$?

25. How many fifths in $\frac{8}{10}$? In $\frac{40}{100}$? $\frac{60}{100}$? .6?

26. How many fourths in $\frac{25}{100}$? In .50? In .75?

27. How many twentieths in $\frac{10}{100}$? In $\frac{15}{100}$? In .20?

28. In .50 how many halves? Fourths? Tenths?

29. How many fourths in .25? In .75?

30. How many eighths in .40? In .80? In 1.20?

How is a decimal changed to the form of a fraction?

31. How many tenths in $\frac{1}{2}$? How many hundredths? How many thousandths?

32. How many tenths in $\frac{1}{5}$? Hundredths in $\frac{2}{5}$? In $\frac{3}{4}$?

33. How many hundredths in $\frac{3}{20}$? In $\frac{4}{25}$? In $\frac{9}{50}$?

34. How many cents in $\frac{1}{2}$ of a dollar? In $\frac{1}{4}$? In $\frac{3}{4}$?

35. How many mills in $\frac{1}{5}$ of a dollar? In $\frac{1}{4}$? In $\frac{3}{5}$?

How is a fraction changed to the form of a decimal?

WRITTEN EXERCISES.

148. Reduce

1. \$615 to cents.
2. \$24.06 to cents.
3. \$9.206 to mills.
4. 86 cents to mills.
5. \$.763 to mills.
6. \$.47½ to mills.

Change

7. 3467 cents to dollars.
8. 10408 cents to dollars.
9. 46725 mills to dollars.
10. 30200 cents to dollars.
11. 762 mills to cents.
12. 5607 mills to dollars.
13. 3009 cents to dollars.
14. 850 mills to cents.

Change

15. .23000 to hundredths.
16. .10200 to thousandths.
17. .9000 to tenths.
18. .0700 to hundredths.

19. Reduce .7, .05, and .304 each to hundred-thousandths. **(139.)**

20. Reduce 2.5, .107, and .0008 each to ten-thousandths.

21. Change 4, 2.17, .136, and .0408 to equivalent decimals having a common denominator.

22. Change .375 to an equivalent fraction.

Explanation.—The numerator is 375, the denominator 1000, and the decimal expressed as a fraction is $\frac{375}{1000} = \frac{3}{8}$. Hence, $.375 = \frac{3}{8}$.

$$.375 = \frac{375}{1000} = \frac{3}{8}.$$

Change to equivalent fractions in lowest terms:

23. .16.
24. .125.
25. .625.
26. \$.75.
27. \$.375.
28. \$.655.
29. .024.
30. .5625.
31. .3125.
32. \$.875.
33. .0008.
34. .9375.

35. Express by an integer and common fraction 8.25.

36. Express by an integer and common fraction \$12.75; 25.005; \$36.125.

37. Reduce $\frac{2}{125}$ to an equivalent decimal.

Explanation. — Since $\frac{2}{125} = \frac{1}{125}$ of 2 units, and 2 units equal 2000 *thousandths*, $\frac{1}{125}$ of 2000 *thousandths* is 16 *thousandths*, or .016.

$$125\,\overline{)\,2.000}$$
$$.016$$

Or,

$$\frac{2}{125} = \frac{2000}{125000} = \frac{16}{1000} = .016$$

Reduce to equivalent decimals:

38. $\$\frac{3}{4}$.
39. $\$\frac{7}{8}$.
40. $\$\frac{4}{5}$.
41. $\$\frac{5}{16}$.
42. $\frac{14}{25}$.
43. $\frac{97}{160}$.
44. $\frac{1}{320}$.
45. $\frac{49}{125}$.
46. $\frac{7}{8}$ of $\frac{3}{5}$.
47. $\frac{1}{20}$ of $\frac{16}{25}$.
48. $\frac{3}{4}$ of $2\frac{1}{2}$.
49. $\$2\frac{3}{4} \times \frac{3}{125}$.
50. $\$5\frac{3}{5}$.
51. $\$12\frac{5}{8}$.
52. $\$42\frac{14}{70}$.
53. $\$36\frac{15}{16}$.

The sign + is sometimes placed after the result to indicate that there is still a remainder. Thus, $\frac{2}{3} = .666+$, or $.666\frac{2}{3}$.

ADDITION AND SUBTRACTION.

INDUCTIVE EXERCISES.

149. 1. What is the sum of 3 *tenths* and 6 *tenths*? Of $\frac{5}{10}$ and $\frac{7}{10}$? Of .6 and .4? Of .3, .4, and .5?

2. What is the sum of $\frac{8}{100}$ and $\frac{13}{100}$? Of .07 and .12?

3. What is the sum of 16 *tenths* and 9 *tenths*? The difference?

4. What is the sum of 45 *hundredths* and 3 *tenths*? The difference?

5. What is the sum of $\frac{3}{100}$ and $\frac{5}{1000}$? Of .06 and .009?

6. What is the difference between $\frac{3}{5}$ and $\frac{7}{10}$? $\frac{1}{2}$ and .8?

7. Find the sum of $\frac{1}{2}$ and 4 expressed decimally; of $\frac{1}{4}$ and 6.

8. Find the difference between 5 and $\frac{3}{5}$ expressed decimally; between \$.75 and \$.5; $\$\frac{1}{2}$ and \$.25.

9. Find the sum of \$2.5, \$3, and \$.6.

10. Which is greater, \$.35 or $\$\frac{2}{5}$? \$.7 or $\$\frac{3}{4}$? $\$\frac{4}{5}$ or \$.75?

11. Which is greater, 30 cts. or \$.3? $\$\frac{5}{20}$ or 40 cts.?

12. Find the value of $½ — 30 cents; 80 cents — $.6.
13. Find the sum of $2.5, $3, $.4, and $.15.
14. Find the value of $.75 — $.25 + .5; $½ + $.75 — $¼.

How many decimal figures in the *sum* of tenths and hundredths? Of hundredths and thousandths? Of tenths and thousandths?

How many decimal figures in the *difference* between tenths and hundredths? Between tenths and thousandths? Between hundredths and ten-thousandths?

Principle.—***Only decimals of like orders, expressing parts of like units, can be added or subtracted.***

150. Since integers and decimals increase and decrease uniformly by the scale of 10, decimals are *added, subtracted, multiplied,* and *divided* in the same manner as integers, and the methods of proof are the same.

WRITTEN EXERCISES.

151. 1. Find the sum of 2.125, 13.07, 7.8, and .3142.

2.1250	Or,	2.125
13.0700		13.07
7.8000		7.8
.3142		.3142
23.3092		23.3092

2. Find the difference between 18.5 and 2.3476.

18.5000	Or,	18.5
2.3476		2.3476
16.1524		16.1524

The arrangement of the given numbers must be such that the decimal points shall fall in the same vertical line.

The number of decimal places in the result will be equal to that one of the given numbers which contains the greatest number of places.

In like manner, find the value, expressed decimally,

3. Of 16.5 + .0348 + 7⅕ + .07½ + 1.008 + ⅘.
4. Of $.87½ + $5¾ + $86 + $.338 + $⅞ + $102.95.
5. Of $25¾ + $81.09 + $16⅛ + $.87½ + $150¾.

6. Of \$125.75 — \$41.095.
7. Of \$1.375 — \$.88.
8. Of \$.875 — \$$\frac{5}{8}$.
9. Of \$.75 — \$$\frac{11}{16}$.
10. Of 578 tenths — 7.36.
11. Of 7.005 — .7005.
12. Of 4 — 24 millionths.
13. Of 10 — 10 thousandths.
14. Of $1\frac{7}{8}$—1875 millionths.
15. Of \$600 — 600 cents.

16. Bought a farm for \$2560.50; paid at one time \$1046 and at another time \$807.87. What remains unpaid?

17. Bought groceries to the amount of \$7.18; how much change must I receive for two five-dollar bills?

18. Bought a ton of coal for \$$7\frac{3}{8}$, a barrel of sugar for \$$28\frac{7}{10}$, a chest of tea for \$23.08, and a barrel of flour for \$$10.87\frac{1}{2}$. What was the cost of all?

19. A man bought an overcoat for \$$36\frac{3}{4}$, a sack for \$$18\frac{3}{8}$, and pants for \$$8.12\frac{1}{2}$, and gave in payment one fifty and two ten-dollar bills. What change should he receive?

20. A coal dealer bought 570.5 tons of coal, and sold at different times 80.54 tons, $120\frac{1}{8}$ tons, and $114\frac{3}{8}$ tons. How much had he left?

21. Deposited in the bank \$2000, and drew out at one time \$$250\frac{1}{4}$, at another \$$322.62\frac{1}{2}$; how much remains in the bank?

22. From a piece of cloth containing 56.25 yards, $24\frac{5}{8}$ yards were cut. How many yards remained?

23. Find the sum of 45 units, 25 tenths, 360 hundredths, 75 thousandths, 52 ten-thousandths, and 406 millionths.

24. A speculator having 7346 acres of land, sold at different times $364\frac{1}{2}$ acres, 1235.125 acres, $2700\frac{3}{8}$ acres, and 850.65 acres How much had he left?

25. A grocer has $2\frac{1}{2}$ barrels of A sugar, $5\frac{3}{4}$ barrels of B sugar, $3\frac{5}{8}$ barrels of C sugar, 3.0642 barrels of crushed sugar, and 8.925 barrels of pulverized sugar; how many barrels of sugar has he?

26. A man having \$14725, gave \$3560 for a store, and \$$7015.87\frac{1}{2}$ for goods. How much money had he left?

27. Bought a ton of coal for \$6.08, a barrel of sugar for \$26.625, a box of tea for \$16, and a barrel of flour for \$7.40. How much less than \$$75\frac{7}{8}$ was the cost of all?

28. A man bought a farm for \$6736.75, which was \$$325\frac{3}{8}$ less than he sold it for. What did he sell it for?

29. How many rods of fence will enclose a field, the sides of which are respectively 34.72 rods, $48\frac{1}{25}$ rods, 152.17 rods, $95\frac{3}{8}$ rods, and $56\frac{5}{8}$ rods?

Find the *decimal* value

30. Of (\$$.87\frac{1}{2}$ + \$$14\frac{3}{8}$) — (\$5.10 + \$.75).
31. Of (155.006 — .32) + (80.0032 + 55.1).
32. Of \$70 — (\$25.4 — \$$10.12\frac{1}{2}$) + \$$7\frac{3}{5}$.
33. Of $.37\frac{1}{2} + \frac{7}{8} + 4.2 - (2 - .68\frac{3}{4})$.
34. Of \$250 — (\$$170\frac{20}{100}$ — \$$14\frac{1}{4}$) + \$$\frac{19}{20}$.
35. Of \$$48\frac{15}{100}$ + \$.97 — (\$$\frac{4}{5}$ + \$$.62\frac{1}{2}$ + \$$\frac{5}{8}$).

MULTIPLICATION.

INDUCTIVE EXERCISES.

152. 1. What is 5 times $\frac{1}{10}$? 6 times .3? 4 times .5?
2. What is 7 times $\frac{4}{100}$? 5 times .08? 6 times .09?
3. What is $\frac{7}{10} \times 3$? $3 \times .7$? $4 \times .6$? $.5 \times 7$?
4. What is $\frac{3}{100} \times 5$? $5 \times .04$? $.05 \times 7$? $8 \times .06$?
5. What is $\frac{5}{10} \times \frac{4}{10}$? $.4 \times .3$? $.8 \times .7$? $.6 \times .9$?
6. What is $\frac{5}{100} \times \frac{5}{10}$? $.5 \times .05$? $.12 \times .6$? $.7 \times .11$?
7. What is $\frac{6}{100} \times \frac{7}{100}$? $.03 \times .07$? $.15 \times .06$?
8. What is 8 times \$.6? 7 times $\frac{8}{100}$ of a dollar?
9. What is $8 \times .5$? $8 \times .05$? $8 \times .005$? $8 \times .0005$?

How many decimal places in the product of *units* multiplied by *tenths?* *Tenths* by *tenths?* *Tenths* by *hundredths?* *Hundredths* by *hundredths?*

Principle.—*The number of decimal places in any product is equal to the decimal places in both factors.*

WRITTEN EXERCISES.

153. 1. Multiply .64 by .8.

.64
.8
.512

Explanation.—Multiply as in fractions (**120**, Ex. 5). Thus, $.64 \times .8 = \frac{64}{100} \times \frac{8}{10} = \frac{512}{1000} = .512$. Or,

Multiply as in integers, and since *hundredths* multiplied by *tenths* produce *thousandths*, the product must contain *three* decimal places. (Prin.)

	(2.)	(3.)	(4.)	(5.)
Multiply	4.64	53.062	.1346	675.1
By	3.35	4.43	.203	.008

Find the product of

6. 6 times $2.45.
7. 9 times .326.
8. 26 times 4.007.
9. 15 times .0038.
10. .5 of $6.25.
11. .22 of 3.84.
12. .07 of $12.031.
13. .042 of .506.
14. $3.6 × .045.
15. .723 × 6.04.
16. 28.7 × .029.
17. .186 × $4.02.

If there are not so many figures in the product as there are decimals in both factors, supply the deficiency by prefixing ciphers.

To multiply by 10, 100, 1000, etc., remove the decimal point in the multiplicand as many places toward the *right* as *there are ciphers* in the multiplier (**138**).

Multiply, and express the product decimally:

18. $324½ by .324.
19. $175.64 by .205.
20. 5.728 by 100.
21. .6207 by 1000.
22. 5½ hundredths by 25.
23. 26000 by 26 thousandths.
24. 84 tenths by 244 hundredths.
25. 7⅝ tenths by .06¼.

Find the value

26. Of 28 × .25 × 6.
27. Of .014 × 6.2 × .007.
28. Of $37½ × .08 × ½.
29. Of $200 × 3⅜ × .006.
30. Of .304 × 100 × 10¼.
31. Of $59.36 × 2½ × .9.

What is the value

32. Of 46 mules, at $75.375 each?
33. Of 100 barrels of flour, at $9.62½ each?
34. Of 14⅛ yards of cloth, at $4½ a yard?
35. Of 125 bushels of oats, at $.625 a bushel?
36. Of 25¾ bushels of timothy seed, at $3.25?
37. Of 3000 pounds of wool, at 37½ cents?
38. Of 170 barrels of apples, at $2⅝ a barrel?
39. Of 100 cords of wood, at $4.38 a cord?
40. Of 204$\frac{7}{10}$ acres of land, at $72¾ an acre?
41. A farmer sold 125 bushels of oats at $.37½ a bushel, and received in payment 75 pounds of sugar at $.09 a pound, 12 pounds of tea at $.60 a pound, and the remainder in cash; how much cash did he receive?

DIVISION.

INDUCTIVE EXERCISES.

154. 1. What is $\frac{1}{3}$ of $\frac{6}{10}$? $\frac{1}{4}$ of $\frac{16}{100}$? $\frac{1}{5}$ of $\frac{25}{1000}$?

2. What is $\frac{1}{2}$ of .8? $\frac{1}{5}$ of .35? $\frac{1}{6}$ of .048?

3. Divide $\frac{6}{10}$ by $\frac{2}{10}$; $\frac{36}{100}$ by $\frac{9}{100}$; $\frac{48}{1000}$ by $\frac{8}{1000}$.

4. Divide .8 by 4; .8 by .4; .08 by .04; .008 by .004.

5. Divide $\frac{6}{10}$ by $\frac{12}{100}$; $\frac{25}{100}$ by $\frac{25}{1000}$; .25 by .025.

6. Divide 3.6 by 6.

Analysis.—3.6 equals 36 tenths, and $\frac{1}{6}$ of 36 tenths is 6 tenths. Hence, 3.6÷6 = .6.

7. Divide .36 by 6; .36 by .06; .036 by .006.

8. Divide $\frac{3}{10}$ by $\frac{6}{100}$; .3 by .06; 1.5 by .05.

Reduce the fractions to a common denominator.

9. Multiply $\frac{7}{10}$ by $\frac{5}{10}$; .7 by .5. Divide .35 by .7.

10. Multiply $\frac{7}{10}$ by $\frac{5}{100}$; .7 by .05. Divide .035 by .7.

Of what two factors is the dividend the product? Then how many decimal places must the dividend contain?

11. The product of two factors is .35, one of which is .5; what is the other?

12. The product of two factors is .035, one of which is .07; what is the other?

13. Divide .6 by .3; .42 by .6; .025 by .05; .0072 by .12.

14. Divide .14 by 7; .48 by .04; .0060 by .012.

How many decimal places in the quotient when *tenths* are divided by *units*? *Tenths* by *tenths*? *Hundredths* by *tenths*?

See Drill Table, page 269, **404**.

PRINCIPLE.—*The quotient must contain as many decimal places as the number of decimal places in the dividend exceeds those in the divisor.*

The number of decimal places in the dividend must first be made equal to the number in the divisor before the division is possible.

WRITTEN EXERCISES.

155. 1. Divide .952 by .7.

EXPLANATION.—Divide as in fractions (**124**, Ex. 4). Thus, $.952 \div .7 = \frac{952}{1000} \div \frac{7}{10} = \frac{952}{1000} \times \frac{10}{7} = \frac{136}{100} = 1.36$. Or,

$$\begin{array}{r|l} .7 & .952 \\ \hline & 1.36 \end{array}$$

Divide as in integers, and since the dividend contains *three* decimal places, and the divisor *one*, the quotient must have *two* decimal places. (PRIN.)

Divide
2. 81.6 by 3.6.
3. 675 by .15.
4. .952 by 4.76.

Divide
5. $41.25 by 33.
6. $518.70 by $14.25.
7. 345.15 by .075.

If the number of figures in the quotient is *less* than the excess of the decimal figures in the dividend over those in the divisor, supply the deficiency by *prefixing* ciphers.

If there is a remainder, the division may be continued, the ciphers annexed being so many decimals of the dividend.

The quotient carried to four decimal places is sufficiently exact for most business transactions.

To divide by 10, 100, 1000, etc., remove the decimal point in the dividend as many places to the *left* as *there are ciphers* in the divisor (**137, 404**).

Find the value of

8. 75÷.75.
9. .75÷75.
10. 7.5÷.75.
11. 645.5÷1000.
12. $56÷.007.
13. .56÷1.12.
14. 17.6÷10.
15. .0992÷.32.
16. 3÷18¾.
17. 17½÷1000.
18. .084÷8.
19. $68÷32.
20. 44÷.4.
21. $27÷37½.
22. 100÷.001.

Find the *price* of each,

23. If 125 bushels of oats cost $62.50.
24. If 35 pounds of sugar cost $2.80.
25. If 144 bushels of wheat cost $168.48.
26. If 100 acres of land cost $3156½.
27. If 9 turkeys cost $7.87½.
28. If 396 pounds of sugar cost $44.748.
29. If 2500 pounds of butter cost $625.
30. If 2450 pounds of pork cost $153⅛.
31. If 894 pine-apples cost $80.46.

32. How many coats can be made from 32.4 yards of cloth, allowing 2.7 yards for each coat?

33. If 125 bushels of potatoes cost $82½, how many barrels, each containing 2½ bushels, can be bought for $224.40 ?

34. A man bought 26 boxes of lemons at $4⅛ a box, and sold them at $3.90 a box. How much did he lose ?

35. At $.31¼ a bushel, how many bushels of potatoes can be bought for $9 ?

36. If 18.24 yards of cloth cost $27.36, what will 7.25 yards cost ? 9¾ yards ? 15⅜ yards ?

37. If 4¾ tons of coal cost $32.3, what will 9⅜ tons cost?

Find the value

38. Of $1.12 \times 5 \div 14$.
39. Of $\$\overline{15.5 \div 8} \times 100$.
40. Of $7.1 \times 8.2 - \overline{34\frac{3}{4} \div 2.5}$.
41. Of $25 \times .5 \times 12 + 20 \div 100$.

42. A merchant bought 14 boxes of tea for \$560; it being damaged, he was obliged to lose \$106.75 on the cost of it; how much did he receive a box?

43. A man bought 4 yards of cloth at \$3.20 a yard, and 37 pounds of sugar at \$.08 a pound; he paid \$6.80 in cash, and the remainder in butter at \$.16 a pound; how many pounds of butter did it take?

44. A farmer sold 40 bushels of barley at \$.75 a bushel 50 bushels of oats at \$$\frac{5}{8}$ a bushel, 5 tons of hay at \$$15\frac{3}{4}$ a ton, for all of which he received \$102.75 in money and 5 barrels of flour. What was the cost of the flour a barrel?

For additional exercise in decimals, treat each set of numbers in both the double columns as follows:

First, add the *four* numbers on every line of the two double columns; thus, 324 + .324 + 7.75 + 5.5 = 337.574. Then take the first set of two numbers, 324 and .324, find their *difference*, their *product*, and the quotient of the *first* divided by the *second*. All processes and results should be in *decimals*.

In like manner, treat each set of two numbers in each double column.

45.	324,	.324.	53.	7.75,	$5\frac{1}{2}$.
46.	.23,	.009.	54.	75.64,	.225.
47.	952,	4.76.	55.	723,	.0156.
48.	.003,	.0026.	56.	$87\frac{1}{2}$,	$.08\frac{4}{5}$.
49.	1.245,	.27.	57.	242.5,	$\frac{3}{4}$.
50.	$506\frac{1}{8}$,	$.048\frac{1}{2}$.	58.	41.625,	$18\frac{3}{8}$.
51.	9.6188,	3.46.	59.	218.4375,	$21\frac{9}{16}$.
52.	3.6,	.00006.	60.	2.39015,	.007.

Find the *second member* of the following equations:

61. $(1.008 \div 18 + \overline{63 \div 4000} \times 100) - \frac{4}{5} = ?$

62. $(714 - .714) \div (\overline{.34 - .034} \times .25 \text{ of } 6) = ?$

63. $(\overline{.48 \div 800} \times 10000 + \overline{6.4 \div .08}) \div .125 = ?$

64. $(\$262.64 \div \$.56) \times \overline{.0084 + .02\frac{1}{4}} \times 100 = ?$

65. $(\$1260 \times 3.49) \div \$10.47 - \$850 \div \$6.80 = ?$

SHORT METHODS.

INDUCTIVE EXERCISES.

156. 1. What will be the cost of 18 pounds of butter, at 25 cents a pound?

ANALYSIS.—Since 1 pound cost 25 cents, or $¼, 18 pounds will cost 18 times $¼, or $\$\frac{18}{4}$, equal to $4½, or $4.50. Or,

At $1 a pound, 18 pounds will cost $18, and at $¼ a pound, ¼ of $18, which is $4½, or $4.50.

2. What will 24 table books cost, at 12½ cents each? At 16⅔ cents each? At 20 cents each?

3. What will be the cost of 15 bushels of oats, at 33⅓ cents a bushel? At 50 cents a bushel?

4. At 16⅔ cents each, what will be the cost of 12 slates? Of 20 slates? Of 24 slates?

5. What will be the cost of 16 pounds of rice, at 6¼ cents a pound? At 8⅓ cents? At 10 cents?

6. How many pounds of raisins, at 16⅔ cents a pound, can be bought for $5?

ANALYSIS.—Since 16⅔ cents are $⅙, $5 will buy as many pounds of raisins as $⅙ is contained times in $5, which are 30 times.

7. At $.50 a bushel, how many bushels of oats can be bought for $15? For $16½? For $25?

8. What is the cost of 40 pairs of shoes, at $1.25 a pair?

ANALYSIS.—At $1 a pair, the cost would be $40; but since the price is $1 + $¼, the whole cost is $40 + ¼ of $40, or $50.

9. At $1.50 each, what is the cost of 48 chairs?

10. What is the cost of 60 yards of cloth, ot $1.12½ a yard? At $1.16⅔? At $1.25? At $1.33⅓? At $2.50?

11. At $2.25 a pair, what is the cost of 12 pairs of shoes? Of 16 pairs? Of 18 pairs? 20 pairs? 25 pairs?

157. ***Quantity,*** in commercial transactions, is the amount of anything bought or sold.

158. ***Price*** is the value of a *unit* of any commodity.

159. ***Cost*** is the value of the *entire quantity* bought or sold.

160. An ***aliquot part*** of a number is *one* of the *equal parts* of a number; or, it is an *exact divisor* of a number.

Thus, 3, 5, and $7\frac{1}{2}$ are *aliquot* parts of 15; 2, $2\frac{1}{2}$, $3\frac{1}{3}$, and 5, of 10.

161. ALIQUOT PARTS OF ONE DOLLAR.

5 cents	=	$\frac{1}{20}$	of \$1.	$6\frac{1}{4}$ cents	=	$\frac{1}{16}$	of \$1.
10 cents	=	$\frac{1}{10}$		$8\frac{1}{3}$ cents	=	$\frac{1}{12}$	
20 cents	=	$\frac{1}{5}$		$12\frac{1}{2}$ cents	=	$\frac{1}{8}$	
25 cents	=	$\frac{1}{4}$		$16\frac{2}{3}$ cents	=	$\frac{1}{6}$	
50 cents	=	$\frac{1}{2}$		$33\frac{1}{3}$ cents	=	$\frac{1}{3}$	

Count to 100 by each of these aliquot parts

GENERAL FORMULAS:
1. *Price × quantity = cost.*
2. *Cost ÷ quantity = price.*
3. *Cost ÷ price = quantity.*

WRITTEN EXERCISES.

162. To find the cost of a quantity, when the price is an aliquot part of one dollar.

1. What cost 951 bushels of oats, at $\$.33\frac{1}{3}$ a bushel?

EXPLANATION.—Since $33\frac{1}{3}$ cts. is $\$\frac{1}{3}$, 951 bushels cost 951 times $\$\frac{1}{3}$, or $\$\frac{951}{3}$, or \$317.

Or, the cost is $\frac{1}{3}$ as many dollars as there are bushels, and $\frac{1}{3}$ of 951 is 317.

$$\$\tfrac{1}{3} \times 951 = \$\tfrac{951}{3} = \$317.$$

Or, $951 \div 3 = 317$, or \$317.

Price × quantity = cost.

2. At 16⅔ cents per dozen, what cost 1935 dozen of eggs?

3. What cost 56480 yards of calico, at 12½ cents per yard?

4. At 20 cts. each, find the cost of 1275 salt barrels?

5. What cost 750 slates, at 33⅓ cents each? At 25 cents?

6. At $.50 each, what cost 631 shad? 1250? 1605?

7. What is the cost of 12 sacks of coffee, each sack containing 43 pounds, at 33⅓ cents a pound?

8. A merchant sold 5 pieces of prints, each containing 28 yards, at 16⅔ cents per yard, 6 pieces of sheeting, each containing 34 yards, at 8⅓ cents per yard, and received in payment 41 bushels of oats at $.50 a bushel, and the balance in money. How much money did he receive?

9. At $1.12½ a foot, what is the cost of 324 feet of wire fence?

Analysis.—At $1 a foot, the cost would be $324; but since the cost is $1 + $⅛, the entire cost is $324 + ⅛ of $324, which is $364.50.

8) 324
40.5
$364.50

10. At $1.33⅓ each, what will 642 steel shovels cost?

11. What is the cost of 288 bushels of wheat, at $1.33⅓ a bushel? At $1.25? At $1.50?

12. Find the cost of 720 yards of cloth, at $1.12½ a yard. At $1.16⅔. At $1.20. At $1.33⅓.

163. To find the quantity when the cost is given, and the price is an aliquot part of one dollar.

1. How many barrels, at $.50, can be bought for $213?

Explanation.—Since $½ will pay for 1 barrel, $213 will pay for as many barrels as $½ is contained times in $213, or 426 barrels. Or, since $1 will pay for 2 barrels, $213 will pay for 213 times 2, or 426 barrels.

$213 ÷ $½ = 426.

Or, 213 × 2 = 426.

Cost ÷ price = quantity.

2. How many baskets of pears can be bought for $318, at $.33⅓ each? At $.50 each?

3. How many pine-apples can be bought for $240, at 16⅔ cents each? At 20 cents? At 25 cents?

4. How many pounds of cheese can be bought for $350, at 6¼ cents a pound? At 8⅓? 10 cents? 12½ cents?

5. How many cocoa-nuts, at $.25, can be bought for $150.75?

6. How many yards of flannel can be bought for $450, at $.50 a yard? At $.33⅓? At $.25?

164. To find the cost, when the quantity and price of 100 or 1000 are given.

1. What cost 325 feet of boards, at $4.36 per 100 feet?

$$325 \div 100 = 3.25 \textit{ hundreds.}$$

$$\$4.36 \times 3.25 = \$14.17.$$

$$\textit{Price} \times \textit{quantity} = \textit{cost.}$$

Explanation.—Since the unit of the commodity is 100, 325 feet is equal to 3.25 hundreds.

Since the price of 1 hundred feet is $4.36, 3.25 hundreds cost 3.25 times $4.36 or $14.17.

When the price is by the *thousand*, change the quantity to thousands and a decimal, before multiplying.

Per C. or @ C. means *by the hundred;* per M. or @ M. means *by the thousand.*

2. What will 3240 feet of timber cost, at $6.20 per 100?

3. What is the cost of 12345 bricks, at $8.50 per thousand?

4. Find the cost of 2465 pounds of lead, at $.75 per 100?

5. What cost 9320 envelopes, at $3.75 per thousand?

6. What cost 5800 shingles, at $5.12 per M.?

7. What cost 6700 laths, at $.84 per C.?

8. What will be the freight on 784 pounds of castings, at $1.12 per 100 pounds?

9. What is the cost of 3200 feet of boards, at $90 per M.; 952 feet of scantling, at $11.50 per M.; and 1624 pickets, at $1.85 per C.?

10. What will be the cost of 1476 pounds of beef, at $4.37½ per hundred pounds?

11. A lumber dealer bought 212500 ft. of lumber at $14.375 per M., and retailed it out at $1.75 per C. What was his whole gain?

165. To find the cost, when the quantity and the price of a ton of 2000 pounds are given.

1. What cost 2460 pounds of hay, at $16.50 per ton?

EXPLANATION.—2460 pounds are 2.460 *thousand* pounds.

Since 2000 pounds are 1 ton, there are $\frac{1}{2}$ as many tons as there are thousand pounds, or 1.23 tons.

At $16.50 a ton, 1.23 tons will cost 1.23 times $16.50, or $20.205.

$$2.460 \div 2 = 1.230 \text{ tons.}$$

$$\$16.50 \times 1.230 = \$20.205.$$

Price × quantity = cost.

2. At $12.75 a ton, what will 450 pounds of hay cost?

3. What cost 1400 pounds of coal, at $6.80 a ton?

4. What is the freight on 3244 pounds of iron, at $1.80 per ton?

5. At $18.50 a ton, what will 1580 pounds of hay cost?

6. At $6.84 a ton, what cost 3142 pounds of plaster?

7. What cost 48 sacks of guano, each sack containing 156 pounds, at $20.25 a ton?

8. Find the value of 6340 pounds of Lehigh coal, at $7.50 a ton, and 5080 pounds of soft coal, at $6.25 a ton.

9. At $26.44 a ton, what will be the cost of 1526 pounds of bone dust?

REVIEW.

WRITTEN EXERCISES.

166. 1. Reduce .16, .625, .5625, and .0008 to the form of common fractions.

2. Change $\frac{3}{32}$, .62$\frac{1}{2}$, .37$\frac{1}{16}$, and $\frac{3}{8}$ to the form of decimals, and find their sum; .41$\frac{3}{16}$, 1.00$\frac{4}{5}$, .15$\frac{1}{4}$, and $\frac{13}{20}$.

3. Bought 27$\frac{1}{2}$ barrels of sugar for $453.75, and sold it at a profit of 4.62\frac{1}{2}$ a barrel. At what price was it sold?

4. At $2.50 a yard, how much cloth can be bought for $2?

5. At $1.37 a bushel for wheat, $.95 for rye, and $.73 a bushel for corn, how much of each, an equal number of bushels, can be bought for $70.15 ?

Find the *cost*

6. Of 14¼ yards of cloth, if 36.48 yards cost $54.72.
7. Of 5 tons of iron, if 46 tons cost $3461.50.
8. Of 7½ barrels of flour, if 4¾ barrels cost $38.
9. Of 25.42 acres of land, if .125 of an acre cost $15⅞.
10. Of 20⅛ cords of wood, if 3½ cords cost $11.37½.
11. Of 9¼ tons of coal, if .875 of a ton cost $5.635.
12. Of .8 of a pound of tea, if 1 pound cost $.62½.
13. Of 35 yards of cloth, if 29 yards cost $101½.
14. Of 3240 peach trees @ $16½ per C.
15. Of 842¾ tons of railroad iron @ $66.44 a ton.

16. Of 840 feet of plank @ $1.94 per C., and 1262 fence pickets @ $12½ per M.

17. Of 140 sacks of guano, each sack containing 162½ pounds @ $17¾ a ton.

18. Of the transportation of 18962 pounds of pork from Cincinnati to New Orleans @ $10 a ton.

19. Of 3700 cedar rails @ $5¾ per C.

20. Of freight on 3840 pounds, from New York to Baltimore, at $.96 per 100 pounds.

21. How much lumber, at $22.50 per M. can be bought for $150 ?

22. At $⅞ a yard, how much of silk can be bought for $15¾ ?

23. At $22½ an acre, how many acres of land can be bought for $4234½ ?

24. At $.42 a pound, how much wool may be bought for $80.745 ?

25. Bought 1200 bushels of corn @ $.56¼, and sold 375½ bushels of it @ $.60. At what price must the remainder be sold, to gain $168.675 on the whole ?

ACCOUNTS AND BILLS.

167. An ***account***, in commercial transactions, is a record of *debits* and *credits*.

168. A ***debtor*** is a person who owes another money, goods, or services.

169. A ***creditor*** is a person to whom money, goods, or services are due from another.

170. A ***bill*** is a written statement of money paid, of goods sold or delivered, or of services rendered. It is sometimes called an *invoice*.

An account or bill should always state the place and the time of each transaction, the names of both the parties, the price or value of each item, and the entire cost.

A *bill* is **receipted** when the words "*Received Payment*" or "*Paid*" are written at the bottom, and the creditor's name is signed either by himself, or by some authorized person.

171. The following abbreviations are in general use:

@	At.	Dft.	Draft.	Mdse.	Merchandise.
℀ or Acc't	Account.	Disc't.	Discount.	Net	Without disc't.
Am't	Amount.	Do.	The same.	No.	Number.
Bal.	Balance.	Doz.	Dozen.	Pay't	Payment.
Bbl.	Barrel.	Dr.	Debtor.	Pd.	Paid.
Bo't	Bought.	Exch.	Exchange.	Per	By.
B. L.	Bill of Lading.	Fol.	Folio.	Prem.	Premium.
¢, ct., or cts.	Cents.	Fwd.	Forward.	Rec'd	Received.
%	Per cent.	Fr't	Freight.	Sund's	Sundries.
Co.	Company.	Ins.	Insurance.	Ult.	Last month.
Cr.	Creditor.	Inst.	This month.	Yd.	Yard.
Com.	Commission.	Int.	Interest.	Yr.	Year.

The character @ is always followed by the *price of a unit.* Thus, 5 yd. of cloth @ $3.25, signifies, 5 yards of cloth *at* $3.25 *a yard;* ½ lb. of tea @ $.90, signifies, ½ a pound of tea *at* $.90 *per pound.*

172. Extend the items, and find the footings and balances of the following bills and accounts.

(1.)

SPRINGFIELD, April 10, 1884.

MR. O. M. BAKER,

To WILLIAM E. TURNER, *Dr.*

1883			
Nov. 15	For Repairing House, as per contract, . . .	$60	
" "	" 600 ft. Pine Boards, . . @ $4.50 per C.		
" 26	" 6 days work of self, . . " 3.25		
" "	" 4 days work, ... " 1.75		
" "	" Nails, Hinges, etc., ...	3	75
	Received Payment,	$	

WM. E. TURNER.

(2.)

BOSTON, June 20, 1882.

JAMES H. RICE & SON,

Bo't of ROBERTS BROS.

216 pairs Boys' Kip Boots, ...	@ $2.25		
160 " " Brogans, ...	" 1.12½		
75 " Women's Fox'd Gaiters, . . .	" 1.25		
110 " " Enameled Boots, .	" 1.37½		
6 cases Men's Calf Boots, ...	" 75.50		
1 case Drill, 648 yd., ...	" .14½		
36 gross Silk Buttons, . . . , ...	" .87½		
Received Payment,		$	

ROBERTS BROS.

(3.)

CHARLESTON, S. C., Oct. 4, 1883.

JOHN COGSWELL, *Bo't of* WALKER & EVANS.

8 bales, ea. 485 lb., Ordinary Tex. Cotton,	@ 18¼¢		
6 " " 506 " Upland, Middlings, .	" 21½¢		
215 gal. N. O. Molasses (N. Crop), . . .	" 60¢		
Rec'd Payment by draft on N. Y.		$	

WALKER & EVANS.

(4.)

New Orleans, Jan. 2, 1885.

Mr. George Soule,

To **Stevens & Seymore,** *Dr.*

1884						
Sept.	3	To 5 Blank Books, ...	@	$2.30		
"	"	" 7 gross Spencerian Pens, . . .	"	1.12½		
"	26	" 15 B. & S. Book-keeping, . . .	"	1.75		
Oct.	10	" 4 reams Cap Paper, ...	"	3.40		
"	19	" 20 Townsend's Com. Law, . .	"	2.87½		
Nov.	22	" 12 packs Plain Cards, ...	"	.37½		
Dec.	4	" Note Paper and Ink, ...			2	78
					$	

Received payment by note at 30 days,

Stevens & Seymore.

(5.)

Statement of Account.

Bismark, Nov. 10, 1884.

Mr. Asa Slayton,

In acc't with **John Mark & Co.**

1884						
March	6	To 5 doz. Scythes, ...	@	$9.		
"	"	" 16 Buckeye Ploughs, ...	"	10.37½		
April	25	" 4½ doz. Hoes, ...	"	5.48		
June	16	" 564 lb. Chain, ...	"	.16⅔		
"	"	" 6¼ doz. Steel Shovels, ...	"	11.72		
Sept.	7	" 120 sets Blind Hinges, . . .	"	.33⅓		
"	"	" 14 gross Screws, ...	"	1.62½		
		Cr.				
July	12	By 10 bbl. Flour, ...	@	$7.50		
Aug.	20	" 1470 ft. Pine Lumber, ...	"	$2 per C.		
Sept.	1	" Draft on New York, ...			200	
Oct.	26	" 146 bu. Oats, ...	@	$.40		
		Bal. due **John Mark & Co.,**			$	

Put in the form of bills and find the amount of each of the following:

6. Bought 25 pounds of sugar @ 12¢; 5 pounds of tea @ $.62½; 4 gallons of molasses @ 42¢; 46 yards of sheeting @ 9¢; 30 yards bleached shirting @ 14¢; and 4 dozen buttons @ 12¢. What was the amount of the bill?

7. Bought 7 yards broadcloth @ $3.60; 9 yards satinet @ $1.12½; 12 yards vesting @ $.90; 24 yards cassimere @ $1.37½; and 32 yards flannel @ $.65. Find the amount.

8. Bought 67 pairs of calf boots at $3.75; 108 pairs of thick boots @ $2.62; 75 pairs gaiters @ $1.12; 27 pairs buskins @ $.86; 35 pairs slippers @ $.70; and 50 pairs rubbers @ $1.04. What was the amount of the bill?

9. Sold 726 pounds butter @ 28¢; 972 pounds of cheese @ 9¢; 481½ pounds lard @ 12¢; 509¾ pounds tallow @ 16¢; 81 dozen eggs @ $.26; 15 barrels salt @ $2.40; and 963 pounds hams @ 14¢. Make out the bill for the amount.

10. Oct. 20, 1885, J. Clark bought of Hart & Co. 12 tons furnace coal @ $5.50 a ton; 7 tons stove coal @ $5.75; and 2 loads of pine wood @ $4.25. Make out Mr. C's bill.

11. The following items were sold in Troy, N. Y., by Chas. Kemp to H. B. Nims, Dec. 7, 1883: 21000 feet of pine boards @ $16 per M.; 9420 feet of plank @ $12.50 per M.; 7075 feet of scantling @ $1.70 per C.; 762 feet of timber @ $2.62½ per C.; 6840 laths @ $.87½ per C.; and 4790 fence-pickets @ $15.60 per M. Make out a receipted bill.

12. Wm. Gallup and Co. bought of H. B. Claflin & Co., New York, Sept. 14, 1882: 15 yards broadcloth @ $4.20; 24 yards satinet @ $1.37½; 10 yards vesting @ $.90; 42 yards flannel @ $.75; 60 yards drilling @ $.16; 12 yards silk @ $2.62½; and 38 yards ticking @ $.37½. Make out a receipted bill, using Bill 2 (p. 134) as a model.

MEASURES.

173. A ***measure*** is a *standard unit* established by law or custom, by which *extent, dimension, capacity, amount,* or *value* is estimated.

Thus, *length* is ascertained by applying *some unit* of linear measure; *capacity*, by applying *some unit* of volume or capacity; *weight*, by applying *some unit* of weight, etc.

MEASURES OF EXTENSION.

174. ***Extension*** is that which has one or more of the dimensions, *length, breadth,* and *thickness,* and may be a *line,* a *surface,* or a *solid.*

A **line** has only one dimension—*length.*

The **standard unit** of measures of extension, whether *linear, surface,* or *solid,* is the *yard,* which is subdivided into feet and inches.

175. ***Linear*** or ***line measures*** are used in measuring *lines* and *distances.*

Table of Units.					Equivalents.	
12	Inches (*in.*)	=	1 Foot, . . .	*ft.*	1 *mi.* =	63360 *in.*
3	Feet	=	1 Yard, . . .	*yd.*		5280 *ft.*
5½	Yards, or 16½ ft.	=	1 Rod, . . .	*rd.*		1760 *yd.*
320	Rods	=	1 Mile, . . .	*mi.*		320 *rd.*

Cloth measures are practically out of use. In measuring goods sold by the yard, the yard is divided into *halves, fourths, eighths,* and *sixteenths.*

At Custom Houses, in estimating duties, the yard is divided into *tenths* and *hundredths.*

176. ***Surface*** or ***square measures*** are used in computing *areas* or *surfaces*.

A **surface** has two dimensions—*length* and *breadth*.

Table of Units.			Equivalents.	
144 Sq. In. (*sq. in.*)	= 1 Sq. Foot,	*sq. ft.*	1 *sq. mi.* =	4014489600 *sq. in.*
9 Square Feet	= 1 Sq. Yard,	*sq. yd.*		27878400 *sq. ft.*
30¼ Square Yards	= 1 Sq. Rod,	*sq. rd.*		3097600 *sq. yd.*
160 Square Rods	= 1 Acre, . .	*A.*		102400 *sq. rd.*
640 Acres	= 1 Sq. Mile,	*sq. mi.*		640 *A.*

177. ***Cubic*** or ***solid measures*** are used in computing the *contents* of solids, also the capacity of bins, boxes, etc.

A **solid** is a body, volume, or space that has *three* dimensions—*length, breadth,* and *thickness.*

Table of Units.			Equivalents.	
1728 Cubic In. (*cu. in.*)	= 1 Cu. Foot,	*cu. ft.*	1 *cu. yd.* =	46656 *cu. in.* 27 *cu. ft.*
27 Cubic Feet	= 1 Cu. Yard,	*cu. yd.*		
16 Cubic Feet	= 1 Cord Ft.,	*cd. ft.*		
8 Cord Feet 128 Cubic Feet	= 1 Cord,	*Cd.*	1 *Cd.* =	128 *cu. ft.* 8 *cd. ft.*

ORAL EXERCISES.

178. 1. How many inches in 3 feet? In 5 feet?

2. How many feet in 36 inches? In 72 inches?

3. How many feet in 4 yd.? In 7 yd.? 10 yd.?

4. How many feet in 4 rd.? In 5 rd.? In 3 rd.?

5. How many yards in 2 rd.? In 4 rd.? In 5 rd.?

6. In 4 ft. 6 in., how many inches? In 1 yd. 2 ft.?

7. In 6 yd. 2 ft., how many feet? In 9 yd. 4 ft.?

8. How many quarters in 3 yd. 2 qr.? Eighths in 5 qr.

9. What part of a foot are 9 in.? Of a yard are 12 in.?

10. How many rods is ⅛ of a mile? ¼? ½? ¾?

11. What part of a mile are 80 rods? 32 rd.? 64 rd.?

12. At $½ a foot, what will 6 yd. 1 ft. of lead pipe cost?

13. At 4 cents a foot, what will 5 yards of wire cost?

14. At 16 cents a yard, what will 2¼ yards of ribbon cost? 3⅛ yd.? 5¾ yd.? 4⅞ yd.?

15. How many square yards in 36 sq. ft.? In 72 sq. ft.?

16. How many square rods in ½ of an acre? In ¼? In .75?

17. What part of an acre is 40 sq. rd.? 20 sq. rd.? 80 sq. rd.?

18. In 54 sq. ft., how many surface yards? In 84 sq. ft.?

19 How many acres in ½ sq. mi.? In .25? In ⅛? In .5?

20. In 2⅔ square yards, how many square feet?

21. How many cubic feet in 2 cu. yd.? In 3 cu. yd.?

22. How many cubic inches in 1 cu. ft. 20 cu. in.?

23. What part of a cubic yard are 9 cu. ft.? Are 12 cu. ft.?

24. How many cubic feet in 3 cd. ft.? In 4 cd. ft.?

25. In ¼ of a cord, how many cord feet? Cubic feet?

26. How many cord feet in 64 cu. ft.? In 96 cu. ft.? In 3½ cords? In 5⅜ Cd.?

27. What part of a cord is 6 cd. ft.? 32 cu. ft.?

28. What part of a cubic yard is 18 cu. ft.? 21 cu. ft.?

MEASURES OF CAPACITY.

179. ***Capacity*** signifies extent of room or space.

Measures of capacity are divided into two classes: *measures of liquids* and *measures of dry substances.*

The units of capacity are the *gallon* for liquid and the *bushel* for dry measure.

180. ***Liquid measures*** are used in measuring *liquids*

Table of Units.					Equivalents.	
4	Gills (*gi.*)	=	1 Pint, . . .	*pt.*	1 *gal.* =	32 *gi.*
2	Pints	=	1 Quart . . .	*qt.*		8 *pt.*
4	Quarts	=	1 Gallon . .	*gal.*		4 *qt.*

In estimating the capacity of *cisterns*, *reservoirs*, etc. :

31½ Gallons = 1 Barrel, . *bbl.*
63 Gallons = 1 Hogshead, *hhd.*

bbl. *gal.* *qt.* *pt.*
1 *hhd.* = 2 = 63 = 252 = 504.

The *barrel* and *hogshead* are not fixed measures, but vary in different States. A barrel of kerosene contains 42 gallons.

181. ***Dry measures*** are used in measuring dry substances; such as grain, seeds, fruit, etc.

TABLE OF UNITS.

2 Pints (*pt.*) = 1 Quart, . . *qt.*
8 Quarts = 1 Peck, . . *pk.*
4 Pecks = 1 Bushel, . . *bu.*

EQUIVALENTS.

1 *bu.* = 64 *pt.*, 32 *qt.*, 4 *pk.*

ORAL EXERCISES.

182. 1. How many gills in 3 pt.? In 1 qt. 1 pt.? In 3½ qt.? In ¾ qt.? In 2.5 pt.?

2. How many pints in 1 gal.? In 2.5 gal.? In 1 gal. 2 qt. 1 pt.?

3. In 2 gal. how many quarts? Pints? Gills?

4. In 32 pt. how many quarts? How many gallons?

5. What part of a quart are 6 gi.? What part of a gal.?

6. What part of 2 gal. are 4 pt.? Are 8 pt.? 2 qt.? 3 qt.?

7. At 4 cts. a pint, what will 6 qt. of milk cost?

8. If a gallon of wine cost $4, what will 1 pt. cost?

9. How many pint bottles will be required to hold 3 gal. 1 qt. of syrup? 2 gal. 3 qt. 1 pt.?

10. How many quarts in 3 pk. 6 qt.? In 1 bu. 2 pk.?

11. In 96 qt. how many pecks? How many bushels?

12. How many quart boxes will 1 bu. 2 pk. 6 qt. fill?

13. At 5 cents a pint, what is a bushel of chestnuts worth?

14. At 20 cts. a quart, what will .5 bu. of plums cost?

15. Bought a peck of peanuts for $.80, and sold them at 6 cts. a pint. What was the gain?

MEASURES OF WEIGHT.

183. ***Weight*** is the measure of gravity, and varies according to the quantity of matter a body contains.

The **unit** of weight is the *Troy pound of the Mint,* and contains 5760 grains.

184. ***Troy weights*** are used in weighing gold, silver, jewels, and in philosophical experiments.

Table of Units.				Equivalents.	
24 Grains (*gr.*)	=	1 Pennyweight, .	*pwt.*	1 *lb.* =	5760 *gr.*
20 Pennyweights	=	1 Ounce, ...	*oz.*		240 *pwt.*
12 Ounces	=	1 Pound, ...	*lb.*		12 *oz.*

185. ***Avoirdupois weights*** are used for all the ordinary purposes of weighing.

Table of Units.				Equivalents.	
16 Ounces (*oz.*)	=	1 Pound,	*lb.*	1 *T.* =	32000 *oz.*
100 Pounds	=	1 Hundredweight,	*cwt.*		2000 *lb.*
20 Cwt., or 2000 Pounds	=	1 Ton,	*T.*		20 *cwt.*

The Long Ton of 2240 lb. is seldom used, except in U. S. Custom Houses, to estimate duties.

186. The weight of the *bushel* of certain kinds of grains, seeds, and vegetables has been fixed by statute in many of the States. The following are the prevailing standards:

Wheat, . .	60 lb.	Beans, ...	60 lb.	Wheat Bran, . .	20 lb.
Rye, . . .	56 "	Buckwheat, . .	42 "	Salt, ...	56 "
Corn,. . .	56 "	Flax Seed, . .	56 "	Corn Meal, . . .	50 "
Barley, . .	48 "	Hemp Seed, . .	44 "	Corn in Ear, . .	68 "
Oats, . . .	32 "	Potatoes, . . .	60 "	Clover Seed, . .	60 "
Peas, . . .	60 "	Onions, . . .	60 "	Timothy Seed,. .	45 "

The following denominations are also in common use:

100 lb. of	Grain or Flour	= 1 Cental.	196 lb. of	Flour	= 1 Barrel.
100 "	Dry Fish	= 1 Quintal.	200 "	Beef or Pork	= 1 Barrel.
100 "	Nails	= 1 Keg.	240 "	Lime	= 1 Cask.

280 lb. of Salt at N. Y. Salt Works = 1 Barrel.

ORAL EXERCISES.

187. 1. How many grains in 3 pwt.? In 2 pwt. 12 gr.?

2. How many ounces in 16 pwt.? In 100 pwt.? In 120 pwt.? In 150 pwt.?

3. How many ounces in 5 lb.? In 3 lb. 10 oz.? In 4½ lb.?

4. What will a gold chain, weighing 2 oz. 12 pwt., cost at $1 a pennyweight?

5. What part of a pound Troy is 4 oz.? 6 oz.? 8 oz.?

6. How many spoons, each weighing 2 oz., can be made of 2 lb. 6 oz. of silver? Of 3¾ pounds?

7. How many ounces in 4 lb. Avoir.? In 5 lb. 6 oz.?

8. In 56 oz. how many pounds Avoir.? In 64 oz.? 80 oz.?

9. How many pounds in 7 cwt.? In 8½ cwt.? In 3.5 cwt.?

10. How many cwt. in 600 lb.? In 450 lb.? In 875 lb.?

11. In 2 T. how many hundredweight? How many lb.?

12. How many tons are 2500 lb.? 3000 lb.? 4000 lb.?

13. What part of a cwt. are 25 lb.? 50 lb.? 75 lb.?

14. How many hundred pounds in .25 of a ton? In ⅕?

15. At 8 cts. a pound, what will 3 cwt. of sugar cost?

16. What is the cost of a ton of hay, at 1 cent a pound?

17. How many pounds in ½ bbl. of pork? In .5 bbl. of flour? In ¼ of a keg of nails? In .25 cental of wheat?

18. What is the cost of 2 bags of wheat, each containing 120 lb., at $2 a bushel?

19. What will 20 lb. of clover-seed cost, at $10 a bushel? 30 lb.? 48 lb.? 80 lb.?

20. What is the cost of .5 of a quintal of fish, at 6 cts. a lb.?

21. At $3 a bushel, what is the cost of 30 lb. of beans?

22. If 25 lb. of cheese cost $3½, what will 2 cwt. cost?

23. How many barrels are 1200 lb. of beef? 1000 lb. of pork? 8 cwt. of beef? 5 cwt. of pork?

24. At $12 a barrel, what will a sack of flour weighing 49 pounds cost?

MEASURES OF TIME.

188. ***Time*** **is a measured portion of duration.**

The unit of time is the *mean solar day.*

TABLE OF UNITS.

60 Seconds (*sec.*) = 1 Minute, . . *min.*
60 Minutes = 1 Hour, . . . *hr.*
24 Hours = 1 Day, . . . *da.*
7 Days = 1 Week, . . . *wk.*
365 Days, or 12 Calendar Mo. = 1 Common Year, *yr.*
366 Days = 1 Leap Year, . *yr.*
100 Years = 1 Century, . . *Cen.*

EQUIVALENTS.

COMMON YEAR.

1 *yr.* = 525600 *min.*
8760 *hr.*
365 *da.*
52 *wk.*
12 *mo.*

The *months* in the year, and the number of *days* in each:

MONTHS.		NO. DAYS.	MONTHS.		NO. DAYS.
1. January,	*Jan.*,	*31.*	7. July,	*July,*	*31.*
2. February,	*Feb.*,	*28* or *29.*	8. August,	*Aug.*,	*31.*
3. March	*Mar.*,	*31.*	9. September,	*Sept.*,	*30.*
4. April,	*Apr.*,	*30.*	10. October,	*Oct.*,	*31.*
5. May,	*May,*	*31.*	11. November,	*Nov.*,	*30.*
6. June,	*June,*	*30.*	12. December,	*Dec.*,	*31.*

In most business transactions, 30 days are counted as a *month,* and 12 months a *year.*

The *civil day* begins and ends at 12 o'clock, midnight. The *astronomical day,* used by astronomers in dating events, begins and ends at 12 o'clock, noon.

When the number denoting the year is divisible by 4 and not by 100, and also when it is divisible by 400, the month of February has 29 days, and the year is called a LEAP YEAR.

Thus, 1880 is a leap year but 1883 is a common year; the year 2000 is a leap year, but 1800 and 1900 are common years.

189. In ***counting*** certain articles, the following is used:

Table of Units.			
12 Units	= 1 Dozen,	. . .	*doz.*
12 Dozen	= 1 Gross,	. . .	*gro.*
12 Gross	= 1 Great Gross,	.	*gt. gro.*
20 Units	= 1 Score,	. . .	*sc.*

Equivalents.	
1 *G. Gross* =	1728 *units.* 144 *doz.* 12 *gro.*

Two things of a kind are often called a *pair*, and six things a *set;* as a *pair* of horses, a *set* of chairs, etc.

190. Used by ***stationers*** and by the ***paper trade:***

Table of Units.			
24 Sheets	= 1 Quire,	. . .	*qr.*
20 Quires	= 1 Ream,	. . .	*rm.*
2 Reams	= 1 Bundle,	. .	*bun.*
5 Bundles	= 1 Bale,	. . .	*b.*

Equivalents.	
1 *B.* =	4800 Sheets. 200 Quires. 10 Reams. 5 Bundles.

191. A ***book*** formed of sheets folded

In 2	Leaves	is called	a Folio,	and makes	4 pp. (pages).
In 4	"	"	a Quarto, 4to,	"	8 pp.
In 8	"	"	an Octavo, 8vo,	"	16 pp.
In 12	"	"	a Duodecimo, 12mo,	"	24 pp.
In 16	"	"	a 16mo,	"	32 pp.

ORAL EXERCISES.

192. 1. How many hours in 2 da.? In 3⅔ da.? In 5⅓ da.?

2. How many hours from 6 A. M. to 5 P. M.?

3. In 4 wk. 3 da., how many days? In 5 wk. 4 da.?

4. How many minutes from 10 min. past 9 o'clock to 25 min. past 10 A. M.?

5. How many days from May 10 to July 16, inclusive?

6. How many things in 3 doz.? In 6 doz.? In 1 gro.?

7. How many dozens in 3½ gro.? In 5 gro.?

8. How many dozens in ½ of a great gross?

9. Find the cost of a gross of pens, at 10 cts. a dozen?

10. How many sheets of paper will be required to make a 12mo book of 320 pages? Of 480 pages?

DENOMINATE NUMBERS.

193. ***Denomination*** is the name of a unit of measure.

Thus, *feet* is the *denomination* of 6 ft.; *pounds*, of 25 lb., etc.

194. A ***denominate number*** is a number composed of one or more units of any denomination, and may be either *simple* or *compound*.

1. A **simple denominate number** is a number composed of but one denomination.

Thus, 7 miles, 10 acres, 5 tons are *simple denominate numbers*.

2. A **compound denominate number** is a number composed of two or more denominations of the same nature.

Thus, 10 lb. 6 oz., 5 yr. 4 mo., 4 ft. 2 in., are *compound denominate numbers*.

195. A ***denominate fraction*** is a fraction of a denominate number.

Thus, $\frac{3}{4}$ of a pound, .9 of a ton, are *denominate* fractions.

REDUCTION.

196. ***Reduction*** of denominate numbers is the process of changing their *denomination* without changing their *value*.

Denominate numbers may be changed from higher to lower, or from lower to higher denominations.

In integral numbers and in decimals, the law of increase and decrease is by the *uniform* scale of 10; but in *compound numbers* the scale *varies*.

WRITTEN EXERCISES.

197. To change a denominate number from higher to lower denominations.

1. How many inches in 26 rd. 3 yd. 2 ft. 9 in.?

Explanation.—Since 1 rod equals 5½ yards, 26 rd. 3 yd. equal 26 times 5½ yd., plus 3 yd.; 5½ yd. × 26 + 3 yd. = 146 yards.

Since 1 yard equals 3 feet, 146 yd. 2 ft. equal 146 times 3 ft., plus 2 ft.; 3 ft. × 146 + 2 ft. = 440 feet

Since 1 foot equals 12 inches, 440 ft. 9 in. equal 440 times 12 in., plus 9 in.; 12 in. × 440 + 9 in. = 5289 in. the number of *inches* in 26 rd. 3 yd. 2 ft. 9 in.

26 rd. 3 yd. 2 ft. 9 in.
5½
146 yd.
3
440 ft.
12
5289 inches.

Principle.—*A denominate number is changed to lower denominations by* MULTIPLICATION.

2. Reduce 7 lb. 10 oz. 16 pwt. 11 gr. to grains.
3. In 3 T. 6 cwt. 21 lb. 12 oz., how many ounces?
4. Change 6 wk. 5 da. 9 hr. 25 min. to minutes.

Rule.—1. *Multiply the highest denomination of the given number by that number of the scale that will reduce it to the next lower denomination, and to the product add the given number, if any, of that denomination.*

2. *Proceed in like manner with this and each successive denomination obtained, until the reduction is brought to the required denomination.*

Reduce	Change
5. 12 mi. 36 rd. 10 ft. to ft.	8. 7 T. 9 cwt. 18 lb. to lb.
6. 10 rd. 5½ ft. to inches.	9. 22 lb. 10 oz. to pwt.
7. 1 A. 15 sq. yd. to sq. ft.	10. 1 common year to min.

Change

11. 75 Cd. 6 cd. ft. to cu. ft.
12. 12 hhd. 21 gal. to pt.
13. 24 bu. 3 pk. to quarts.

Reduce

14. 5 bundles to quires.
15. 6 G. gro to dozens.
16. 31½ gal. to gills.

17. How much is 5 lb. 9 oz. 14 pwt. of gold dust worth, at $.75 a pwt.?

18. How many rods of fence will enclose a farm ¾ of a mile square?

19. If 1 barrel will hold 2 bu. 3 pk., how many barrels will be required to hold 1548 bu. 1 pk.?

20. How many boxes, each containing 12 lb., can be filled from a hogshead containing 9 cwt. 60 lb. of sugar?

21. If I buy 9 bu. of chestnuts at $4¾ a bushel, and retail them at 12½ cents a pint, what is my whole gain?

22. If it require 4 reams 10 quires of paper to print a book, how many sheets are required?

23. At 12½ cents each, what will be the cost of 2 great gross of writing books?

24. If a clock tick seconds, how many times will it tick during February, 1877?

25. How many pint, quart, and 2-quart bottles, of each an equal number, can be filled from a barrel of 31½ gallons?

26. What will be the cost of 1 hhd. molasses at $.28 per gal.?

27. How many rods of fence will inclose a farm 1½ mi. square?

28. If 14 A. of land are sold from a field containing 50 A., how many square rods does the remainder contain?

29. A man returning from Pike's Peak has 36 lb. 8 oz. of pure gold. What is its value, at $1.04 per pwt.?

30. In the eighteenth century, how many hours?

31. How large an edition of a 12mo book can be printed from 2 bales 2 bundles 15 quires of paper, allowing 8 sheets to the volume?

32. How many pages, 2 pages to each leaf, will there be in an 8vo book, containing 16 fully printed sheets?

How many pounds

33. In 36¼ centals of grain?
34. In 42⁴⁄₇ bbl. of flour?
35. In 29.5 quintals of fish?
36. In 116⅓ bbl. of salt?
37. In 63.25 kegs of nails?
38. In .75 of 75 bu. of salt?
39. In 125¾ bu. of wheat?
40. In ⅔ of 21648 bu. oats?
41. In .7 of 40 bu. corn?
42. In 7.5 casks of lime?

198. 1. Reduce ⅚ bu. and .645 da. each to integers of lower denominations.

SOLUTION.

5
4
6) 20 pk.(3 pk. 2 qt. 1⅓ pt.
18
2
8
6) 16 qt.
12
4
2
6) 8 pt.
6
2

⅚ bu. = 3 pk. 2 qt. 1⅓ pt.

SOLUTION.

.645 da.
24
2580
1290
15.480 hr.
60
28.800 min.
60
48.000 sec.

.645 da. = 15 hr. 28 min. 48 sec.

Observe, that the *solution*, *explanation*, and *rule* for the preceding examples are the same as in Art. **197.**

Reduce to integers of lower denominations:

2. ⅗ of a day.
3. ⅗ of a pound Troy.
4. $\frac{2}{10}$ of a yard.
5. ⅘ of a month.
6. ⅚ of an acre.
7. ⅕ of a week.
8. ⅞ of a rod.
9. .125 of a barrel.
10. .92 of a day.
11. .4156 of a cwt.
12. .625 of a bushel.
13. .008 of a mile.
14. .659 of a week.
15. .9 of a pound Avoir.

199. To change a denominate number from lower to higher denominations.

1. Change 5722 inches to rods.

Explanation.—Since 12 in. make 1 ft. there must be $\frac{1}{12}$ as many feet as inches; $\frac{1}{12}$ of 5722 in. is 476 and 10 in. remaining.

$$5722 \text{ in.} = \tfrac{5722}{12} \text{ ft.} = 476 \text{ ft. } 10 \text{ in.};$$
$$476 \text{ ft.} = \tfrac{476}{3} \text{ yd.} = 158 \text{ yd. } 2 \text{ ft.};$$
$$158 \text{ yd.} = \frac{158}{5\frac{1}{2}} \text{ rd.} = 28 \text{ rd. } 4 \text{ yd.}$$

Since 3 ft. make 1 yd., there must be $\frac{1}{3}$ as many yards as feet; $\frac{1}{3}$ of 476 ft. is 158 and 2 ft. remaining.

Since $5\frac{1}{2}$ yd. make 1 rd., there must be $\frac{1}{5\frac{1}{2}}$ or $\frac{2}{11}$ as many rods as yards; $\frac{2}{11}$ of 316 half yd. is 28 and 4 yd. remaining.

Hence, 5722 in. = 28 rd. 4 yd. 2 ft. 10 in.

In order to divide by $5\frac{1}{2}$, both dividend and divisor may be reduced to *halves* before dividing. In this case the remainder, if any, is *halves*, which may be reduced to integers.

Principle.—*A denominate number is changed to higher denominations by* DIVISION.

2. Reduce 6048 gills to hogsheads.
3. In 3199 pints, how many bushels?
4. In 31556926 seconds, how many days?

Rule.—1. *Divide the given denomination by that number of the scale which is equal to a unit of the next higher denomination.*

2. *In like manner, divide this and each successive quotient, until the denomination required is reached. The last quotient, with the remainders annexed, will be the required result.*

How many

5. Pounds Troy are 85894 gr.?
6. Tons are 51570 pounds?
7. Cwt. are 40607 ounces?
8. Pounds are 3000 pwt.?
9. Bu. are 12060 lb. of wheat?

Reduce

10. 120400 pens to gro.
11. 2734 eggs to dozens.
12. 5020 balls to scores.
13. 10738 sheets to reams.
14. 6048 quires to bundles.

How many

15. Yards are 1242 inches?
16. Miles are 21120 feet?
17. Yards are 325 eighths?
18. Acres are 25600 sq. rd.?
19. Sq. rd. are 346720 sq. in.?
20. Cu. yd. are 786 cu. ft.?
21. Cords are 43860 cu. ft.?
22. Cd. ft. are 165888 cu. in.?
23. Quin. are 3172 lb. of fish?

Change

24. 1236 pints to pecks.
25. 23597 gr. to lb. Troy.
26. 86400 min. to weeks.
27. 28635 sec. to hours.
28. 10800 hr. to mo. of 30 da
29. 17647 sheets to reams.
30. 14000 lb. to bu. of corn.
31. 27072 qt. to bushels.
32. 76742 pt. to barrels.

What will be the cost

33. Of 1 T. 15 cwt. 36 lb. of sugar, at 9 cents a pound?
34. Of 2 lb. 8 oz. 12 pwt. of gold dust, at $.72 a pwt.?
35. Of 3 mi. of telegraph wire, at 12 cents a foot?
36. Of 1 hhd. of wine, at 40 cents a pint?
37. Of 2 bu. 1 pk. 6 qt. grass seed, at 14 cents a quart?
38. Of 2760 lb. of wheat, at $1.50 a bushel?
39. Of 12 weeks board, at $2.62½ a day?
40. Of 1 half acre of land, at 25 cents a square foot?
41. Of 128 pints of chestnuts, at $3.50 a bushel?
42. Of 240 cord feet of wood, at $4½ a cord?
43. Of 4000 lb. of hay, at $.75 a hundredweight?
44. Of 360 quarts of milk, at 32 cents a gallon?
45. Of 16 reams of paper, at 20 cents a quire?
46. Of 5 bbl. of pork, at 9 cents a pound?
47. Of 8 gross of lead pencils, at 5 cents a piece?
48. Of 3 pecks of clover-seed, at $.12½ a lb.?
49. Of 1 ton of corn-meal, at $1.20 a bu.?
50. Of 3 T. 17 cwt. 20 lb. of hay, at $22½ a ton?

51. At 12 cents a pint, how much molasses can be bought for $8.52?

52. What is the cost of a load of oats weighing 1960 lb., at $.56 a bushel?

200. 1. What decimal of a pound Troy are 2 oz. 14 pwt.?

EXPLANATION.—Since 20 pwt. make 1 oz., there are $\frac{1}{20}$ as many ounces as pennyweights; and $\frac{1}{12}$ as many pounds as ounces (**199**). Hence, 2 oz. 14 pwt. = .225 lb., or changed to a *fraction*, by (**148**, Ex. 22), is $\frac{9}{40}$ lb.

14 pwt. = $\frac{14}{20}$ oz. or .7 oz.;
2.7 oz. = $\frac{2.7}{12}$ lb. or .225 lb.
.225 lb. = $\frac{9}{40}$ lb.

In order to find *what part* one compound number is of another, *both* must be *like numbers*, and must be reduced to the lowest denomination in *either*. Thus, 2 oz. 14 pwt. are equal to 54 pwt. and 1 lb. is equal to 240 pwt. Hence, 2 oz. 14 pwt. = $\frac{54}{240}$ lb. = $\frac{9}{40}$ lb., or, reduced to a *decimal*, by (**148**, Ex. 37), .225 lb.

2 oz. 14 pwt. = 54 pwt.
1 lb. = 240 pwt.
$\frac{54}{240} = \frac{9}{40}$ lb. = .225 lb.

1. If the given number contain a fraction, the *denominator* of this fraction must be regarded as the *lowest* denomination.

2. The pupil may be required to give the answers either in the form of a fraction, or of a decimal, or both.

What part of

2. 1 wk. is 3 da. 2 hr. 40 min.?
3. 2 lb. Troy is 7 oz. 4 pwt.?
4. 4 bu. is 1¾ pk.?
5. 1 bbl. is 15 gal. 3 qt.?
6. 1 yd. is 1 ft. 9.6 in.?
7. 3 pk. is 2 pk. 4 qt.?
8. 1 da. is 7 hr. 12 min.?
9. 2 lb. Avoir. is 10⅔ oz.?

What decimal of

10. 2 cd. is 4 cd. ft. 8 cu. ft.?
11. 3 bu. is 3 pk. 1.12 qt.?
12. 2 rd. 8 ft. is 4 yd. 1½ ft.?
13. 1 T. 5 cwt. is 25 lb.?
14. 6 gal. is 3 qt. 1 pt. 2 gi.?
15. 1 wk. 3 da. is 4 da. 9 hr.?
16. 5 yd. 1 ft. is 2 yd. 2 ft.?
17. 1 ream is 150 sheets?

18. From a hhd. of molasses 28 gal. 2 qt. were drawn. What part of the whole remained?

19. What part of 1 barrel of flour is 24½ lb.?

20. What part of 15 cwt. 21 lb. is 2 cwt. 11 lb.?

21. What part of 4 gal. 3 qt. is 3 qt. 2 gi.?

22. What part of 54 cords of wood are 4800 cu. ft.?

23. What part of 3 bbl. of flour are 110 lb. 4 oz.?

201. To find the sum of two or more denominate numbers, or denominate fractions.

Denominate numbers are *added*, *subtracted*, *multiplied*, and *divided* by the same general methods as are employed for like operations in Simple Numbers.

The corresponding processes are based upon the *same principles*. The only modification of the rules needed is that which is required by a *varying* scale instead of the *uniform* or *decimal scale* of 10.

The *principles* will be made sufficiently plain in the solutions and explanations to enable the pupil readily to construct a *rule* for each.

WRITTEN EXERCISES.

1. What is the sum of 32 bu. 2 pk. 6 qt.; 24 bu. 1 pk. 4 qt.; 16 bu. 3 pk. 7 qt.?

EXPLANATION.—Write the numbers so that units of the same denomination stand in the same column, and begin at the right to add.

The sum of the quarts is 17 qt., equal to 2 pk. 1 qt. Write the 1 qt. under the column of quarts, and add the 2 pk. to the column of pecks.

Add, in like manner, the column of pecks and bushels.

bu.	pk.	qt.
32	2	6
24	1	4
16	3	7
74	0	1

2. What is the sum of $\frac{7}{10}$ wk., $\frac{3}{5}$ da., and $\frac{3}{8}$ hr.?

EXPLANATION.—First find the value of each denominate fraction in integers of lower denominations (**198**), and then add the resulting compound numbers.

	da.	hr.	min.	sec.
$\frac{7}{10}$ wk. =	4	21	36	00
$\frac{3}{5}$ da. =		14	24	00
$\frac{3}{8}$ hr. =			22	30
	5	12	22	30

3. A miller bought 4 loads of wheat; the first contained 46 bu. 36 lb.; the second 43 bu. 50 lb.; the third 49 bu. 23 lb.; and the fourth 50 bu. 56 lb. What was the amount of the 4 loads?

4. A farmer received \$.75 a bushel for 4 loads of corn; the first contained 48.4 bu., the second 2626 lb., the third $36\frac{3}{4}$ bu., and the fourth 41 bu. 52 lb. What did he receive for the whole?

5. Bought 3 loads of hay at \$15 a ton. The first weighed 1.125 T., the second $1\frac{3}{5}$ T., and the third 2750 lb. What did the whole cost?

6. A tailor used in one year, 2 gross 5 doz. 10 buttons, another year 3 gross 7 doz. 9, and another year 4 gross 6 doz. 11; how many did he use in the three years?

7. What is the sum of $\frac{5}{8}$ of a day added to $\frac{7}{15}$ of an hour?

8. To $\frac{4}{7}$ of a hhd. add $\frac{5}{8}$ of 10 gal.

9. What is the sum of $22\frac{4}{7}$ cwt., $26\frac{7}{8}$ lb., and 14 oz.?

10. Add 5 Cd. 7 cd. ft., 2 Cd. 2 cd. ft. 12 cu. ft., 6 cd. ft. 15 cu. ft., $7\frac{3}{8}$ Cd., and 3 Cd. 2 cu. ft.

11. Add 7 yd. 2 ft., 5 yd. $1\frac{1}{4}$ ft., 2 ft. $9\frac{1}{2}$ in., 3 yd. 1 ft. $6\frac{1}{2}$ in., $2\frac{3}{4}$ ft., and $4\frac{1}{2}$ yd.

12. What is the sum of $1\frac{2}{3}$ hhd., 36 gal. 3 qt. $1\frac{1}{4}$ pt., $\frac{7}{8}$ gal., 2 qt. $\frac{3}{4}$ pt., and 1.75 pt.?

202. To find the difference between any two denominate numbers, or denominate fractions.

1. From 16 lb. 8 oz. 6 pwt. 10 gr., take 7 lb. 4 oz. 12 pwt. 6 gr.

lb.	oz.	pwt.	gr.
16	8	6	10
7	4	12	6
9	3	14	4

EXPLANATION.—Write the numbers as in addition.

Subtract 6 gr. from 10 gr., and write the difference, 4 gr., under the grains.

Since 12 pwt. cannot be subtracted from 6 pwt., take 1 oz., equal to 20 pwt., from the 8 oz., leaving 7 oz., and add it to the 6 pwt., making 26 pwt.; 12 pwt. from 26 pwt. leaves 14 pwt., which write under the pennyweights.

Since 1 oz. was taken from 8 oz., subtract 4 oz. from 7 oz., and write the difference, 3 oz., under the ounces. 7 lb. from 16 lb. leaves 9 lb., which write under the pounds. Hence, etc.

2. From $1\frac{5}{8}$ bu. subtract .8 of a bushel.

Explanation.—First find the value of each denominate fraction in integers of lower denominations (**198**); then subtract as in integers.

	bu.	pk.	qt.	pt.
$1\frac{5}{8}$ bu. =	1	2	4	0
.8 bu. =		3	1	$1\frac{1}{5}$
		3	2	$\frac{4}{5}$

(3.)

	rd.	yd.	ft.	in.
From	25	2	2	6.3
Take	12	4	0	11.6
	12	3 ($\frac{1}{2}$)	1	6.7
		$\frac{1}{2}$ =	1	6
	12	4	0	.7

(4.)

yr.	mo.	wk.	da.	hr.	min.
45	1	3	0	17	30
10	10	0	1	6	48
34	3	2	6	10	42

5. From a tub of butter containing 1 cwt. 28 lb., was taken 56 lb. 8 oz.; how much remained?

6. From a pile of wood containing 42 Cd. 5 cd. ft., take 16 Cd. 6 cd. ft. 12 cu. ft., and how much remains?

Find the result

7. Of 7 T. 5 cwt. 18 lb. 6 oz. — 2 T. 9 cwt. 12 lb. 10 oz.

8. Of 29 sq. yd. 6 sq. ft. 84 sq. in. — 16 sq. yd. 2 sq. ft. 96 sq. in.

Find the difference between

9. $8\frac{9}{10}$ cwt. and $48\frac{3}{8}$ lb.
10. $\frac{3}{40}$ lb. and 5 lb. 4 oz. 8 pwt.
11. .659 wk. and 2 wk. $3\frac{5}{6}$ da.
12. .9 da. and $\frac{1}{4}$ wk.
13. .0625 bu. and $\frac{3}{4}$ pk.
14. $5\frac{1}{2}$ bbl. and $\frac{4}{7}$ hhd.

15. If from a hhd. of molasses 14 gal. 1 qt. 1 pt. be drawn at one time, 10 gal. 3 qt. at another, and 29 gal. 1 pt. at another, how much will remain?

16. From a pile of wood containing $125\frac{3}{4}$ Cd., was sold at time 26 Cd. 7 cd. ft.; at another, 30 Cd. $4\frac{5}{8}$ cd. ft.; at ther, $37\frac{9}{16}$ Cd. How much remained?

203. To find the interval of time between two dates.

WRITTEN EXERCISES.

1. How many yr., mo., da., and hr., from 3 o'clock P. M. of May 16, 1874, to 9 o'clock A. M. of Sept. 25, 1885?

EXPLANATION.—Since the later date expresses the greater period of time, write it as the *minuend*, and the earlier date as the *subtrahend*, writing first the year, next the *number* of the month, next the *number* of the day, and next the *number* of the hour, reckoning from 12 o'clock at night.

yr.	mo.	da.	hr.
1885	9	25	9
1874	5	16	15
11	4	8	18

1. In finding the difference of time between two dates, 12 mo. are usually considered a year, and 30 days a month.

2. When the time is less than a year, to be *exact* the *true* number of days in each month and parts of a month are added.

3. The day on which a note, draft, or contract is *dated*, and that on which they *mature*, are *not both* included. The *former* is generally omitted.

Find the time

2. From June 12, 1879, to Sept. 21, 1883.

3. From Jan. 4, 1872, to Oct. 3, 1881.

4. From April 10, 9 o'clock A.M., 1873, to 2 o'clock P.M., July 15, 1883.

5. How many days did a note run, that was dated May 20, 1882, and paid Sept. 14, 1882?

SOLUTION.—In May there were 11 da. remaining, 30 in June, 31 in July, 31 in Aug., and 14 in Sept., or 117 da. in all.

6. How long has a note to run that is dated Jan. 16, 1883, and made payable July 10, 1883?

7. A note dated May 28, 1885, was paid Feb. 10, 1886. What length of time did it run?

8. The construction of the Brooklyn Suspension Bridge was commenced Jan. 3, 1870, and opened for travel May 24, 1883. How long was it in building?

204. To multiply a denominate number by an abstract number.

WRITTEN EXERCISES.

1. Multiply 12 T. 15 cwt. 27 lb. 9 oz. by 8.

12 T.	15 cwt.	27 lb.	9 oz.
			8
102 T.	2 cwt.	20 lb.	8 oz.

EXPLANATION. — Write the multiplier under the lowest denomination, and multiply as in simple numbers.

Thus, 8 times 9 oz. are 72 oz., equal to 4 lb. 8 oz. Write the 8 oz. under the ounces, and reserve the 4 lb. to add to the product of pounds.

8 times 27 lb. are 216 lb., and 4 lb. added make 220 lb., equal to 2 cwt. 20 lb. Write the 20 lb. under the pounds, and reserve the 2 cwt. to add to the product of cwt.

Proceed in the same manner until all the denominations are multiplied.

When the multiplier is large, and a *composite* number, multiply successively by its factors (**57**).

What is the result

2. Of 6 hhd. 20 gal. 3 qt. × 5? by 8? by 15?

3. Of 2 rd. 4 yd. 2 ft. 6 in. × 6? by 18?

4. Of 9 cu. yd. 15 cu. ft. 520 cu. in. × 7? by 14?

5. Of 5 hr. 42 min. 50 sec. × 12? by 36?

6. Of 26 cd. 3 cd. ft. 12 cu. ft. × 18? 24?

7. What is the weight of 1 doz. spoons, each weighing 1 oz. 12 pwt. 16 gr.?

8. What is the weight of 48 loads of hay, each weighing 1 T. 3 cwt. 50 lb.?

Find the product in integers of lower denominations of

9. .36 cwt. × 12.
10. .12 hr. × 18.
11. $\frac{5}{12}$ lb. × 24.
12. .875 hhd. × 9.
13. $\frac{4}{5}$ of .225 mi. × 8.
14. $6\frac{5}{16}$ A. × 6.
15. 3.96 in. × 36.
16. .216 gr. × 15.
17. $\frac{4}{7}$ mo. × 21.

18. At $1.37½ a gallon, what will be the cost of 5 casks of wine, each containing 28 gal. 2 qt. 1 pt.?

19. A farmer sold 4 loads of oats, averaging 41 bu. 3 pk. each, at $.75 a bushel. What did he receive for the whole?

205. To divide a denominate number into equal parts.

WRITTEN EXERCISES.

1. Divide 56 lb. 9 oz. 12 pwt. by 6.

	lb.	oz.	pwt.
6)	56	9	12
	9	5	12

EXPLANATION.—Write the divisor at the left of the dividend. The object is to find 1 *sixth* of a compound number.

⅙ of 56 lb. is 9 lb. and a remainder of 2 lb. Write the 9 lb. in the quotient, and reduce the 2 lb. to ounces, which, added to 9 oz., make 33 oz.

⅙ of 33 oz. is 5 oz. and a remainder of 3 oz. Write the 5 oz. in the quotient, and reduce the 3 oz. to pwt., which, added to 12 pwt., make 72 pwt. ⅙ of 72 pwt. is 12 pwt., which write in the quotient.

When the divisor is large, and is a *composite number*, the work may be shortened by dividing successively by its factors.

Find the result of

2. 376 gal. 3 qt. 1 pt. ÷ 9.
3. 328 yd. 1 ft. 3 in. ÷ 6.
4. 9 hhd. 28 gal. 2 qt. ÷ 12.
5. 192 bu. 3 pk. 1 qt. 1 pt. ÷ 9.
6. 45 T. 15 cwt. 25 lb. ÷ 7.
7. 196 Cd. 4 cd. ft. 12 cu. ft. ÷ 36.

8. Divide 282 bu. 3 pk. 1 qt. 1 pt. by 9; by 10; by 12.

9. Divide 254 yd. 4 ft. 3⅓ in. by 21; by 42.

10. A teamster drew 19 Cd. 2 cd. ft. 11 cu. ft. of wood in 15 loads. How much did he average per load?

11. Bought 6 spoons, which weighed 11 oz. 3 pwt. What was the weight of each spoon?

12. How many iron rails, each 18 ft. long, will be required to lay 3 miles of railroad track?

13. The total weight of 18 hhd. of sugar is 7 T. 15 cwt. 66 lb. 4 oz. What is their average weight?

14. A man traveled by railroad 1000 miles in one day. What was the average rate per hour?

15. If a town 4 mi. square is divided into 62 farms of equal size, how much land will each farm contain?

MEASUREMENTS.

206. ***Measurements*** involve a practical application of measures to the industrial and mechanic arts, and the common business of life.

DEFINITIONS.

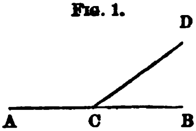
207. ***Dimensions*** are *length, breadth,* and *thickness.*

208. A ***line*** has only one dimension, *length.*

A **straight line** is the shortest distance between two points.

209. An ***Angle*** is the difference in the direction of two straight lines proceeding from a common point, called the ***vertex*** of the angle.

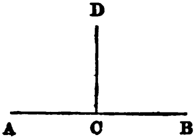
FIG. 1.



Thus, in Fig. 1, ACD and DCB are *angles,* and C is their *vertex.*

210. When two straight lines meet, so as to form *equal adjacent* angles, they are said to be perpendicular to each other, and the angles are called ***right angles.***

FIG. 2.



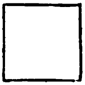
Thus, in Fig. 2, the lines AB and CD are *perpendicular* to each other, and form the two *right angles* ACD and DCB.

211. A ***surface*** has two dimensions, *length* and *breadth.*

212. A ***plane figure*** is a portion of a plane surface bounded by straight or curved lines.

213. A ***rectangle*** is a plane figure bounded by four straight lines, having four right angles.

Rectangle.



Square.

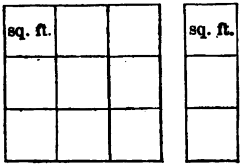
A **square** is a rectangle whose sides are equal.

214. The ***unit of measure*** for surfaces is a *square* each side of which is a unit of some known length.

Thus, the unit of square inches is 1 *square inch;* of square feet, 1 *square foot;* of square yards, 1 *square yard*, etc.

215. The ***area*** of a rectangle is the surface included within the lines which bound it, and is expressed by the *product* of the two numbers representing the two dimensions, or by the number of times it contains a given *unit of measure.*

The diagram represents a square yard, each side of which is 1 yd. or 3 ft. long, and the whole is divided into square feet, 1 sq. ft. being the *unit of measure.* In one row there are 3 sq. ft., in 3 rows there are 3 times 3 sq. ft., or 9 sq. ft. Hence the *area* of 1 sq. yd. is 9 sq. ft.



3 sq. ft. × 3=9 sq. ft.

FORMULAS FOR RECTANGULAR SURFACES.

216.
1. *Length* × *breadth* = *area.*
2. *Area* ÷ *length* = *breadth.*
3. *Area* ÷ *breadth* = *length.*

The two *given* dimensions must be expressed in units of the *same denomination.*

217. ***Artificers,*** in estimating materials and labor, make use of the following *units:*

The **square foot** for glazing, stone-cutting, flagging, and lumber.
The **square yard** for plastering, paving, ceiling, and masonry.
The **square** of 100 *square feet*, for roofing, flooring, and bricklaying.

Shingles are estimated to average 16 in. long, and 4 in. wide, and are put up 4 bundles to the 1000.

Allowing for waste, 1000 shingles, laid 4 in. to the weather, are estimated to cover a *square.*

Laths are estimated at 4 ft. in length, and are put up 10 bundles to the 1000.

WRITTEN EXERCISES.

218. 1. What is the area of a floor 27 ft. by 18 ft.?

SOLUTION.—27 × 18 = 486; hence, the *area* is 486 sq. ft. (**216**, 1.)

2. What is the width of a hall that is 36 ft. long, and contains 252 square feet?

SOLUTION.—256÷36 = 7; hence, the *width* is 7 ft. (**216**, 2.)

3. What is the length of a hall that is 8 ft. 4 in. wide, and contains 450 square feet?

4. How many tiles 8 in. square will be required to lay a floor 48 ft. by 10 ft.?

5. What will be the cost of flagging a sidewalk 312 ft. long and 6½ ft. wide, at $2.70 a square yard?

6. What will it cost to cement a cellar bottom 48 ft. 6 in. long and 27 ft. wide, at $.45 a square yard?

7. How many acres in a piece of land 60 rd. square?

8. What is the difference between the area of a floor 25 ft. square and that of two others, each 12 ft. 6 in. square?

9. What will be the cost of a piece of land 80 rd. long and 75 rd. wide, at $68½ an acre?

10. A rectangular field 120 rd. long contains 24 A. What is its width?

11. Find the cost of covering the floor of a hall 45 ft. long and 6 ft. 6 in. wide with oil-cloth, at $1.35 a square yard?

NOTE.—In buying carpeting to carpet a room, it is necessary to decide whether the strips are to run lengthwise or across the room; also, how much must be turned under or cut off, at the side or end of the room, or both, on account of the dimensions of the room, or to properly match the figures. Multiplying the number of yards in each strip by the number of strips will give the whole number of yards required.

12. How many yards of carpeting 30 inches wide will carpet a floor 16½ ft. long and 15 ft. wide, if the strips run lengthwise, and there be no loss from matching?

13. Find the cost of covering the floor of a hall 46½ ft. long and 14 ft. 9 in. wide, with matting 1¼ yd. wide, at $.25 a yard.

It will require 4 strips of matting, each 15½ yd. long.

14. How many yards of velvet carpeting, 1¼ yd. wide, will cover a parlor floor 24 ft. 9 in. long and 17½ ft. wide, if the strips run lengthwise, and the matching of the figures requires 9 in. to be turned under at one end of the room?

15. How many yards of Brussels carpeting, ¾ yd. wide, will cover the same floor, and what will be its cost, at $1.65 a yard, if the strips run lengthwise, and the matching of the figures requires 6 in. to be turned under?

16. How many squares are there in a partition 104 ft. 9 in. long, and 20 ft. 4 in. high?

17. Find the cost of plastering the sides and ceiling of a room 40 ft. long, 36½ ft. wide, and 22¼ ft. high, at $.36 a sq. yd., allowing 1375 sq. ft. for doors, windows, and baseboard.

18. How many planks 16 ft. long and 9 in. wide, will be required to floor a room 36 ft. long and 24 ft. wide?

19. Find the cost of glazing 6 windows, each 8 ft. 3 in. by 5 ft. 4 in., at $.75 a square foot.

20. How many sods, each 16 in. square, will be required to turf a yard 53 ft. 4 in. long and 28 ft. wide?

21. How many shingles averaging 4 in. wide, laid 6 in. to the weather, will cover the roof of a building 46 ft. long, each of the two sides being 20 ft. wide from the eaves to the ridge, the first course on each side being double?

22. What will be the cost of wainscoting a room 21 ft. 8 in. by 14 ft. 10 in., and 10 ft. 6 in. high, at $.30 a sq. yd.?

23. Find the cost of slating a roof 64 ft. 9 in. long and 45 ft. wide, at $15.37½ per square?

24. What is the area of a square field, the distance around which is 560 rods?

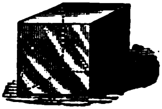
25. Find the cost of painting a roof 52 ft. long and 30 ft. wide, at $.75 a square.

SOLIDS.

219. A ***solid*** has three dimensions, *length, breadth,* and *thickness* or *height.*

220. A ***rectangular solid*** is a body bounded by six rectangular *faces.*

Rectangular Solid.



Cube.

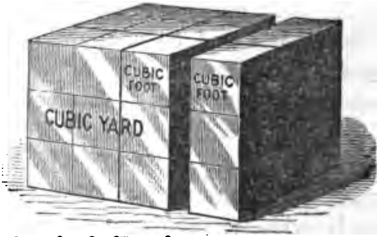
A **cube** is a rectangular body whose faces are *all equal.*

The opposite sides are equal and parallel.

221. The ***unit of measure*** for solids is a *cube,* the edge of which is a unit of some known length.

Thus, the unit of cubic inches is a *cube,* the edge of which is 1 *inch,* or 1 *cubic inch;* of cubic feet, 1 *cubic foot,* etc.

222. The ***volume*** or ***solid contents*** of a rectangular body is the space included within the surfaces which bound it, and is expressed by the *product* of the numbers representing its three dimensions, or by the number of times it contains a given *unit of measure.*



9 cu. ft. × 3=27 cu. ft.

The diagram represents a cubic yard, each face being a square yard, containing 9 sq. ft. If a section 1 ft. thick is cut from one side, it may be divided into 3 times 3 cu. ft., or 9 cu. ft., 1 cu. ft. being the *unit of measure.* And since the cubic yard is 3 ft. thick, it contains 3 such sections, or 3 times 9 cu. ft., which are 27 cu. ft. Hence the *volume* of 1 cu. yd. is 27 cu. ft.

So the volume of a *solid,* formed of two adjacent sections, is expressed by 3 cu. ft. × 3 × 2 = 18 cu. ft.

FORMULAS FOR RECTANGULAR SOLIDS.

223. 1. *Length* × *breadth* × *height* = *volume.*
2. *Volume* ÷ (*length* × *breadth*) = *height.*
3. *Volume* ÷ (*length* × *height*) = *breadth.*
4. *Volume* ÷ (*breadth* × *height*) = *length.*

The three *given* dimensions must be expressed in units of the *same denomination.*

For measures of *volume,* the following *units* are used:

The **cubic foot** for bricklaying, masonry, and hewn timber.

The **cubic yard** for embankments, excavations, and masonry.
A *cubic yard* of common earth is sometimes called a *load.*

The **perch** of stone, 16½ ft. long, 1½ ft. wide, and 1 ft. high, equal to 24¾ cu. ft. It is customary, however, to call 25 cu. ft. a perch.

Brick.—The size of a *common* brick is 8 × 4 × 2 in., and for ordinary calculation it is sufficiently accurate to reckon 27 bricks to the cubic foot, laid dry, or 22½ laid in mortar.

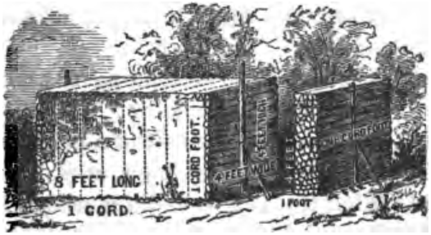
Brickwork is generally estimated by the *thousand bricks.* In estimating *material,* allowance is made for openings in walls, as doors, windows, etc.

In estimating *labor,* the length of each wall is measured on the *outside,* and thus each corner is measured twice.

Sometimes, by *special contract,* an allowance is made for *one half* the openings and corners.

A pile of wood 8 feet long, 4 ft. wide, and 4 ft. high is a **Cord.**

A cord foot is 1 foot in length of such a pile; that is, 1 ft. long, 4 ft. wide, and 4 ft. high.



WRITTEN EXERCISES.

224. 1. What is the volume of a rectangular solid 22 ft. long, 13 ft. wide, and 5 ft. thick?

SOLUTION.—$22 \times 13 \times 5 = 1430$; hence, the *volume* is 1430 cu. ft. (**223**, 1).

2. A tank $14\frac{1}{2}$ ft. long and 12 ft. wide contains 1566 cu. ft. What is its depth?

SOLUTION.—$1566 \div \overline{14\frac{1}{2} \times 12} = 9$; hence, the *depth* is 9 ft. (**223**, 2.)

3. In a solid 3 ft. 2 in. long, 2 ft. 2 in. wide, and 1 ft. 8 in. thick, how many cubic inches?

4. A vat 12 ft. square contains 1224 cu. ft. What is its depth?

5. What is the volume of a bin, the inside dimensions of which are 8 ft. 6 in. by 6 ft. by 4 ft. 4 in.?

6. How many cubic yards of earth must be removed in digging a cellar 36 ft. long, 24 ft. wide, and $6\frac{1}{2}$ ft. deep?

Find the volume of rectangular solids having the following dimensions:

7. Of a cube the edge of which is 1 yd. 1 ft. 9 in.

8. Of a solid 6 yd. 2 ft. 7 in. by 3 ft. 4 in. by 2 ft. 11 in.

9. Of a solid 5 ft. square and the height 6.4 ft.

10. My sleeping room is 10 ft. long, 9 ft. wide, and 8 ft. high. If I breathe 10 cu. ft. of air in one minute, in how long a time will I breathe as much air as the room contains?

11. How many cords in a pile of wood 30 ft. long, 8 ft. wide, and 6 ft. 6 in. high?

12. A pile of wood containing $67\frac{1}{2}$ cords, is 90 ft. long and 12 ft. wide. How high is it?

13. What will be the cost of a pile of wood 12 ft. 6 in. long, 8 ft. wide, and 4 ft. 6 in. high, at $3.75 a cord?

14. What will it cost to dig a cellar 45 ft. long, 28 ft. wide, and 8 ft. 6 in. deep, at $.42 a cubic yard?

15. How much wood is there in a pile 8 ft. long, 6 ft. wide, and 5 ft. 4 in. high?

16. How many perches of masonry in a wall 120 ft. long, 6 ft. 9 in. high, and 18 in. thick?

17. At $3½ a cord, what is the value of the wood that can be piled under a shed 50 ft. long, 25 ft. wide, and 12 ft. high?

18. A cellar wall, 32 ft. by 24 ft., is 6 ft. high and 1½ ft. thick. What did it cost, at $1.25 a perch?

19. What must be the length of a load of wood that is 3 ft. high and 5 ft. 4 in. wide, to contain a cord?

20. At $.56 a cu. yd., what will it cost to remove an embankment 240 ft. long, 38 ft. wide, and 8.5 ft. high?

21. What will it cost to build a wall 240 feet long, 6 feet high, and 3 ft. thick, at $3.25 per 1000 bricks, each brick being 8 in. long, 4 in. wide, and 2 in. thick?

22. How many cans, 6 in. by 4 in. by 3 in. can be packed in a box 24 in. by 18 in. square, in the clear?

23. Find the cost of digging and walling the cellar of a house, whose length is 41 ft. 3 in., and width 33 ft.; the cellar to be 8 ft. deep, and the wall 1½ ft. thick. The excavating will cost $.50 a cu. yd., and the stone and mason work $3.75 a perch.

24. In a school-room 32 ft. long, 18 ft. wide, and 12½ feet high, are 60 pupils, each breathing 10 cu. ft. of air in a minute. In what time will they breathe as much air as the room contains?

25. How many perches of stone will be required to enclose a lot 16 rd. long and 12 rd. wide, with a wall 6 ft. high and 3 ft. thick?

26. A street 650 ft. long and 72 ft. wide, averages 4.5 feet below grade. Find the cost of filling it in, at $.42 a cu. yd.?

27. How many cubic feet of masonry in the wall of a cellar 37½ ft. long, 26 ft. wide, and 9 ft. deep, the wall being 2 ft. thick; and what will be the cost, at $3.85 a perch?

BOARDS AND TIMBER.

225. A ***board foot*** is 1 ft. long, 1 ft. wide, and 1 inch thick. Hence, 12 board feet make 1 cubic foot.

Board feet are changed to cu. ft. by dividing by 12, and cubic feet are changed to board feet by multiplying by 12.

1. In *board measure*, all boards are assumed to be 1 in. thick; if less than 1 in., the difference is disregarded; if more than 1 in., the quantity is increased *one fourth* for every additional ¼ in. in thickness. Thus,

1200 sq. ft., 1 in. thick or less = 1200 ft. board measure.
1200 sq. ft., 1¼ in. thick = 1500 ft. " "
1200 sq. ft., 2 in. thick = 2400 ft. " "

2. *Lumber* and *sawed timber*, such as plank, scantling, joists, etc., are usually estimated in *board* measure.

3. *Hewn* and *round timber* are commonly estimated in *cubic* measure.

WRITTEN EXERCISES.

226. To find the contents of a piece of lumber.

1. Find the contents of a board 15 ft. long and 8 in. wide.

SOLUTION.—$\overline{15 \times 8} \div 12 = 10$ board feet.

2. What are the contents in board measure of a joist 16 ft. long, 10 in. wide, and 3 in. thick?

SOLUTION.—$\overline{3 \times 10 \times 16} \div 12 = 40$ board feet. Hence,

RULE.—*Multiply the length in feet by the width and thickness in inches, and divide the product by 12 for board feet, or by 144 for cubic feet.*

1. If *one* of the dimensions is *inches*, and the other *two* are *feet*, the product will be *board feet.*

2. If a board tapers regularly, multiply the length by the *mean width*, found by taking half the sum of the two ends.

Find the contents of lumber, measuring :

3. 16 ft. by 10 in.
4. 17 ft. by 11 in.
5. 22 ft. by 20 in.
6. 18 ft. by 15 in.
7. 26 ft. by 14½ in.
8. 20 ft. by 1¾ ft.
9. 12 ft. × 10 in. × 2½ in.
10. 18 ft. × 2 ft. × 8 in.
11. 16 ft. × 1 ft. × 4½ in.

12. Find the contents of a board 18 ft. long, 1 ft. 8 in. wide at one end, and 14 in. at the other.

SOLUTION:—$\overline{20\text{ in.} + 14\text{ in.}} \div 2 = 17$ in.; $\overline{18 \times 17} \div 12 = 25\frac{1}{2}$ board ft.

13. Find the cost of 5 boards 12 ft. long, 17 in. wide at one end and 11 in. at the other, at 6 cents a square foot.

14. Find the cost of 10 planks, each 15 ft. long, 16 in. wide, and 3½ in. thick, at $2.25 per hundred feet.

15. Find the cost of 3 pieces of timber, each 26 ft. long and 6 in. by 9 in., at $1.75 per hundred board feet.

16. Find the cost of 8 pieces of scantling, 3 in. by 4 in. and 14 ft. long, at $9.50 per thousand board feet.

17. What amount of inch lumber will make a box 4 ft. by 3 ft. 6 in. by 2 ft. 6 in. on the outside.

Find the cost of the following :

18. Of 36 boards, 12 ft. long, 11 in. wide, @ $2½ per C.

19. Of 16 planks, 14½ ft. long, 10 in. wide, and 3 in. thick, @ $16½ per M.

20. How many board feet in a stick of timber 36 ft. long, 10 in. thick, 12 in. wide at one end and 9 in. wide at the other end? How many cubic feet?

21. How much flooring 1½ in. thick will be required to lay the floors of a two-story house, 48 ft. by 38 ft., no allowance being made for waste; and what will be its cost, at $35 per M.?

22. A field 64 rd. by 32 rd. is enclosed by a board fence; the posts are set 8 ft. apart, the boards are 16 ft. long, and the fence is 4 boards high. The bottom board is 12 in., the top board 6 in., and the others each 9 in. wide. The posts cost $25 per C., and the boards $14.80 per M. Find the number of posts, the amount of lumber, and the cost of both.

RECTANGULAR BINS, CISTERNS, ETC.

227. *Cubic measures* are used to ascertain the *capacity* of rectangular cisterns, bins, etc., but the results are generally expressed in *units* of *liquid* and *dry* measures.

228. COMPARATIVE TABLE OF MEASURES OF CAPACITY.

	Cubic inches in 1 gal.	Cubic inches in 1 qt.	Cubic inches in 1 pt.	Cubic inches in 1 gi.
Liquid measure,	231	$57\frac{3}{4}$	$28\frac{7}{8}$	$7\frac{7}{32}$
Dry measure ($\frac{1}{8}$ pk.),	$268\frac{4}{5}$	$67\frac{1}{5}$	$33\frac{3}{5}$	$8\frac{2}{5}$

A *cubic foot* of pure water weighs 1000 oz., equals $62\frac{1}{2}$ lb. Avoir.

The *standard bushel* of the United States contains 2150.42 cu. in.

The *stricken* or *even* bushel is the unit of measure for grain, seeds, and small fruits.

The *heaped* bushel is the unit of measure for coal, lime, corn in the ear, large fruits, root crops, etc., and contains 2747.7 cu. in. Hence,

A cubic foot may be considered .8 of a bushel STRICKEN *measure, or .63 of a bushel* HEAPED *measure.*

WRITTEN EXERCISES.

229. To find the exact capacity of a vessel or space in gallons.

1. How many gallons of water will a cistern hold, that is 4 ft. square and 6 ft. 9 in. deep?

SOLUTION.—$(4 \times 4 \times 6\frac{3}{4} \times 1728) \div 231 = 807\frac{69}{77}$ gal.

2. How many cubic feet in a space that holds 48 hhd.?

SOLUTION.—$(48 \times 63 \times 231) \div 1728 = 404\frac{1}{4}$ cu. ft.

RULE.—*Divide the contents in cubic inches by 231 for* LIQUID *gallons, or by 268.8 for* DRY *gallons.*

Conversely, to reduce gallons to cubic inches:

Multiply the given number of liquid gallons by 231; then reduce to higher denominations, if required.

3. How many gallons will a tank 6 ft. long, 3 ft. 4 in. wide, and 2 ft. 6 in. deep contain?

4. How many barrels of water will a vat hold that contains 43659 cubic inches?

5. How many hogsheads will a cistern 11 ft. long, 6 ft. wide, and 7 ft. deep contain?

6. Find the weight of water in a bath-tub 6 ft. long, 3 ft. wide, and 1 ft. 9 in. deep.

7. How many gallons will a space contain that is 22.5 ft. long, 3.25 ft. wide, and 6.4 ft. deep?

8. A cellar 35 ft. long and 24 ft. wide, contains 3 ft. of water. What will it cost to pump it out, at 5 cents a hogshead?

9. A tank in the attic of a house is 4 ft. long, 3 ft. wide, and 1 ft. 8 in. deep. How many gallons of water will it hold, and what will be its weight?

10. How many cubic feet in a space that holds 52 barrels?

11. How many more cubic inches in 89.5 gal., dry measure, than in the same number of gallons liquid measure?

230. To find the exact capacity of a bin, or any rectangular space, in bushels.

1. A bin is 6 ft. long, 5 ft. wide, and 4 ft. deep. How many *stricken*, or how many *heaped* bushels will it hold?

Solution.—
(6 ft. × 5 × 4 × 1728)÷2150 42 = 96.423, *stricken* bushels.
(6 ft. × 5 × 4 × 1728)÷2747.7 = 75.46, *heaped* bushels.

2. What is the volume of a rectangular bin that will hold 128 bu., *stricken* measure; also of one that will hold 128 bu. *heaped* measure?

Solution.—
1st, *stricken* meas., 128 × 2150.42÷1728 = 159:29 cu. ft.
2d, *heaped* " 128 × 2747.7 ÷1728 = 203.53 cu. ft.

Rule.—*Divide the contents in cubic inches by 2150.42 for* STRICKEN, *or by 2747,7 for* HEAPED *bushels.* Conversely, To find the cubic contents in a given number of bushels:

Multiply the number of bushels by 2150.42, if STRICKEN, *or by 2747.7, if* HEAPED; *the product will be the number of cu. in., which may be reduced to higher denominations, if required.*

For all practical purposes, the following is sufficiently exact:

1. *Multiply the contents in cubic feet by .8 for* STRICKEN, *or by .63 for* HEAPED *bushels, and the product will be the required number of bushels.* Conversely,

2. *Divide the number of* STRICKEN *bushels by .8, or the number of* HEAPED *bushels by .63, and the quotient will represent the capacity in cubic feet.*

The following examples are all solved by these last rules.

3. A bin 10 ft. long, 6 ft. wide, and 4 ft. deep, will hold how many bushels of oats? Of potatoes?

$$\text{Solution.—}\begin{cases}10\times6\times4\times.8 = 192 \text{ bu. of } \textit{oats.}\\ 10\times6\times4\times.63 = 151.2 \text{ bu. of } \textit{potatoes.}\end{cases}$$

4. What is the volume in cubic feet of a bin that will hold 320 bushels of grain? Of corn in the ear?

$$\text{Solution.—}\begin{cases}320\div.8 = 400 \text{ cu. ft.}\\ 320\div.63 = 507.93 \text{ cu. ft.}\end{cases}$$

5. What must be the depth of a bin to contain 240 bu. of grain, its length being 10 ft. and its width 5 ft.?

Solution.—$240\div.8 = 300$; $300 \text{ cu. ft.}\div\overline{10\times5} = 6 \text{ ft.}$, the *depth.*

6. How many bushels of grain will a bin 10 ft. square and 4 ft. high contain?

7. A rectangular box will hold 256 bu. of wheat. What is its volume in cubic feet?

8. How many bushels of wheat can be put in a bin 8 ft. long, 6 ft. 6 in. wide, and 3 ft. 4 in. deep?

9. What must be the length of a bin that is 6 ft. wide and 4½ ft. deep, to contain 324 bushels of oats?

10. How many bushels will a bin hold, that is 16 ft. long, 4 ft. 6 in. wide, and 3 ft. 4 in. deep?

11. A bin that holds 100.8 bu. of corn is 7 ft. long and 6 ft. deep. How wide is it?

12. How many bushels of grain will fill a bin that is 8.5 ft. long, 4.25 ft. wide, and 3¾ ft. deep?

13. How many bushels of apples will a wagon-box hold, that is 12 ft. long, 3 ft. wide, and 2 ft. 6 in. deep? How many bushels of barley?

14. A bin 20 ft. long, 12 ft. wide, and 5 ft. deep, is full of wheat. What is its value at $2 a bushel?

15. A bin 7 ft. long, 6 ft. wide, and 5 ft. deep, is ¾ full of rye. What is its value at $1.37½ a bushel?

16. A farmer's entire crop of barley just filled a bin 10 ft. long. 6 ft. wide, and 5 ft. deep. What was its value, at $1.78 per cental?

17. A crib, the inside dimensions of which are 15 ft. long, 7 ft. 4 in. wide, and 8 ft. high, is full of corn in the ear. If 2 bu. of ears make 1 bu. of shelled corn, what is the value of the whole, when shelled, at $.92 a bushel?

18. If one bushel or 60 lb. of wheat make 48 lb. of flour, how many barrels of flour can be made from the contents of a bin 10 ft. long, 5 ft. wide, and 4 ft. deep, filled with wheat?

19. How many tons of ice can be packed in a building 40 ft. long, 30 ft. wide, and 20 ft. high, a cubic foot of ice weighing 58⅛ pounds?

20. A vat that will hold 5000 gallons of water will hold how many bushels of corn?

21. Required the number of bushels of shelled corn in a crib 16 ft. long, 6 ft. 9 in. wide, and 7 ft. high, full of ears, if 2 bu. of ears make 1 bu. of shelled corn?

22. If a man buy 6 bu. of chestnuts at $4 a bushel, dry measure, and sell the same at 20 cents a quart, liquid measure, how much does he gain?

The only *accurate* method of measuring coal, hay, corn in the crib, and like products is by *weight;* but for the ordinary purposes of *estimation,* the following units of measure *in bulk* may be used:

1. **Coal.**—Ordinary anthracite coal measures from 36 to 40 cu. ft. to the ton; bituminous coal, from 36 to 45 cu. ft. to the ton.

2. Lehigh, white ash, egg size, measures about 34½ cu. ft. to the ton (2000 lb.); Schuylkill, white ash, 35 cu. ft., and of gray or red ash, 36 cu. ft. to the ton.

3. Coal is bought and sold in *large* quantities by the *ton;* in *small* quantities by the *bushel,* the conventional rate being 28 heaped bushels to a ton, or about 44.5 cu. ft.

Hay.—When loose, or in loads, or upon scaffolds, 500 cu. ft. is estimated to the ton of 2000 lb., in bays, or mows, 400 cu. ft.; and in large, well-settled stacks, or in bales packed for shipping, 10 cu. yd.

For *clover-hay,* deduct about *one-fifth* from the weight.

23. How many tons of red ash coal, egg size, will a bin 17 ft. long, 6 ft. wide, and 3 ft. deep, contain?

24. A bin 6 ft. long, 4 ft. deep, and 5 ft. 9 in. wide, is full of Lehigh white ash coal. Find its value at $6.75 a ton?

25. A large crib 10 yd. long, 6 yd. wide, and 6 yd. deep, is filled with Schuylkill red ash coal. Find the number of tons it contains, and its value at $5½ a ton?

26. A bin 7 ft. long, 5 ft. wide, and 5 ft. deep, is half full of Schuylkill white ash coal. Find its value at $5.90 a ton.

How many tons

27. Of old hay, in a mow 86 ft. × 24 ft. × 16 ft.?

28. Of hay in a load, 16 ft. long, 8 ft. wide, and 7 ft. high?

29. Of hay in a well-settled stack, 15 ft. square and 16 ft. high; and what is its value at $14½ a ton?

30. What is the value of a well-settled mow of clover-hay, measuring 60 ft. × 30 ft. × 18 ft., at $12.75 a ton?

PERCENTAGE.

INDUCTIVE EXERCISES.

231. 1. What is $\frac{1}{100}$ of $100? $\frac{5}{100}$? $\frac{25}{100}$? $\frac{84}{100}$?

2. What is $\frac{5}{100}$ of $600? $\frac{8}{100}$? $\frac{24}{100}$? $\frac{45}{100}$?

3. How many *hundredths* of $1 are 25 cts.? 40 cts.? 63 cts.?

4. How many *hundredths* of anything is $\frac{1}{2}$ of it? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{10}$?.

5. What part of $100 is $7? $25? $33? $75?

6. If I pay $100 for a clock, and sell it for $109, how many dollars do I gain? How many hundredths of the *cost*?

7. If I sell the clock for $93, how many dollars do I lose? How many *hundredths* of the *cost*?

The number of *hundredths* gained or lost is called the *rate*.

232. ***Per cent.*** means *by the hundred.*

Thus, 5 *per cent.* means 5 of every 100; 10 per cent. of any number of miles means 10 mi. of every 100 mi., or 1 mi. for every 10 mi.

233. The ***sign*** *%* is used for the words *per cent.*

Thus, 6*%* means 6 *per cent.*; $4\frac{1}{2}$*%* means $4\frac{1}{2}$ *per cent.*
5 *per cent.*, 5*%*, $\frac{5}{100}$ are equivalent expressions.
Per cent. may be expressed in the form of a common fraction.

1. How many *hundredths* of a number is 4 per cent. of it? 6 per cent.? 7*%*? 9*%*? 15*%*? 34*%*?

2. What *per cent.* of a number is $\frac{3}{100}$ of it? $\frac{7}{100}$ of it? $\frac{8}{100}$? $\frac{11}{100}$? $\frac{26}{100}$? $\frac{42}{100}$? $\frac{75}{100}$?

3. How many *hundredths* of a number is $2\frac{1}{2}$*%* of it? $5\frac{1}{4}$*%*?

4. What *%* of a number is $\frac{1}{2}$ of it? $\frac{1}{3}$ of it? $\frac{1}{5}$? $\frac{1}{8}$? $\frac{1}{12}$? $\frac{3}{4}$? $\frac{2}{3}$? $\frac{4}{5}$? $\frac{5}{8}$? $\frac{7}{10}$? $\frac{9}{20}$? $\frac{12}{25}$?

DRILL TABLE No. 3.

500	5000	50	5	4250
700	7000	70	7	4225
600	6000	60	6	4275
400	4000	40	4	4285
900	9000	90	9	4299

Let the pupil recite, thus, by line: 1% *of anything is* 1 *one-hundredth of that thing, and* 1 *one-hundredth of* 500 *is* 5 ; $\frac{1}{100}$ *of* 5000 *is* 50 ; $\frac{1}{100}$ *of* 50 *is* .5 ; *of* 5 *is* .05 ; *of* 4250 *is* 42.5.

Then 10% *of anything is* $\frac{1}{10}$ *of that thing, and* $\frac{1}{10}$ *of* 500 *is* 50 ; $\frac{1}{10}$ *of* 5000 *is* 500 ; *of* 50 *is* 5, *etc.*

Read the above, by line and by column, as miles, rods, days, etc. Thus, 10% of 500 mi. is 50 mi. ; of 5000 mi. is 500 mi. ; etc. See **402**, **403**, and **404**.

DEFINITIONS.

234. In the applications of percentage, at least three elements or parts are involved, viz.; the *base* or multiplicand, the *rate* or multiplier, and the *percentage* or product.

Any *two* of these being given, the *third* is readily found.

235. The ***base*** is the number on which the percentage is computed.

236. The ***rate*** is a decimal which denotes how many *hundredths* of the base are to be taken.

237. ***Percentage*** has two significations :

1st. It is the process of computing by *hundredths*.

2d. It is such a part of the base, as is indicated by the *rate*.

Thus, in the statement, 5% of $40 is $2, the *rate* is .05, the *base* $40, and the *percentage* $2.

238. The ***amount*** is the base, *plus* the percentage.

239. The ***difference*** is the base, *minus* the percentage.

Thus, if the base is \$80, and the percentage is \$5, the *amount* is \$80 + \$5 = \$85; and the *difference* is \$80 − \$5 = \$75.

240. PRINCIPLES.—I. *The* BASE *is either an abstract, or denominate number.*

II. *The* RATE *is always abstract.*

III. *The* PERCENTAGE, AMOUNT, *and* DIFFERENCE *are always like the* BASE.

241. The base and rate given to find the percentage.

ORAL EXERCISES.

1. What is 10% of 80?

ANALYSIS.—10% of 80 is $\frac{10}{100}$, or $\frac{1}{10}$ of 80, which is 8.

What is

2. 5% of \$80?
3. 7% of 200 lb.?
4. 6% of 150 men?
5. 25% of 120 mi.?

What is

6. 12½% of 72 gal.?
7. 40% of 60 sheep?
8. 8% of 50 bu.?
9. 50% of \$240?

10. What is the amount of \$60 increased by 5% of itself? By 10%? By 20%? 12½%? 25%? 33⅓%?

11. What is the difference of 360 feet diminished by 4% of itself? By 8⅓%? By 12½%? 16⅔%? 20%? 33⅓%? 50%?

12. What is the amount of \$200 increased by 5% of itself?

13. A farmer having 120 bu. of wheat, sold 25% of it. How many bushels did he sell? What % did he keep?

14. Bought 25 bbl. of apples, and upon opening them found 4% of them worthless. How many were good?

PRINCIPLE.—*Percentage is always a product, of which the base and the rate are factors.*

WRITTEN EXERCISES.

242. 1. What is 12½% of $1260? The am't and difference?

$1260	Base.	$1260 + $157.50 = $1417.50, *Amt.*
.125	Rate.	
$157.50	Percentage.	$1260 − $157.50 = $1102.50, *Diff.*

EXPLANATION.—Since 12½% of any number is $\frac{12\frac{1}{2}}{100}$, equal to ⅛, or .125 of that number, 12½% of $1260 is $1260 × .125 = $157.50; or, it is ⅛ of $1260 = $157.50.

FORMULAS:
1. *Base × rate = percentage.*
2. *Base + percentage = amount.*
3. *Base − percentage = difference.*

Formulas will take the place of *rules*, whenever practicable, and the pupil may be required to construct a rule in ordinary language from each formula. Thus, for the preceding:

1. *The base multiplied by the rate equals the percentage.*
2. *The percentage added to the base equals the amount.*
3. *The percentage subtracted from the base equals the difference.*

What is
2. 35% of 695 lb.?
3. 75% of $8428?
4. 4½% of 318.8 rd.?
5. 105% of $5728?
6. $3140.75 + 1¼%?
7. 2¾ mi. + 7½%?
8. 400 ft. − 3⅛%?

Find
9. 33⅓% of 8736 bu.
10. ⅛% of $35000.
11. 120% of $171.24?
12. 84% of 25$\frac{4}{7}$ bu.
13. 25% of ⅞ of a ton.
14. ¾% of 16400 men.
15. 3⅘% of 2640 lb.

16. The bread made from a barrel of flour weighs 35% more than the flour. What is the weight of the bread?

17. A man having a yearly income of $4550 spends 20% of it the first year, 25% of it the second year, and 37½% of it the third year. How much does he save in 3 years?

18. Bought a bill of goods amounting to $875.50, from which was deducted 5%. What was the percentage allowed, and the amount paid?

19. Having $10720, I invested 25% of it in land, and 12½% of the remainder in fencing it. What remained?

20. Two men engaged in trade, each with $3540. One of them gained 33⅓% of his capital, and the other gained 60%. How much more did the one gain than the other?

243. The base and percentage given, to find the rate.

ORAL EXERCISES.

1. What per cent. of 84 is 7?

Analysis.—Since 7 is $\frac{1}{12}$ of 84, it is $\frac{1}{12}$ of 100%, or 8⅓% of 84.

What *per cent*
2. Of 21 lb. are 7 lb.?
3. Of 25 bbl. are 15 bbl.?
4. Of $40 are $30?
5. Of 12½ rd. are 2½ rd.?
6. Of 1 A. are 40 sq. rd.?
7. Of 1 da. are 8 hr.?

What *per cent.*
8. Are $6¼ of $12½?
9. Are 36 bu. of 60 bu.?
10. Are 3¼ qt. of 13 qt.?
11. Are 8⅓ cents of $1?
12. Is $¼ of $25?
13. Is ¾ of 3¾?

14. $\frac{3}{10}$ of 40 is what per cent. of 40?

15. ⅓ of a farm is what per cent. of it?

16. ⅘ of a cargo is what per cent. of it?

17. 1½ times a number is what per cent. of it?

18. If a miller take 4 qt. of every bushel he grinds, what per cent of it does he take?

19. If $5 is paid for the use of $20 one year, what is the rate per cent.?

Principle.—*The rate is the quotient of the percentage divided by the base.*

WRITTEN EXERCISES.

244. 1. What per cent. of 320 is 80?

Base. Percentage.
320) 80.00
.25 Rate.

Explanation.—Since the percentage is the *product* of the base and rate, the rate is the quotient found by dividing the percentage by the base; and 80÷320 = .25, the required *rate*. Or, since 80, the percentage, is $\frac{80}{320}$ or ¼ of the base, the rate is ¼ of 100%, which is 25%.

Formula: *Percentage ÷ base = rate.*

What *per cent.*

2. Of 450 is 90? 180?
3. Of 56 gal. are 7 gal.?
4. Of $480 are $26.40? $60?
5. Of 192 A. are 120 A.?
6. Of 15 mi. are 10.99 mi.?
7. Of 46 gal. are 5 gal. 3 qt.?
8. Of 6 bu. 1 pk. are 4 bu. 2 pk. 6 qt.

What *per cent.*

9. Are 45 cents of $9?
10. Are 40 lb. of 250 lb?
11. Are 448 da. of 5600 da.?
12. Are 5 lb. 10 oz. of 15 lb.?
13. Are 77½ bu. of 500 bu.?
14. Are 5 dimes of $100?
15. Is 4.5 of 75? 13.5 of 225?
16. Is .45 of .60? 3⅝ of 18⅛?

17. A regiment went into battle with 600 men, and came out with 320. What % were lost?

18. Of 4000 A. of land, I sell 140 A. What % do I retain?

19. A grocer sold from a hogshead containing 600 lb. of sugar, ¼ of it at one time, and ⅓ of the remainder at another time. What per cent. of the whole remained?

20. A merchant owes $15120, and his assets are $9828. What per cent. of his debts can he pay?

21. If $52.50 is paid for the use of $750, 1 yr., what is the rate %? If $56.70 is paid for the use of $1260?

22. A man shipped 2600 bushels of grain from Chicago, and 455 bushels were thrown overboard during a gale; what was the rate per cent. of his loss?

245. The rate and percentage given, to find the base.

ORAL EXERCISES.

1. 84 is 7% of what number?

Analysis.—Since 7% or $\frac{7}{100}$ of some number is 84, 1% or $\frac{1}{100}$ of the number is $\frac{1}{7}$ of 84, or 12; and 100% is 100 times 12, or 1200.

Of what number

2. Is 15 25%? 48 8%?
3. Is 24 75%? 1.2 6%?
4. Is 30 lb. 20%? 25%?
5. Is $84 12%? 21%?
6. Is 15 bu. 30%? 50%?
7. Is 16 doz. 12½%? 8⅓%?

8. 12½% of 96 is 33⅓% of what number?

9. 25% of 800 bu. is 2½% of how many bushels?

10. Sold 105 bbl. of potatoes, which was 35% of all I raised. How many did I raise?

11. A farmer sold 7.5 A. of land, which was 15% of all he owned. How many acres did he own?

Principle.—*The base is the quotient of the percentage divided by the rate.*

WRITTEN EXERCISES.

246. 144 is 75% of what number?

Explanation.—Since the percentage, 144, is the *product* of the base by the rate, the base is the quotient of the percentage divided by the rate, or 144÷.75 = 192, the required *base*.

Rate. Percentage.
.75) 144.00
192 Base.

Formula: *Percentage* ÷ *rate* = *base.*

2. 6.5 gal. are 5% of what?
3. $18.75 are 6¼% of what?
4. 420 bbl are 12½% of what?
5. $300 are 48% of what?
6. 18 yd. are 36% of what?
7. $975 are 15% of what?
8. 78.5 is 25% of what?
9. 19.8 lb. are 7½% of what?

10. 2% of $125.50 is 8% of what sum?

11. 12½% of 400 ft. is 2½% of how many feet?

12. A man spends $825.60, which is 33⅓% of his salary. How much is his salary?

13. A man drew out 9% of his bank deposit to pay a debt of $243.72. How much had he in bank?

14. If a man, owning 40 per cent. of an iron foundry, sells 25 per cent. of his share for $1246.50, what is the value of the whole foundry?

15. A farmer sold 3150 bushels of grain and had 30% of his entire crop left. What was his entire crop?

16. If a man owning 45% of a steamboat sells 16⅔% of his share for $5860, what is the value of the whole boat?

247. The amount, or difference, and rate given, to find the base.

ORAL EXERCISES.

1. What number increased by 25% of itself amounts to 60?

ANALYSIS.—Since 60 is 25% more than the number, 60 is 125% or $\frac{125}{100}$, equal to $\frac{5}{4}$ of the number, and the number is 4 times $\frac{1}{5}$ of 60, or 48.

2. What number diminished by 20% of itself is 36?

ANALYSIS.—Since 36 is 20% less than the number, 36 is 80%, or $\frac{80}{100}$, equal to $\frac{4}{5}$ of the number, and the number is 5 times $\frac{1}{4}$ of 36, or 45.

3. What number increased by 10% of itself is 55?

4. What sum increased by 12½% of itself is $18.90?

5. $45 is 25% more than what sum?

6. A tailor sold a coat for $26, which was 30% more than it cost him. What did it cost him?

7. What number diminished by 5% of itself is 57?

8. What sum diminished by 50% of itself is $12.50?

9. 60 cwt. are 40% less than what number?

10. 35 yd. are 12½% less than what number?

11. A boy spent 25% of his money, and had 75 cents left? How much had he at first?

WRITTEN EXERCISES.

248. 1. What number increased by 37% of itself amounts to 2055?

Explanation.—Since 2055 is the number increased by 37% of itself, 2055 is 137%, or 1.37 times the number. Hence, 2055 ÷ 1.37 = 1500, the required number.

1 + rate. Amount.
1.37) 2055.00
1500 Base.

2. What number increased by 18% of itself equals 2950?

3. What sum increased by 15% of itself is $6900?

4. What number diminished by 12% of itself is 2640?

Explanation.—Since the number is diminished by 12% of itself, 2640 is 88%, or .88 of the number. Hence, 2340 ÷ .88 = 3000, the required number.

1 — rate. Difference.
.88) 2640.00
3000 Base.

5. What number diminished by 30% of itself is 350?

6. What number diminished by 4½% of itself is 1640?

Formulas: $$\left.\begin{cases} 1.\ \textit{Amount} \div (1 + \textit{rate}) \\ 2.\ \textit{Difference} \div (1 - \textit{rate}) \end{cases}\right\} = \textit{base}.$$

What number *increased*

7. By 15% of itself is 345?
8. By 36% is 238 A.?
9. By 100% is 84.6 cwt.?
10. By 6% is 1272 bu.?
11. By 22% is $549?

What number *diminished*

12. By 65% of itself is $2590?
13. By 50% is 28.5 ft.?
14. By 16⅔% is 1035 mi.?
15. By 4% is $465.60?
16. By 37½% is $203.375?

17. Sold a horse for $340, which was 15% less than his value. What was his value?

18. A man having increased his bank deposit 40%, it amounted to $840. How much had he at first?

19. My income this year is $2232, which is 7% less than it was last year. How much was it last year?

20. A man sold 160 A. from his farm, which was 12½% less than the number of acres he retained. How many acres in his farm?

249. The *applications* of percentage are *twofold:*

1st. Those in which *time* is *not* an element; as, Profit and Loss, Commission and Brokerage, Insurance, Taxes, and Duties.

2d. Those in which *time is* an element; as Interest, Discount, Stocks, Equation of Payments, and Exchange.

Since the *principles*, and some one of the *four formulas* of percentage already considered will apply to some extent to all these topics, the pupil should carefully observe what *elements* or *parts* are given in the example, and what required, and then apply the corresponding rule or formula.

TRADE DISCOUNT.

250. ***Trade discount*** is a percentage deducted from the face of bills, the list price of goods, or from the amount of debt, without regard to *time*, and is expressed by the term "*per cent. off.*"

Thus, 20% off, or 20 off, means a deduction of 20% from the nominal or asking price. 20 and 5% off means a discount, first of 20%, and then 5% from the remainder. 2 *tens* and 5% off means *two* successive discounts of 10%, and 5% from the remainder, etc.

251. The ***net price*** is the price *received* for goods, etc.

The **invoice price,** or the **debt,** corresponds to the *base* in percentage; the **per cent. off** to the *rate;* and the **sum** of the **discounts** to the *percentage.*

WRITTEN EXERCISES.

252. 1. What is the net cost of a bill of goods amounting to $975, bought on 4 mo., at 10% discount, and 5% off for cash?

This means that the goods bought are at the regular rates, on the usual time, at 10% off, but for *cash* a further discount of 5% is made.

Find the *discount* and *net cost* of the following bills of goods:

2. Bought for $750, on 3 mo., at 20% discount, and 4% off : cash.

3. Bought for $1584.50, on 60 da., at 12½% discount, and 2½% off for cash?

4. Bought for $365.75, on 4 mo., at 20, 10, and 5% off for cash.

5. Bought for $260, on 90 da., at 2 tens and 3% off for cash.

6. What is the difference on a bill of $650, between a discount of 30%, and a discount of 25 and 5% off?

7. School slates, marked to sell at $12.75 a case, were sold at 50, 10, and 5% off. What was the *net* price?

8. What is the difference between 15, 2 *tens*, and 5% off, and 40% off?

PROFIT AND LOSS.

253. ***Profit and loss*** are *commercial* terms used to express gain or loss in business transactions.

254. The *terms* corresponding to those of *percentage* are as follows:

1. The **cost**, or capital invested, is the *base*.
2. The **rate** of profit or loss is the *rate*.
3. The **profit** or **loss** is the *percentage*.
4. The **selling price** is the *amount* or *difference*.

ORAL EXERCISES.

255. To find the profit or loss, and the selling price.

1. A watch that cost $80 sold for 20% more than cost. What was the gain, and the selling price?

ANALYSIS.—Since the *gain* was 20%, it was $\frac{20}{100}$, or ⅕ of $80, equal to $16, and the *selling price* was $80 + $16 = $96. (**241.**)

Find the *profit* or *loss*, and *selling price* of

2. Cloth bought at $4 a yard, and sold at 25% advance; 10%.

3. A cow bought for $48, and sold at a loss of 12½%; 15%.

4. A carriage that cost $120, and sold at 10% loss; 16⅔%.

5. Gloves bought at $12 a doz., and sold at a gain of 50% a pair.

6. Bought a carriage for $160, and, after paying 10% for repairs, sold it at 12½% profit. What was the gain, and the selling price?

7. If butter bought at 36 cents a pound is sold at a loss of 16⅔%, what is the selling price?

8. What must be the selling price of coffee that cost 25 cts. a pound, in order to gain 20%?

9. At what price must an article that cost $5 be sold, to gain 100%? 120%? 150%? 200%?

WRITTEN EXERCISES.

256. 1. A house and lot bought for $4750 was sold at a profit of 37½%. What was the gain, and the selling price?

SOLUTION.—$4750 × .375 = $1781.25, *gain*; $4750 + $1781.25 = $6531-25, the *selling price*. (**242**, 2.)

$$\text{FORMULAS:}\left\{\begin{array}{l}1.\ \textit{Cost} \times \textit{rate} = \textit{profit} \text{ or } \textit{loss}.\\ \left.\begin{array}{l}2.\ \textit{Cost} + \textit{profit}\\ 3.\ \textit{Cost} - \textit{loss}\end{array}\right\} = \textit{selling price.}\end{array}\right.$$

Find the *profit* or *loss*, and the *selling price*,

2. Of goods that cost $3½ a yard, and sold at 25% gain.

3. Of a house bought for $3270, and sold at 7% gain.

4. Of sugar that cost $623.75, and sold at 6¼% loss.

5. Of land that cost $1745, and was sold at a gain of 20%.

6. Of goods that cost $3120, and were sold at 27% gain.

7. Of a boat bought for $2545½, and sold at 25% loss.

8. A miller bought 500 bushels of wheat at $1.15 a bushel, and he sold the flour at 16⅔ per cent. advance on the cost of the wheat. What was his gain?

9. Bought 76 cords of wood at $3.62½ a cord, and sold it so as to gain 26 per cent. Find the whole gain, and the selling price per cord.

10. A hatter bought 40 hats at $1.75 apiece, and sold them at a loss of 14²⁄₇ per cent. What was his whole loss?

11. A merchant pays $7650 for a stock of spring goods; if he sells at an advance of 20% upon the purchase price, what are his profits, after deducting $480 for expenses?

12. A grocer bought 3 barrels of sugar, each containing 230 pounds, at 8¼ cents a pound, and sold it at 18²⁄₁₁ per cent. profit. What was his whole gain, and what the selling price per pound?

13. Bought a house for $4380. For what must it be sold to gain 14½%?

SOLUTION.—$4380 × (1 + .14½ or 1.145) = $5015.10.

14. At what price must pork, bought at $18.40 a barrel, be sold, to lose 15%?

SOLUTION.—$18.40 × (1 − .15) = $15.64.

$$\text{FORMULAS: } \left\{ \begin{array}{l} 1.\ \textit{Cost} \times (1 + \textit{rate of gain}) \\ 2.\ \textit{Cost} \times (1 - \textit{rate of loss}) \end{array} \right\} = \textit{selling price.}$$

Find the *selling price* of goods,

15. Bought at $126.50, and sold at 9% gain.

16. Bought at $3424, and sold at 12½% loss.

17. Bought at $1032.50, and sold at 14% gain.

18. Bought at $3000, and sold at 8¼% loss.

19. At what price must goods that cost $3⅓ a yard be marked, to gain 25%? To lose 20%?

20. Sold a lot of damaged goods at a loss of 15%. What was the selling price of those that cost $.62½? $1.25?

21. Bought a hogshead of sugar containing 9 cwt. 56 lb. for $86.04, and paid $4.78 freight and cartage. At what price per pound must it be sold to gain 20%?

22. A merchant bought 15 pieces of broadcloth, each piece containing 23⅓ yards, for $840, and sold it so as to gain 18¾%. What did he receive a yard?

257. To find the rate of profit or loss.

ORAL EXERCISES.

1. Flour was bought for $8, and sold for $10 a barrel. What was the gain per cent.?

ANALYSIS.—Since the whole gain is the difference between $8 and $10, which is $2, or $\frac{1}{4}$ the cost, the *gain* % is $\frac{1}{4}$ of 100%, or 25%. (**243.**)

Find the rate of profit or loss on

2. Coal, bought at $6 a ton, and sold at $7; at $8.
3. Tea, bought at $.80 a pound, and sold at $.90; at $1.
4. Sugar, bought at 10 cts. a pound, and sold at 12 cts.
5. A sewing-machine, sold at a loss of $\frac{1}{3}$ the cost; $\frac{1}{5}$; $\frac{1}{4}$.
6. What % is gained on goods sold at *double* the cost?
7. What % is lost on goods sold at *one-half* the cost?

WRITTEN EXERCISES.

258. 1. Bought wool at $.48 a pound, and sold it at $.60 a pound. What was the gain per cent.?

SOLUTION.—$.60−$.48 = $.12; and $.12÷$.48=.25=25%. (**244.**)

FORMULA: *Profit* or *loss* ÷ *cost* = *rate.*

Find the *rate* % of profit or loss

2. On produce, bought for $640, and sold for $\frac{7}{4}$ as much.
3. On a carriage, bought for $250, and sold for $\frac{3}{5}$ as much.
4. On coal, sold for $72.96, at a gain of $9.12.
5. On cloth, bought for $4.25 a yard, and sold for $4.93.
6. On sugar bought at 8 cents and sold at 9$\frac{1}{3}$ cents.
7. A drover bought 150 head of cattle for $42 per head, and sold them for $5400; what was his loss per cent.?
8. Bought 108 bbl. of flour at 4.62\frac{1}{2}$ a bbl. and sold it so as to gain 114.88\frac{1}{2}$; what per cent. profit did I make?
9. If I sell $\frac{1}{3}$ of an article for $\frac{3}{4}$ of its cost, what % is gained?
10. If $\frac{4}{5}$ of an article is sold for $\frac{1}{3}$ its cost, what % is lost?

259. To find the cost.

ORAL EXERCISES.

1. A fruit-dealer sold pears at a profit of $3 a barrel, which was a gain of 20%. What did they cost?

Analysis.—Since the gain, 20%, was $\frac{20}{100}$, or ⅕ of 100%, $3 is ⅕ of the cost; $3 is ⅕ of 5 times $3, or $15. (**245.**)

Find the *cost* of

2. Hats, sold for $1 less than cost, or at a loss of 16⅔%.
3. Flour, sold at a profit of $2 a barrel, or a gain of 25%.
4. Boots, sold at $1.50 a pair, above cost, or a gain of 30%.
5. A cow, sold at 33⅓%, or $12 less than cost.
6. A drover sells sheep at a profit of $1½ a head, which is a gain of 25%. Find the cost, and the selling price?
7. Sold muslin at 45 cents a yard, and gained 12½%. What did it cost?

Analysis.—Since the gain, 12½%, is ⅛ the cost, 45 cents, the selling price is $\frac{9}{8}$ the cost. $\frac{1}{9}$ of 45 cents, or 5 cents, is ⅛ the cost, and $\frac{8}{8}$ is 8 times 5 cents, or 40 cents. (**247.**)

Find the *cost* of

8. Tea, sold at $.90 a pound, or a gain of 25%; a loss of 20%.
9. Turkeys, sold at $.30 a pound, or a profit of 20%.
10. Flour, sold at a loss of 20%, or $8 a barrel.
11. Two pictures were sold for $99 each; on one there was a gain of 10%, on the other a loss of 10%. Was there a gain or loss on the sale of both, and how much?

WRITTEN EXERCISES.

260. 1. Sold a sewing-machine at an advance of $22.65, which was 25% of its cost. What did it cost?

Solution.—$22.65 ÷ .25 = $90.60, the *cost*. (**246.**)

Formula: *Profit* or *loss* ÷ *rate* = *cost.*

Find the *cost*

2. Of goods sold at $1500 profit, or a gain of 16%.

3. Of flour sold at a loss of $.88, or 10% on a barrel.

4. Of wheat sold at a loss of 6 cents, or 4% on a bushel?

5. Of lumber sold at $4.95 gain per M., or 35%.

6. A furniture dealer sold two parlor sets for $450 each; on one he made 15%, on the other he lost 15%. What did each cost him?

SOLUTION.— $\left\{\begin{array}{l}\$450 \div (1+.15) = \$391.30+, \textit{ cost of one.}\\ \$450 \div (1-.15) = \$529.41+, \textit{ cost of the other.}\end{array}\right.$ (248.)

$$\text{FORMULA}: \textit{Selling price} \div \left\{\begin{array}{l}1 + \textit{rate of gain}\\ 1 - \textit{rate of loss}\end{array}\right\} = \textit{cost.}$$

7. Sold 25 barrels of apples for $69.75, and made 24%. What did they cost per barrel?

8. Sold 9⅛ cwt. of sugar at $8¼ per cwt., and thereby lost 12%. What was the whole cost?

Find the *cost*,

9. Of coal sold at $6, being at a loss of 12½%.

10. Of grain sold at $.96 a bushel, at a gain of 28%.

11. Of silk sold for $5.40 a yard, at a profit of 10%.

261. To find the marking price.

1. At what price must a grocer mark tea that cost $.76 a pound, so that he may abate 5%, and yet gain 25%?

First find the *selling price* at 25% gain; then regard the selling price as *cost*, and find the *asking price*.

SOLUTION.— $\left\{\begin{array}{l}\$.76 \times (1+.25) = \$.95, \text{ the } \textit{selling price.}\\ \$.95 \div (1-.05) = \$1, \text{ the } \textit{asking price.}\end{array}\right.$

Find the *marking price*, when the cost

2. Is $120, so as to abate 4%, and yet make a profit of 20%.

3. Is $60, so as to abate 25%, and still gain 20%.

4. Is $20000, so as to fall 20%, and still make 10% profit.

COMMISSION.

262. ***Commission*** is the *percentage* or compensation allowed an agent for transacting business.

The **agent** may be known as a *factor*, *broker*, or a *commission merchant.*
A **consignment** is a quantity of goods sent by a party to his agent.
The **consignor** is the party who sends the goods.
The **consignee** is the party to whom the goods are sent.

263. The ***net proceeds*** is the balance due the *consignor*, after deducting the commission and other charges.

264. The *terms* corresponding to *percentage* are as follows:

1. The **money invested** or **collected** is the *base.*
2. The **rate %** allowed for services is the *rate.*
3. The **commission** is the *percentage.*

WRITTEN EXERCISES.

265. Find the *commission*

1. On a sale of merchandise for $2150, at 2¼%.

SOLUTION.—$2150 × .02¼ = $48.37½, *commission* (**242**, 1).

FORMULA : *Amt. of sales × rate = commission.*

2. On the sale of a farm for $4650, at 2⅛%.
3. On the sale of 375 bbl. of flour, at $6.25 a bbl., at 3¼%.
4. On the purchase of a farm for $13750, at 2¾%.
5. On the sale of a mill for $9384, at ⅞%.
6. On the sale of $21680 worth of wool, at 1¾%.
7. On the sale of 250 bales of cotton, averaging 520 lb., at 14⅝ cents a pound, at 1⅛%.
8. A commission merchant sells 225 bbl. of potatoes at $3.25 per bbl., and 316 bbl. of apples at $4⅛ per bbl. What is his commission at 4⅛%?

266. Find the *rate* of commission,

1. When $52.20 is paid for selling goods to the amt. of $2320.

Solution.—$52.20 ÷ $2320 = .0225 = 2¼%, *rate.* (**244.**)

Formula: *Commission ÷ amt. of sales = rate.*

2. When $78 is paid for selling goods for $5200.
3. When $189 is paid for selling a farm for $7560.
4. When $63 is paid for collecting a debt of $1260?
5. Paid my N. O. agent $74.25 for buying 26400 lb. of rice, at 4½ cts. a lb. What was the rate of his commission?

267. Find the *amount of sales,*

1. When a commission of $105 is charged, at 2½%.

Solution.—$105 ÷ .025 = $4200, *amt. of sales.* (**246.**)

Formula: *Commission ÷ rate = amount of sales.*

2. When $378 commission is paid, at 2¼%?
3. When $210 commission is charged, at 6%.
4. If an agent charges $59.50 commission, at 1¾%.
5. Paid an agent $394 for selling a piece of property, at 2½%. What did the property bring?

268. Find the *amount to be invested,*

1. If $9500 is remitted to a correspondent to be invested in woolen goods, after deducting 5% commission.

Solution.—$9500 ÷ 1.05 = $9047.62, *amt. invested.* (**248**, 1.)

Formula: *Amt. remitted* ÷ (1 + *rate*) = *sum invested.*

2. If $4455 is remitted, deducting 1¼% commission.
3. If $6617.70 is remitted, deducting 2⅛% commission.
4. If $9909.30 is remitted, deducting ½% commission.
5. Sent to my agent in Chicago $1508.80, to invest in flour at $5¾ a barrel, after deducting his commission at 2½%. How many barrels can he buy?

INSURANCE.

269. ***Insurance*** is security against *loss*.

The **policy** is the written contract between the insurer and the insured.

The **premium** is the sum paid for the insurance.

270. The corresponding *terms* are as follows:

1. The **amount insured** is the *base*.
2. The **rate %** of premium is the *rate*.
3. The **premium** is the *percentage*.

WRITTEN EXERCISES.

271. Find the *premium* for insuring

1. A house for \$3500, at 1½%.

SOLUTION.—\$3500 × .015 = \$52.50, the *premium*. (**242**, 1.)

FORMULA: *Amount insured* × *rate* = *premium*.

2. A house and furniture valued at \$9700, at ¾%.

3. Dry goods valued at \$3840, at ⅞%.

4. A cargo of 5000 bu. of grain valued at \$1.20 a bushel, at 2¼% on ⅔ of its value.

5. A gentleman has a house insured for \$8000, and the furniture for \$4000, at 2⅜ per cent.; what is the premium?

272. Find the *rate* of insurance,

1. If \$48.75 is paid for an insurance of \$3250.

SOLUTION.—\$48.75 ÷ \$3250 = .015, or 1½%, the *rate*. (**244.**)

FORMULA: *Premium* ÷ *sum insured* = *rate*.

2. If \$172.20 is paid for an insurance of \$9840?

3. If \$38.25 is paid for an insurance of \$6120?

4. A house valued at \$6800 was insured for ½ its value, at a cost of \$46.75. What was the rate of insurance?

273. Find the *amount* of insurance,

1. If it cost $38.25 to insure a certain house and furniture, at ⅝%.

Solution.—$38.25 ÷ .00625 = $6120, the *amt ins.* (246.)

Formula: *Premium ÷ rate of ins. = sum insured.*

2. If it cost $93.50 to insure a store for one-half of its value, at 1⅜%, what is the store worth ?

3. Paid $245 insurance at 4⅜% on a shipment of pork, to cover ⅝ of its value. What was its total value?

4. A merchant paid $1500 premium on a cargo of cotton from New York to Belgium, which was 2½% on its valuation. What was the amount insured?

5. A steam-mill was insured at 4½%, to cover ¾ of its value. The premium paid was $526.50 : what was the value of the mill ?

6. A person insured his house for ¾ of its value at 40 cents per $100, paying a premium of $73.50. What was the value of the house ?

TAXES.

274. A *tax* is a sum of money assessed upon persons, property, or incomes, for public purposes.

A **property tax** is a tax on personal property and real estate.
A **poll tax** is a tax on the person.

275. An *assessor* is an officer appointed to estimate the taxable value of property, prepare the assessment rolls, and apportion the taxes.

276. The corresponding *terms* are as follows :

The **valuation** of the property is the *base.*
The **tax rate,** or **tax** on $1, is the *rate.*
n to be raised is the *percentage.*

WRITTEN EXERCISES.

277. 1. For the purpose of building a bridge, a tax of $2628 was levied upon the taxable property of a town, valued at $584000. What was the tax rate?

SOLUTION.—$2628 ÷ $584000 = .004½, or 4½ mills, the *tax rate* (**244**)

$$\text{FORMULA: } \frac{\textit{Sum to be raised}}{\textit{valuation}} = \textit{rate of tax.}$$

2. At what rate must property, valued at $1475000, be assessed to raise a tax of $7743.75? What will be the tax upon a farm assessed at $6840?

The amount of poll tax, if any, must first be subtracted from the whole sum to be raised, and the *rate* of tax found on the remainder.

3. In a certain town a tax of $5000 is to be assessed. There are 500 polls, each assessed $.75, and the valuation of the taxable property is $370000. What is the rate of property tax, and what is A's tax, whose property is valued at $7500, and who pays for 2 polls?

EXPLANATION.—The sum to be raised is $5000 less 500 polls at $.75, equal to $375; $5000 − $375 = $4625, *sum to be raised;* $4625 ÷ $370000 = .0125, the *tax rate;* A's tax is $7500 × .0125 + $1.50 = $95.25.

4. What is B's tax, who is assessed for $8530, and 3 polls?

5. What is C's tax, who is assessed for $987, and 1 poll?

6. In a certain district, a school house is to be built at a cost of $18527. What amount must be assessed to cover this and the collector's fees at 3%?

SOLUTION.—$18527 ÷ .97 = $19100, the *sum assessed.*

$$\text{FORMULA: } \frac{\textit{Sum to be raised}}{1 - \textit{rate of collection}} = \textit{sum to be assessed.}$$

7. What sum must be assessed to raise a net amount of $11123, and pay the cost of collecting at 2%?

DUTIES OR CUSTOMS.

278. ***Duties*** or ***customs*** are taxes levied upon *imported* goods for revenue purposes.

A **manifest** is a detailed statement, or invoice of a ship's cargo, showing the items, quantity, quality, cost, where shipped, and other details.

279. Duties are either *ad valorem* or *specific*.

280. An ***ad valorem duty*** is a certain per cent. assessed on the cost of goods in the country from which they are brought.

281. A ***specific duty*** is a fixed sum imposed on articles according to their weight or measure, without regard to their value.

282. An ***Appraiser*** is one who examines imported merchandise, and fixes the dutiable value and the rate of duty of the same.

Before estimating specific duties, certain deductions or allowances are made, called *tare*, *leakage*, *breakage*, etc.

Leakage is an allowance for waste of liquids in casks.
Breakage is an allowance for loss of liquids in bottles.
Tare is an allowance for the weight of box, bag, cask, etc.
Gross weight is the weight before any deduction is made.
Net weight is the weight after the deduction is made.

In the U. S. Custom Houses, the *long ton* of 2240 lb. is invariably used. In weights, less than ½ lb. is not regarded, and more than ½ lb. is taken as 1 pound.

283. The corresponding *terms* of *ad valorem* duties and *percentage* are the following:

1. The **net value** or **quantity** is the *base*.
2. The **rate** % of duty is the *rate*.
3. The **duty** is the *percentage*.

WRITTEN EXERCISES.

284. 1. What is the *specific* duty on 84 bags of coffee, gross weight 136 lb. each, tare 3%, at 3½ cents a pound?

SOLUTION.

136 lb. × 84 = 11424 lb., *gross weight;*
11424 lb. × .03 = 342.72, *tare;*
11424 lb. − 342.72 lb. = 11081.28 lb., *net weight;*
$.035 × 11081.28 = $387.84, *duty.*

FORMULA: *Rate × net quantity = specific duty.*

Find the *duty*

2. On 50 hhd of sugar, each containing 500 lb., at 1¾ cts. per lb., tare 56 lb. per hhd.

3. On 50 bbl. of sperm oil, each containing originally 31½ gal., duty 10 cts. per gal., leakage 2%.

4. On 72 boxes of sperm candles, each weighing 1 cwt., duty 2½ cts. per lb., tare 5½%.

5. On 120 gross London ale, duty $.60 a dozen, breakage 2½%.

6. Imported from Geneva 25 watches invoiced at $125 each, and 15 clocks, at $37.50. What was the duty, the rate being on clocks 25%, and on watches, 35% ad valorem?

FORMULA: *Net invoice cost × rate = ad val. duty.*

Find the *duty*

7. On 175 chests of tea, each 65 lb., invoiced at $.46 a pound, duty 28% ad valorem.

8. On an invoice of French lace, valued at $625, an allowance of 12% being made at the custom-house for damage received since shipped, duty 24%.

9. On 350 boxes of cigars, each containing 100 cigars, invoiced at $7.50 per box; weight, 12 lb. per 1000; duty, $2.50 per lb., and 25% ad valorem.

285. Mixed problems in percentage, in which the element of *time* does not appear.

1. Bought 450 bu. of wheat, at $1.25 a bushel, and sold it at $1.40 a bushel. What was the whole gain, and the gain %?

2. An agent sold a consignment of goods for $5250, and charged 3¼% commission, and 2½% for guaranty. Find the net proceeds.

3. Sold cloth at $3.60 a yard, and gained 12½%. What should the selling price have been, to gain 20%.

4. What is the difference on a bill of $750, between a discount of 40%, and a discount of 30 and 10%?

5. A cargo of 4000 bu. of wheat, worth $1.20 a bushel, is insured at ¾ of 1½% on ⅔ of its value. If the cargo is lost, how much will the owner of the wheat lose?

6. If a stationer marks his goods 50% above cost, and then deducts 50%, what per cent. does he lose?

7. If 20 and 10% are deducted from the retail price of an article, and it then sells for $1.80, what is the retail price, and the rate of discount?

8. A sells his horse for $198, which is 10% less than his asking price, and the asking price is 10% more than the cost. What did the horse cost him?

9. A machinist sold 24 grain-drills for $125 each. On one-half of them he gained 25%, and on the other half he lost 25%. Did he gain or lose on the whole, and how much?

10. Bought land, at $30 an acre. How much must I ask an acre, that I may abate 25% from my asking price, and still make 20% on the purchase money?

11. In a certain district a school-house is to be built at an expense of $9120, to be defrayed by a tax upon property valued at $1536000. What shall be the rate of taxation to cover both the cost of the school-house, and the collector's commission at 5%?

12. A merchant, on opening a case of goods that cost $.80 a yard, finds them slightly damaged. How must he mark them, to fall 25% in his asking price, and sell at cost?

13. Having used a carriage six months, I sold it for $96, which was 20% below cost. What would I have received, had I sold it for 15% above cost?

14. A trader got a shipment of 500 barrels of flour insured for 80% of its cost, at 3¼%, paying $107.25 premium. At what price per barrel did he purchase the flour?

15. Sold ⅝ of a barrel of beef for what the whole barrel cost. What per cent. did I gain on the part sold?

16. A lumberman sold 36840 feet of lumber at $21.12 per M, and gained 28%. What would he have gained or lost, had he sold it at $17 per M?

17. Sold a farm of 106 A. 150 P. for $96 an acre, and gained 18% on the cost. What did the whole farm cost?

18. Find the duty at 15% on 175 bags of Java coffee, each containing 115 lb., valued at 15¢ a pound?

19. A salesman asked an advance of 20% on the cost of some goods, but was obliged to sell at 20% less than his asking price. Did he gain or lose, and what per cent.?

20. If I buy a piece of land, and it increases in value each year at the rate of 50% on the value of the previous year, for 4 years, and then is worth $12000, what did it cost?

21. A hotel valued at $10000 was insured for $6000 at 1¼%, $5.50 being charged for the policy and the survey of the premises; if destroyed by fire, what loss would the owner suffer?

22. A dry-goods merchant sells delaines for 2⅛ cents per yard more than they cost, and realizes a profit of 8%. What was the cost per yard?

23. A speculator gained 30% on ⅔ of his investment, and lost 5% on the remainder, and his net profits were $720. What would have been his profits, had he gained 30% on ⅓, and lost 5% on the remainder?

INTEREST.

INDUCTIVE EXERCISES.

286. 1. At 6%, what decimal part of the money borrowed is equal to the money paid for its use? At 5%? 4½%? 7%? 8%?

2. At 6%, how many dollars must be paid for $100, 1 yr.? For 2 yr.? 4 yr.? 6 yr.?

3. At 5%, how many dollars must be paid for the use of $200, 1 yr.? $200, 2 yr.? $300, 3 yr.?

4. When $5 is paid for the use of $100 one year, what is the annual rate %? If $6? If $7½? If $10?

287. ***Interest*** is a sum paid for the use of money, or its equivalent.

288. The ***principal*** is the sum for the use of which interest is paid.

289. The ***rate*** of ***interest*** is the per cent. of the principal paid for its use *one year*, or some given interval of time.

290. The ***amount*** is the sum of the principal and interest.

Legal interest is the rate established by law.
Usury is a greater rate than the legal rate.

291. In the operations of interest, *four* elements are considered, viz.:

The *principal*, the *rate per annum*, the *interest*, and the *time*.

1. The **principal** is the *base*.
2. The **rate per annum** is the *rate*.
3. The **interest** is the *percentage*.
4. The sum of the principal and interest is the *amount*.

292. ***Time*** is an *additional* element, to be considered always in connection with the rate per annum.

293. The method of finding the interest on any principal, for any rate and time, is represented by the

FORMULA : $I = P \times R \times T$, and read,

INTEREST equals the product of the PRINCIPAL, *the* RATE PER ANNUM, *and the* TIME, *expressed in years.*

Of the various methods in use for computing interest, th most simple, brief, and practical are presented in this work.

Since 6% is the legal rate in the majority of the States, the "six per cent. method" is presented as likely to be largely used.

The method by *Cancellation*, *uniform* in its applications to *any rate* and *time*, is *brief* and *simple*, hence, *practical*, and needs only to be understood to be preferred to all others.

WRITTEN EXERCISES.

294. 1. Find the interest and the amount of $97.50, at 7%, for 2½ yr.

1ST SOLUTION.

$97.50	
.07	
$6.8250	Int. for 1 yr.
2½	
$17.0625	Int. for 2½ yr.
97.50	Prin.
$114.5625	Amt.

2D SOLUTION.

	$97.50
	.17½
Int. for 2½ yr.	$17.0625
Prin.	97.50
Amt.	$114.5625

The int. of any sum, at 7%, for 2 yr. 6 mo. is 2½ times .07, or .17½ of the principal.

2. Find the int of $267.60, for 1 yr. 4 mo. 18 da., at 5%.

EXPLANATION.—Since 30 da. are reckoned as a month, 3 da. is $\frac{1}{10}$, or .1 of a month; hence, 1 yr. 4 mo. 18 da. equals 16.6 mo. So any number of days is readily reduced to *tenths* of a mo. by dividing by 3. Thus, 12 da. is .4 of a mo.; 21 da., .7; 25 da., .8⅓.

$267.60	
.05	
12) $13.3800	Int. for 1 yr.
1.115	Int. for 1 mo.
16.6	Time in mo.
$18.5090	Int. for 1 yr. 4 mo. 18 da.

RULE.—1. *Multiply the principal by the rate, to find the interest for 1 year.*

2. *Divide the interest for 1 yr. by 12, to find the interest for 1 month.*

3. *Multiply the interest for 1 mo. by the time in months and tenths of a month, to find the required interest.*

The above is a very simple and practical rule, and much used.

In like manner, find the *interest* and the *amount*,

3. Of $137.25 for 1 yr. 6 mo. 10 da. at 6%. At 4%.

4. Of $510.50 for 3 yr. 7 mo. 15 da. at 5%. At 8%.

5. Of $1297.60 for 2 yr. 11 mo. 18 da. at 7%. At 7½%.

6. Of $36.40 for 1 yr. 7 mo. at 6%. At 7%. At 7½%.

7. Of $750.50 for 3 yr. 1 mo. at 5%. At 8%. At 9%.

8. Required the amount of $408.60 from Aug. 20 to Dec. 18, 1886, at 10%.

9. What is the interest on a note for $515.62, dated March 1, 1883, and payable July 16, 1885, at 7%?

10. What is the value of a note of $65.75, due with interest for 3 yr. 2 mo., at 6½%?

11. If a person borrow $375 at 5%, what will be due the lender at the end of 2 yr. 6 mo.?

12. A man sold his house and lot for $12500; the terms were, $4000 in cash on delivery, $3500 in 9 mo., $2600 in 1 yr. 6 mo., and the balance in 2 yr. 4 mo., with 6% interest. What was the whole amount paid?

SIX PER CENT. METHOD.

295. The interest of $1 at 6%,

For	12 mo.,	or	1 year,	is	6 cents,	or	.06	of the principal
"	2 "	"	1/6 "	"	1 "	"	.01	" "
"	1 "	"	1/12 "	"	5 mills,	"	.005	" "
"	6 da.	"	1/5 month,	"	1 "	"	.001	" "
"	1 "	"	1/30 "	"	1/6 "	"	.000⅙	" "

Hence, the following

PRINCIPLES.—I. *The interest on any sum at 6%, is* SIX *times as many hundredths of the principal, as there are* YEARS *in the given time;* or,

II. *One* HALF *as many hundredths of the principal, as there are* MONTHS *in the given time;* or,

III. *One* SIXTH *as many thousandths of the principal, as there are* DAYS *in the given time.*

Thus, the interest on any sum, at 6% for 3 yr., is 6 times .03, or .18 of the principal; for 9 mo. it is ½ of .09, or .04½, or .045 of the principal; for 19 da. it is ⅙ of .019, or .003⅙ of the principal. Hence, for 3 yr. 9 mo. 19 da. it is .18 + .045 + .003⅙ = .228⅙ of the principal.

To find the interest at any other rate per cent., divide the interest at 6% by 6, and multiply the quotient by the given rate.

WRITTEN EXERCISES.

296. 1. Find the int. of $427.20, at 6%, for 2 yr. 5 mo. 27 da.

SOLUTION.

Interest for 2 yr. = 6 times .02 = .12 of the principal.
" " 5 mo. = ½ of .05 = .025 " "
" " 27 da. = ⅙ of .027 = .004½ " "
Int. = .149½ " "

Hence, $427.20 × .149½ = $63.8664, the *required int.*

Find the *interest* at 6% of

2. $267.27 for 6 mo. 24 da.
3. $146.18 for 1 yr. 21 da.
4. $256.84 for 2 yr. 4 mo. 12 da.
5. $597.25 for 7 mo. 18 da.
6. $418.75 for 1 mo. 25 da.
7. $309.18 for 2 yr. 24 da.
8. $38.90 for 1 yr. 1 mo. 6 da.
9. $146.48 for 9 mo. 10 da.
10. $275.50 for 11 mo. 27 da.
11. $1298 for 3 yr. 1 mo. 13 da.
12. $2000 for 2 yr. 7 mo. 24 da.
13. $4010 for 1 yr. 1 mo. 13 da.
14. $450 for 63 da.; 33 da.; 93 da.; 120 da.; 47 da.

Find by either of the preceding methods the *int.* of

	Principal,	from,	to,	rate.
15.	$35.61,	Nov. 11, 1881,	Dec. 15, 1883,	6%.
16.	$50,	Sept. 4, 1880,	Jan. 1, 1882,	3½%.
17.	$97.86,	May 17, 1876,	Dec. 20, 1883,	7%.
18.	$325.28,	June 20, 1882,	Sept. 4, 1884,	8%.
19.	$154.75,	Apr. 10, 1883,	Nov. 24, 1883,	6%.
20.	$861.50,	June 3, 1881,	March 25, 1882,	5%.

Find the *amount* of

21.	$450.80,	March 6, 1883,	Dec. 20, 1883,	6%.
22.	$1500,	May 5, 1884,	Jan. 20, 1885,	4%.
23.	$127.36,	Dec. 12, 1879,	July 3, 1881,	4½%.

CANCELLATION METHOD.

297. This *uniform* method of computing interest for any *time* and *rate* by *Cancellation,* so simple and easily understood, is especially commended to the attention of the learner.

All cases of simple interest that can arise are covered by the following

FORMULAS:

1. When the time is expressed in *years.*

$$Principal \times rate \times time = interest \ (\mathbf{293}).$$

2. When the time is expressed in *months.*

$$\frac{Principal \times rate \times time}{12} = interest.$$

3. When the time is expressed in *days.*

$$\frac{Principal \times rate \times time}{360} = interest.$$

WRITTEN EXERCISES.

298. 1. Find the int. of $337 for 2 yr. 8 mo. at 6%; also, of $320 for 3 mo. 15 da. at 4½%; of $76.50 for 2 yr. 2 mo. at 5%.

	$337 Prin.
100	6 Rate.
3	8 Years.
100	$5392
	$53.92 Int.

Or,	$337
100	6
12	32
25	$1348
	$52.92

	$320 Prin.
2	9 Rate.
100	
2	7 Years.
12	
10	$42
	$4.20 Int.

100	$7650 Prin.
100	5 Rate.
12	26 Years.
80	$663
	$8.287 Int.

It will be observed that the principle and the solution of the above examples are essentially the same as before presented; namely, that *the interest is the product of the principal by the rate and time* (**293**).

The *rate* may be expressed *decimally*, but it is more convenient to express it as a fraction—100, expressed or understood, being the denominator.

When the principal contains cents, removing the decimal point multiplies by 100; if mills, by 1000. Hence, if the principal contains cents or mills, place 100 or 1000 on the left, as the case may be, and remove the decimal point.

In the above examples, the *time*, 2 yr. 8 mo., may be expressed as 2⅔ or $\frac{8}{3}$ yr., or as 32 mo. ($\frac{32}{12}$ yr.); 3 mo. 15 da. as 3½ mo., or $\frac{7}{2}$ mo., or 105 da. ($\frac{105}{360}$ yr.); and the *rate*, 4½%, as $\frac{9}{2}$%. All mixed numbers should be changed to improper fractions.

RULE.—1. *Draw a vertical line, and on the right, first write the principal in dollars, next the rate per year, and last the time in years, placing numerators on the right and denominators on the left.*

2. *Cancel equal factors, if any, on opposite sides of the line; and the product of the remaining factors on the right, divided by the factor, or product of the factors on the left, will give the required interest.*

For "Exact Int." and "Annual Int.," see **438** and **439**, Addenda.

The advantage of this method is, that the operation is always *correctly indicated*, and in most examples the work is shortened by cancellation.

2. Find the int. of $264.80 for 3 mo. 21 da., at 8%; also, of $648 for 4 mo. 12 da., at 7½%.

Explanation.—3 mo. 21 da. may be expressed as 111 da.; 4 mo. 12 da. as 4⅖, or 22/5 mo., or as 132 da.; and the *rate* 7½% as 15/2%. Either form may be used, as most convenient.

100	$26480 Prin.
100	8 Rate.
360	111 Years.
1875	$12247
	$6.53 Int.

	$648
2	15
100	
360	132
50	$891
	$17.82 Int.

3. Find the int. of $840 for 18 da., at 1½% a mo.; for 3 mo. 10 da., at 2% a mo.; for 5 mo. 22 da., at ¾% a mo.; and for 93 da., at ⅘% a mo.

	$840 Prin.
100	18 Rate per yr.
360	18 Years.
100	$756
	$7.56 Int.

	$840
100	24
3	10
12	
	$56 Int.

	$840
100	9
360	172
25	$903
	$36.12 Int.

	$840
5	48
100	
360	93
	$20.83 Int.

Explanation.—It is plain that 1½% a mo. is 18% a year; 2% a mo. is 24% a year; ¾% a mo. is 9% a year; and ⅘% a mo. is 48/5% a year.

So many illustrative examples have been given, that the simplicity and practical utility of this method might be apparent.

By any of the preceding methods, find the *interest*

	Of	for	at
4.	$76.50,	2 yr. 2 mo.,	5%.
5.	$1276.25,	11 mo.,	7%.
6.	$1500,	6 mo. 24 da.,	7½%.
7.	$2162.94,	1 yr. 0 mo. 18 da.,	10%.
8.	$2500,	93 da.,	8%.
9.	$2000,	33 da.,	6%.
10.	$54.75,	3 yr. 0 mo. 24 da.,	5%.
11.	$51.10,	10 mo. 3 da.,	4%.
12.	$460.50,	60 da.,	¾% a mo.
13.	$650,	4 mo. 5 da.,	2% a mo.

Find the *amount*

	Of	for	at
14.	$84.25,	1 yr. 5 mo. 10 da.,	6¼%.
15.	$500,	93 da.,	1% a mo.
16.	$2500,	7 mo. 20 da.,	5%.
17.	$1500,	76 da.,	½% a mo.
18.	$370.80,	1 yr. 10 mo. 6 da.,	6%.
19.	$1200,	1 yr. 0 mo. 9 da.,	4½%.
20.	$960,	10 mo. 24 da.,	7%.
21.	$860,	84 da.,	1¼% a mo.
22.	$1280,	7 mo. 10 da.,	¾% a mo.

23. If a man borrow $9700 in N. Y., at 6%, and loan it in Colorado, 10%, what will it gain in 1 yr. 6 mo.?

24. A person borrows $3754.45, being the property of a minor who is 15 yr. 3 mo. 20 da. old. He retains it until the owner is 21 yr. old. How much money will then be due at 6%.

25. A note for $710.50, with interest after 3 mo., at 7%, was given Jan. 1, 1884, and paid Aug. 12, 1886. What was the amount due?

26. A speculator borrowed $9675, at 6%, April 15, 1884, with which he purchased flour at $6.25. May 10, 1885, he he sold the flour at $7⅜ a barrel, cash. What did he gain by the transaction?

27. A man, engaged in business with a capital of $21840, is making 12½% per annum on his capital; but on account of ill health he quits his business, and loans his money at ¾% a month. How much does he lose in 2 yr. 5 mo. 10 da. by the change?

28. Bought 4500 bu. of wheat at $1.12½ a bushel, payable in 6 mo.; I immediately realized for it $1.06 a bushel, cash, and put the money at interest at 10%. At the end of the 6 mo. I paid for the wheat. Did I gain or lose by the transaction, and how much?

PROBLEMS IN INTEREST.

299. Since interest is the product of the three factors, principal, rate, and time (**293**), any *one* of these *three* factors may be found by dividing the interest by the product of the other two. Hence,

The *general formula*, $I = P \times R \times T$, produces the following *formulas*. Observe that I, divided by the product of any *two* letters of the *three*, P, T, R, equals the other *one*.

$$P = \frac{I}{R \times T}; \qquad T = \frac{I}{P \times R}; \qquad \frac{R}{100} = \frac{I}{P \times T}.$$

The above is a brief substitute for the numerous rules given in the books, and is simple and easy to be remembered. The pupil may be required to construct a rule for each.

WRITTEN EXERCISES.

300. Find each *one* of the *four* elements in the following example, from the other *three*, by the above formulas.

	Principal.	Rate %.	Time.	Interest.
1.	$1600,	7%,	2 yr. 6 mo.	$280.

$$\begin{array}{r|l} & \cancel{\$1600}\ ^{8} \\ \cancel{100} & 7 \\ \cancel{2} & 5 \\ \cline{2-2} & \$280 \end{array}$$

1st. $I = P \times R \times T$.

Or, $\$1600 \times .07 \times 2.5 = \$280$, *int.*

$$\begin{array}{r|l} & \cancel{\$280}\ ^{40} \\ \cancel{7} & \cancel{100}\ ^{20} \\ \cancel{5} & 2 \\ \hline & \$1600 \end{array}$$

2d. $P = \frac{I}{R \times T}$.

Or, $\$280 \div (.07 \times 2.5) = \$1600$, *prin.*

If the *amount* is given instead of the *interest,* divide it by the amount of $1 at the given rate and time; thus,

$$P = \frac{A}{\$1 + R \times T}.$$

$$\begin{array}{r|l} {}^{100}\,\cancel{\$1600} & \cancel{\$280}\,{}^{7} \\ \cancel{5} & \cancel{2} \\ \hline 100 & 7 = \tfrac{7}{100} = .07 \end{array}$$

3d. $\frac{R}{100} = \frac{I}{P \times T}.$

Or, $280 ÷ (1600 × 2.5) = .07, *rate.*

$$\begin{array}{r|l} {}^{2}\,\cancel{\$1600} & \cancel{\$280}\,{}^{5} \\ \cancel{7} & \cancel{100} \\ \hline 2 & 5 = 2\tfrac{1}{2}\text{ yr.} \\ & \quad = 2\text{ yr. }6\text{ mo.} \end{array}$$

4th. $T = \frac{I}{P \times R}.$

Or, $280 ÷ ($1600 × .07) = 2.5 yr., *time.*

The preceding brief formulas will enable the pupil readily to solve all problems pertaining to interest.

In using the vertical line and cancellation, it will be remembered, that numerators or *dividends* are placed on the *right,* and denominators or *divisors* on the *left,* and that to divide by a fraction we *invert* the terms.

The time should be expressed in *years;* thus, 2 yr. 6 mo. is $2\frac{1}{2}$ or 2.5 yr.; 1 yr. 4 mo. is $1\frac{1}{3}$ or $\frac{4}{3}$ yr.; 9 mo. is $\frac{9}{12}$, or $\frac{3}{4}$, or .75 yr.; 7 mo. is $\frac{7}{12}$ yr.; 2 mo. 12 da. is $\frac{72}{360}$ yr., etc. The result will be in years and decimals of a year, which may be reduced to months and days (**198**).

In the following examples, find *each one* of the elements:

	Principal.	Rate.	Time.	Interest.
2.	$560,	6%,	2 mo. 12 da.,	$6.72.
3.	$315,	4%,	3 yr. 6 mo.,	$44.10.

What *principal*

4. Will gain $173.97 in 4 yr. 4 mo. at 6%? At 12%?
5. Will gain $153.75 in 3 mo. 24 da. at 7%? At 8%?
6. Will amount to $1596 in 2 yr. 6 mo. at $5\frac{1}{2}$%?
7. Will amount to $1531.50 in 3 mo. 18 da. at 7%?

At what *rate %*

8. Will $350 gain $70 in 2 yr. 6 mo.?
9. Will $550 gain $220 in 5 yr. 4 mo.?
10. Will $2085 gain $68.11 in 5 mo. 18 da.?

In what *time* will

11. $245 gain $2.94 at 6%? | 13. $550 amt. to $576.95 at 7%?
12. $600 gain $132 at 8%? | 14. $204 amt. to $217.09 at 5½%?

15. In what time will $1600 amt. to $2000, at 8%?

16. What sum invested at 8% will yield an average income of $1560?

17. A man invests $15600, which gives him an annual income of $1014. What rate of interest does he receive?

18. What sum of money must be invested in real estate, yielding 10% profit in rents, to produce an income of $3370?

19. A note which had run 4 yr. 6 mo. amounted to $2000, at 7%. What was the principal?

20. B loaned $1600 at 6% until it amounted to $2000. What was the time?

21. I invest $35680 in a business that pays me a profit of $223 a month. What annual rate of interest do I receive?

The *rate % per annum* at which any sum will double itself may be found by dividing 100 yr. by the number of years.

22. At what rate will any number double itself in 16⅔ yr.? In 12½ yr.? In 8 yr. 4 mo.? In 6 yr. 3 mo.?

The *time in years* in which any sum will double itself may be found by dividing 100% by the rate %.

23. In what time will any number double itself, at 3%? 4½%? 5%? 6%? 7%? 8%? 10%? 12½%?

24. In what time will the int. of any number at 8%, equal the principal? Twice the principal? One-half the principal?

25. In what time will $9750 gain $780 int. at 2% a mo.?

26. If I loan $750 at simple interest, and at the end of 1 yr. 8 mo. receive $843.75, what is the rate per cent?

27. If I receive $206.25 each quarter for rent of a house that cost me $15000, what rate % do I receive on the investment?

28. How large a sum in the savings bank, at 4% int., will give a yearly income of $1000?

301. ***Compound interest*** is interest on both principal and interest, when the interest is not paid when due.

WRITTEN EXERCISES.

302. 1. Find the amt. and the comp. int. of $640, for 2 yr. 4 mo. 15 da., at 5% ?

SOLUTION.

	$640	Prin. for 1st year.
$640 × .05 =	32	Int. for 1st year.
	$672	Prin. for 2d year.
$672 × .05 =	33.60	Int for 2d year.
	$705.60	Prin. for 4 mo. 15 da.
$705.60 × .01875 =	13.23	Int. for 4 mo. 15 da.
	$718.83	Amt. for 4 mo. 15 da.
	$640	Given principal.
	$78.83	Comp. interest.

RULE.—*Find the amount of the given principal for the first period of time. Using this amount as a new principal, find its amount for the second period, and so on for the entire time.*

2. *Subtract the given principal from the last amount, and the remainder will be the compound interest.*

If the interest is compounded semi-annually, the rate % is *one-half* the yearly rate ; if quarterly, *one-fourth* the yearly rate. Thus, 2 yr. 6 mo. in semi-annual payments at 6% is the same as 5 yr. at 3% ; and 1 yr. 9 mo. quarterly, at 8%, the same as 7 yr. at 2%.

Find the *amount* and the *compound interest* of

2. $312, for 3 yr. at 6%; $800, for 4 yr. at 4%.
3. $640, for 4 yr. at 5% ; $376, for 3 yr. 8 mo. 15 da. at 6%.
4. $1200, for 2 yr. 4 mo. at 4½%; for 3 yr. 8 mo. at 7%.
5. $400, for 1 yr. 6 mo. at 7%, payable semi-annally.
6. $2000, for 1 yr. at 8%, payable quarterly.
7. What is the annual income from $2500, at 6%, interest added to the principal quarterly ?

NOTES.

303. A ***promissory note*** is a *written promise* to pay a certain sum, either on demand, or at a specified time.

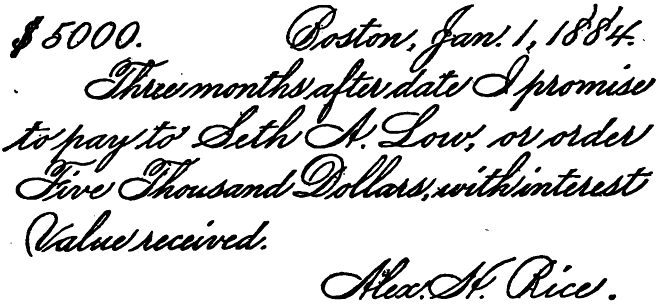
The **face** of a note is the sum of money made payable by it.

The **maker** or **drawer** of a note is the party who signs it.

The **payee** of a note is the party to whom or to whose order the money is made payable.

An **endorser** of a note is the party who writes his name on the back of the note, and thereby makes himself responsible for its payment.

The following is a simple form of a negotiable note:



$5000. Boston, Jan. 1, 1884.
Three months after date I promise
to pay to Seth A. Low, or order
Five Thousand Dollars, with interest
Value received.
Alex. H. Rice.

A note should always contain the words "*value received.*"

A note containing the words "*with interest,*" or "*with use,*" draws interest from date. When these words are omitted, the note will not draw interest until after it is due.

The legal rate prevails when no rate is mentioned.

304. A ***negotiable note*** is one made payable to *bearer*, or to any person's *order*. It can then be sold, or transferred.

If the words "*or order,*" or "*or bearer*" are omitted, the note is not transferable, and is payable only to the person whose name is mentioned in it.

A note made by a minor, or on Sunday, is void.

An endorser of a note may prevent his own liability to be sued, by writing upon it, "*without recourse,*" or similar words.

DISCOUNT.

305. ***Discount*** is an allowance made for the payment of a debt, note, or other obligation before it is due.

306. The ***present worth*** of any debt, note, or other obligation, is the amount of it, less the discount.

307. The ***true present worth*** of a debt payable at a future time without interest, is such a sum as, being put at legal interest, will amount to the debt when it becomes due.

308. The ***true discount*** is the difference between the whole debt and the true present worth.

Time is *always* an element in the operations of true discount.

309. The corresponding *terms* of true discount and *percentage* are as follows:

1. The **true present worth** is the *base.*
2. The **rate %** is the *rate.*
3. The **true discount** is the *percentage.*
4. The **whole debt** is the *amount.*

WRITTEN EXERCISES.

310. To find the present worth and true discount.

1. Find the present worth and the true discount of $362.95, payable in 7 mo. 12 da., at 6%.

EXPLANATION.—The amt. of $1 for 7 mo. 12 da., at 6% = $1.037.
The *present worth* is $362.95 ÷ 1.037 = $350.
The *true discount*, $362.95 − $350 = $12.95.

FORMULAS: { 1. *Debt ÷ amt. of $1 = present worth.*
2. *Debt − present worth = true discount.*

Find the *present worth* and *discount*,

2. Of $614.40, due in 4 mo. 24 da., at 6%.

3. Of $2640, due in 9 mo. 18 da., at 7%.

4. Of $475, due in 7 mo. 6 da., at 5%.

5. A merchant buys goods for $4200 on 4 mo. credit, but is offered a discount of 3% for cash. If money is worth ½% a month, what is the difference?

6. If I sell goods for $1250 on 6 months' credit, which cost me $1195 cash, what is my loss, money being worth to me 10%?

7. What is the present worth of a debt for $1005, on which $475 is to be paid in 10 mo., and the remainder in 1 yr. 3 mo., the rate of interest being 6%?

8. What is the difference between the interest and true discount of $130, due 10 mo. hence, at 10%?

9. A person sold goods to the amount of $3750, 15% payable in cash, 25% in 3 mo., 20% in 4 mo., and the remainder in 6 mo. What ready money would discharge the whole debt, money being worth 6%?

BANK DISCOUNT.

311. ***Bank Discount*** is simple interest paid in advance to a bank, for the payment of a note before it becomes due.

312. ***Days of grace*** are three additional days, usually allowed by law for the payment of a note after the expiration of the time specified in it.

313. The ***maturity*** of a note is the expiration of the whole time, including the days of grace.

314. The ***term of discount*** is the time from the discount of a note to its maturity.

315. The ***proceeds*** of a note is the sum received for it when discounted, or the face of the note less the discount.

316. A ***protest*** is a formal declaration in writing, made by a notary public, giving legal notice to the maker and the indorsers of a note, of its non-payment.

1. The failure to protest a note on the third day of grace releases the indorsers from all obligation to pay it.

2. If the third day of grace or the maturity of a note occurs on Sunday or a legal holiday, it must be paid on the day previous.

317. The corresponding *terms* are as follows:

1. The **face** of the note is the *base.*
2. The **rate %** is the *rate.*
3. The **bank discount** is the *percentage.*
4. The **term of discount** is the *time.*

WRITTEN EXERCISES.

318. To find the bank discount and proceeds of a note.

1. Find the bank discount and proceeds of a note for $450, due in 60 da., at 6%.

EXPLANATION.—The *term* of discount is 63 da.
The *face* of the note is $450.
The *discount* is the int. of $450 for 63 da., at 6% = $4.725.
The *proceeds* are $450 − $4.725 = $445.275 (**242**, 3).

FORMULAS: { 1. *Face × rate × term of disc't = bank disc't.*
2. *Face − bank discount = proceeds.*

Find the *bank discount* and *proceeds* of

2. $368 for 90 da., at 7%.
3. $597.50 for 60 da., at 6%.
4. $684.60 for 4 mo., at 5%.
5. $1080 for 78 da., at 8%.
6. $875 for 3 mo. 10 da., at 6%.
7. $1250 for 2 mo. 15 da., at 5%.
8. $480 for 4 mo. 12 da., at 10%.
9. $940 for 117 da., at 7%.

10. Find the bank discount on a note at 6 mo., for $672.50, at 6%.

11. What will be the proceeds of a note at 3 months for $1500, discounted at a bank, at 7%?

12. Required the bank discount and proceeds of a note for $480, due in 9 mo. 27 da., at 8%.

13. Find the difference between the *bank* and the *true* discount on a note for $1500, due in 60 da., at 6%.

14. Find the proceeds of a 4 mo. note for $560, dated April 1, 1882, and discounted May 15, 1882, at 6%.

Find the *discount* and *proceeds* of notes, using the following conditions, all dates being in the same year:

	Face.	Date.	Time to run.	Day of discount.	Rate.
15.	$900,	Sept. 4,	90 da.,	Oct. 10,	6%.
16.	$680,	Jan. 1,	4 mo.,	[illegible] 4,	8%.
17.	$1250,	May 20,	60 da.,	June 11,	7%.
18.	$1500,	June 12,	30 da.,	June 24,	10%.
19.	$375.60,	Oct. 17,	3 mo.,	Nov. 13,	5%.

20. Find the proceeds of a note for $1615, due in 3 mo., *with interest*, at 7%, discounted at bank.

When a note bears interest, find the discount on the *amount* of the note at maturity.

Find the *maturity*, *term of discount*, and *proceeds* of the following notes:

$1250. New Orleans, June 12, 1882.

21. Six months after date, I promise to pay Jas. Martin, or order, twelve hundred fifty dollars, *with interest* at 5%, value received. Geo. Lane.

Discounted at a broker's, Nov. 15.

15

$$497\tfrac{32}{100}.$$ Chicago, March [illegible], 1882.

22. Three months after date, we promise to pay J. C. Ellis, or order, four hundred and ninety-seven $\frac{32}{100}$ dollars, with interest at 6%. Value received. Hadley Bros.

Discounted at bank, April 20.

916\frac{25}{100}$.   SAN FRANCISCO, Feb. 4, 1883.

23. Two months after date, we jointly and severally agree to pay JOHN BLISS, or order, nine hundred sixteen $\frac{25}{100}$ dollars, *with int.* at 2% a mo. Value received.

T. H. ROSE,
B. F. MARKS.

Discounted at Marine Bank, Feb. 21, at 10%.

319. To find the face of a note.

1. For what sum must a note be drawn, payable in 90 da., at 6%, so that the proceeds may be $5000?

EXPLANATION—The *bank discount* of $1 for 93 da., at 6% is $.0155.
The *proceeds* of $1 is $1 − $.0155 = $.9845.
The *face* of the note is $5000 ÷ $.9845 = $5078.72.

FORMULA: *Proceeds of note ÷ proceeds of $1 = face of note.*

Find the *face* of notes, the proceeds, time, and rate % of which are as follows:

2. $1265 for 60 da., at 6%.
3. $276.84 for 3 mo., at 7%.
4. $1200 for 30 da., at 5%.
5. $357.36 for 30 da., at 8%.
6. $2844 for 8 mo. 9 da., at 7%.
7. $236.28 for 90 da., at 6%.

8. Find the face of a 6 mo. note, the proceeds of which, discounted at 2% a month, are $496.

Find the *face* of notes, answering to the following data:

	Proceeds.	Date.	Time.	Date of disc't.	Rate.
9.	$640,	Mar. 10,	60 da.,	Mar. 10,	6%.
10.	$380,	April 16,	90 da.,	May 19,	6%.
11.	$1260,	Jan. 16,	4 mo.,	Mar. 13,	8%.

12. Owing a man $575, I give him a 60 da. note; what should be the face of the note, to pay him the exact debt, if discounted at $\frac{3}{4}$% a mo.?

13. Find the face of a note which, discounted at a broker's for 110 da., at 1% a mo., will give as its proceeds $187.50?

PARTIAL PAYMENTS.

320. ***Partial payments*** are payments in part of the amount of a note, bond, or other obligation.

321. ***Indorsements*** are the acknowledgment of such payments, written on the back of the note, bond, etc., stating the time and amount of the same.

322. U. S. COURT RULE.—1. *Find the amount of the given principal to a time when the payment, or the sum of the payments, equals or exceeds the interest due.*

2. *Subtract the payment, or sum of the payments, from the amount, and treat the remainder as a new principal.*

3. *Proceed in the same manner with the new principal, and each succeeding one, to the time of settlement.*

WRITTEN EXERCISES.

323. To compute interest on notes and bonds, when *partial payments* have been made.

$1500. NEW YORK, April 1, 1881.

1. *On demand, I promise to pay to the order of* CHAS. RAY, *fifteen hundred dollars, with interest at 6%. Value received.*

S. M. PERKINS.

On this note were endorsed the following payments:

Sept. 16, 1881, $250; May 6, 1882, $375; June 24, 1883, $675. What was due Sept. 9, 1883?

First find the difference between the successive dates, by arranging a "Time Table," as follows:

Write in the first column, in their order, the date of the note, the date of each payment, and the date of settlement.

In the second column, write the difference between each date and the succeeding one, beginning at the top and subtracting downward.

In the third column, write the payments in their order; and in fourth the face of the note, and as the work progresses, write also the amount due after each payment, to be regarded as a new principal.

YR.	MO.	DA.	DIFFERENCE BETWEEN DATES.							PAYMENTS.	PRINCIPALS.
1881	4	1	yr.	mo.	da.		mo.		da.		$1500
1881	9	16		5	15	=	5½	or	165	$250	$1291.25
1882	5	6		7	20	=	7⅔	or	230	$375	$965.747
1883	6	24	1	1	18	=	13.6	"	408	$675	$356.417
1883	9	9		2	15	=	2½	"	75		$360.87

In computing the interest, the time may be reduced to months or to days, as shall best be adapted to the method used.

The *reduction* need not appear in the table, the design of the above being to show the *completed* work.

We give the solution of the preceding example in full, using the U. S. Rule and the method by cancellation, leaving the cancelling to the ingenuity of the pupil. (**74**, NOTE 2.)

After arranging the table, compute the interest on the principal from the date of the note to the time of the first payment, or for 5 mo. 15 da., etc.

FIRST STEP.

	$1500	Prin.
100	6	Rate.
2	11	Time.
12		
	.$41.25	Int.
	$1500	
	$1541.25	Amt.
	250	Payment.
	$1291.25	1st new prin.

SECOND STEP.

100	$129125	1st new prin.
100	6	Rate.
3	23	Time.
12		
	$49.497	Int.
	$1291.25	
	$1340.747	Amt.
	375	Payment.
	$965.747	2d new prin.

THIRD STEP.

100	$96575	2d new prin.
100	6	Rate.
360	408	Time.
	$65.67	Int.
	965.747	
	$1031.417	Amt.
	675	Payment.
	356.417	3d new prin.

FOURTH STEP.

100	$35642	3d new prin.
100	6	Rate.
2 12	5	Time.
	$4.455	Int.
	$356.42	
Ans.	$360.875	Last amt.

The simplicity, brevity, and practical character of the foregoing method, make it vastly superior to the old methods.

$3000. MACON, May 7, 1882.

2. On demand, we promise to pay to Robt. E. Park, or order, three thousand dollars, with interest at 7%. Value received. J. W. BURKE & Co.

Indorsed as follows: Sept. 10, 1882, $25; Jan. 1, 1883, $500; Oct. 25, 1883, $75; April 4, 1884, $1500. What remained due, Feb. 20, 1885?

DATES.			DIFFERENCE BETWEEN DATES.		PAYMENTS.	PRINCIPALS.
1882	5	7	mo.	da.		$3000, principal.
1882	9	10	4	3	$25	$2611.50, 1st new prin.
1883	1	1	3	21	$500	
1883	10	25	9	24	$75	$1266.53, 2d new prin.
1884	4	4	5	9	$1500	
1885	2	20	10	16	*Ans.*	$1344.35, last amount.

Observe, the first payment is *less* than the int. due; hence, compute the int. to the time of the second pay't, or for 7 mo. 24 da., and subtract the sum of the two pay'ts. The same thing occurs in the third pay't.

$500. **San Jose, Feb. 1, 1885.**

3. Three months after date, I promise to pay to the order of C. H. Allen, five hundred dollars, with interest at 1¼% a month. Value received. John Swett.

Indorsements: May 1, 1885, $40; Nov. 13, 1885, $20; May 1, 1886, $75. What was due Sept. 16, 1886?

$475 $\frac{50}{100}$. **Racine, May 1, 1882.**

4. Nine months after date, I promise to pay A. R. Sprague, or order, four hundred seventy-five and $\frac{50}{100}$ dollars, with 7% interest. Value received. B. B. Northrup.

The following payments were made on this note:

Dec. 25, 1882, $50.	Sept. 1, 1883, $25.50.
July 10, 1883, $15.75.	June 16, 1884, $104.

What was due April 13, 1885?

5. Find what would be due on the same note Oct. 10, 1884, at 8%.

6. D. W. McWilliams holds a bond against Jacob Gosler, dated June 20, 1882, for $4800, on interest at 6%. Indorsements: May 5, 1883, $1200; Aug. 14, 1884, $950; May 12, 1885, $2000. What will be due Nov. 24, 1885?

7. A note for $900, dated March 10, 1881, payable on demand, with interest, at 6½%, bears the following indorsements: July 3, $175; Oct. 21, $284.50; Jan. 6, 1882, $310. What is due May 16, 1882?

8. What would be due upon this, if settled Nov. 15, 1883, at ¾% a mo.?

9. A mortgage for $3840, dated Rochester, April 1, 1879, bearing int. at 5%, has the following payments indorsed upon it: Jan. 1, 1880, $550; Aug. 7, 1880, $1000; Feb. 10, 1881, $790; July 13, 1881, $264.50. What amount will be due April 1, 1882?

324. The following method of computation is often used by merchants in the settlement of notes and of interest accounts running a year or less; hence called the

MERCANTILE RULE.

1. *Find the amount of the note or debt from its date to the time of settlement.*

2. *Find the amount of each payment from its date to the time of settlement.*

3. *Subtract the sum of the amounts of payments from the amount of the note or debt.*

An accurate application of this rule requires that the time should be reduced to *days*, allowing 365 da. to the year.

1. On a note for $600 at 7%, dated Feb. 15, 1884, were the following indorsements: March 25, 1884, $150; June 1, 1884, $75; Oct. 10, 1884, $100. What was due Dec. 31, 1884?

SOLUTION.

Amt. of	$600	from	Feb. 15	to Dec. 31,	319 da.,		$636.71
"	$150	"	Mar. 25	" "	281 da.,	$158.08	
"	$75	"	June 1	" "	213 da.,	78.06	
"	$100	"	Oct. 10	" "	82 da.,	101.57	337.71
				Bal. due Dec. 31, 1884,			$299.00

2. A note for $950, dated Jan. 25, 1883, payable in 9 mo., at 7% int., had the following indorsements: Mar. 2, 1883, $225; May 5, 1883, $174.19; June 29, 1883, $187.50; Aug. 1, 1883, $79.15. Find the balance due at the time of its maturity?

3. Payments were made on a debt of $1750, due April 5, 1882, as follows: May 10, 1882, $190; July 1, 1882, $230; Aug. 5, 1882, $645; Oct. 1, 1882, $372. What was due Dec. 31, 1882, interest at 6%?

For methods of computation in the States of Vermont, New Hampshire, and Connecticut, see "Addenda."

STOCKS AND INVESTMENTS.

325. A ***corporation*** is a body of individuals, or a company, authorized by law to transact business as one person.

326. ***Stock*** is the capital or money subscribed by a corporation to carry on its business.

327. A ***share*** is *one* of the equal parts into which the stock of a corporation is divided.

The *value* of a share varies in different companies. When not otherwise stated, $100 will be understood to be the value.

328. A ***certificate of stock*** is a paper issued by a corporation specifying the number of shares to which the holder is entitled, and the original value of each.

329. The ***par value*** of stock is the sum for which the scrip or certificate was issued.

330. The ***market value*** of stock is the sum for which it can be sold.

Stock is **at par** when its market value is 100% ; **above par,** or at a *premium,* when its market value is above 100% ; and **below par,** or at a *discount,* when its market value is below 100%. When stock is at par, it is quoted at 100; when it is 5% above par, at 105; when it is 5% below par, at 95.

331. A ***stock broker*** is one who buys and sells stocks for a commission, called *brokerage.*

332. ***Premium, discount,*** and ***brokerage*** are each a *percentage* computed upon the par value of the stock.

333. A ***dividend*** is a sum paid to the stockholders from the profits of the business.

Dividends and assessments are a *percentage* computed upon the par value of the stock as the *base.*

334. ***Net earnings*** are the moneys left from the profits of a business after paying expenses, losses, and the interest upon the bonds.

335. A ***bond*** is a written instrument securing the payment of a sum of money at or before a specified time.

The principal bonds dealt in by brokers are Government, State, City, and Railroad bonds.

336. A ***coupon*** is a certificate of interest attached to a bond, to be cut off and presented for payment when the interest is due.

Bonds are quoted in commercial language by the rate of interest which they bear. Thus, U. S. bonds bearing 6% int. are quoted as U. S. 6's; U. S. bonds bearing 4½% int., payable in 1891, are quoted as U. S. 4½'s, '91.

Bonds issued by States, cities, etc., are quoted in a similar manner. Thus, S. C. 6's are bonds bearing 6% interest, issued by the State of South Carolina; so, also, Mo. 7's, N. Y. 5's, etc.

337. The corresponding *terms* in stocks are as follows:

1. The **par value** is the *base.*
2. The **rate %** of **premium** or **discount** is the *rate.*
3. The **premium** or **discount** is the *percentage.*
4. The **market value** is the *amount* or *difference.*

WRITTEN EXERCISES.

338. Find the *cost*

1. Of 250 shares of Lake Shore R. R. stock, the market value of which is 104⅞, brokerage ⅛%.

SOLUTION.—(104⅞% + ⅛%) of $100 = $105, cost of 1 share.
$105 × 250 = $26250, cost of 250 shares.

FORMULAS.

1. *Par value + premium,* or *— discount = market value.*
2. (*Market value of 1 share + brokerage*) × *No. shares = cost.*

2. Find the cost of 250 shares of Western Union Telegraph stock, market value 97¾, brokerage ⅛%?

3. A broker bought for me 15 one-thousand-dollar U. S. 5-20 bonds, at 112¼, brokerage ⅛%. What was their cost?

4. Bought 120 shares of Pacific R. R. stock, at a discount of 2⅛%, and sold the same at an advance of 12%. What was my gain?

339. Find the *number of shares*

1. Of bank stock at 105, that can be bought for $25260, including brokerage at ¼%?

Solution.—(105% + ¼%) [redacted] = $105¼, cost of 1 share.
$25260 ÷ $105¼ = 240, *No. of shares.*

Formula: *No. of shares = investment ÷ cost of 1 share.*

2. How many shares of telegraph stock can be bought for $12240, at 10% discount? At 20% premium?

3. How many shares of Mo. 6's, at 97¾, brokerage ¼%, will $21560 purchase?

4. How many shares of the Bank of Commerce, at 110⅛, can be bought for $22567.50, brokerage ⅛%?

340. Find the *amount of investment*

1. In U. S. 5's of '81, at 111, so as to realize therefrom an annual income of $2500?

Solution.—$2500+$5, income on 1 share = 500, No. of shares.
$111, price of 1 share × 500 = $55500, *investment.*

Formulas: 1. *Given income ÷ inc. of 1 share = No. shares.*
2. *Cost of 1 share × No. shares = investment.*

2. What sum invested in Tennessee 6's at 85, will yield an annual income of $1800?

3. What sum invested in any stock at 105⅛, which pays 5% semi-annual dividends, will secure an annual income of $2000?

341. What *annual income*

1. Will be realized by investing $22260 in 4½% bonds, bought at 92½, brokerage ¼%?

SOLUTION.—(92½% + ¼%) [redacted] = $92¾, cost of 1 share;
$22260 ÷ $92.75 = 240, No. of shares;
$4.50, income on 1 share × 240 = $1080, *total income.*

FORMULA: $\frac{\textit{Investment}}{\textit{cost of 1 share}} \times$ *inc. on 1 share = total inc.*

2. What income will $19650 invested in U. S. 3½'s at 97¾ yield, brokerage ⅛%?

3. If $48000 is invested, ½ in 5% stock, at 95½, and ½ in 6% stock at 112, brokerage ⅛%, what annual income is secured?

4. A farm which rents for $411.45 per annum, is sold for $8229, and the proceeds invested in 5% bonds at 105, brokerage ⅛%. Is the yearly income increased or diminished, and how much?

342. Find the *rate of income*

1. Realized from bonds bought at 112, and paying 4% semi-annual dividends.

EXPLANATION.—Since $112, the cost of 1 share, pays $8 annually, the income is $\frac{8}{112}$ of the investment, or $\frac{8}{112}$ of 100% = 7⅐%.

FORMULA: $\frac{\textit{Annual income per share}}{\textit{cost per share}} =$ *rate of income.*

2 What per cent. of his money will a man obtain by investing in 6% stock at 108%? At a discount of 16%?

3. If stock paying 10% dividends is at a premium of 12½%, what % of income will be realized on an investment in it?

4. Which will yield the better income, 8% bonds at 110, or 5's at 75; 5's at 70, or 6's at 80?

5. Which is the more profitable, and how much, to buy N. Y. 7's at 105, or 6% bonds at 84?

EXCHANGE.

343. ***Exchange*** is a method of remitting money from one place to another, or of making payments by written orders, called *bills of exchange* or *drafts.*

344. Exchange is of two kinds—*domestic* and *foreign.*

345. ***Domestic*** or ***inland exchange*** relates to remittances made between different places of the same country.

An *inland bill of exchange* is commonly called a *draft.*

346. A ***draft*** or ***bill of exchange*** is a written request or order upon one person to pay a certain sum to another person, or to his order, at a specified time.

The **drawer** of a draft or bill is the party who signs it; the **drawee** is the party to whom the order is addressed, and when he accepts the order, he becomes the **acceptor.** The **payee** is the party to whom the money is directed to be paid. The **buyer** or **remitter** is the party who purchases it. The buyer and payee may be the same party.

A **sight draft** or **bill** is one that requires payment to be made when *presented.*

A **time draft** or **bill** is one that requires payment to be made at a certain specified time, after date, or after being presented.

347. The ***acceptance*** of a draft or bill is the agreement by the drawee to pay it at maturity. This is done by writing the word "Accepted" across the face of the bill and signing it.

Three days of grace are allowed on time drafts, the same as on notes, but not on sight drafts. When a bill is *protested* for non-acceptance, the drawer is bound to pay it immediately.

348. The ***course of exchange*** is the current price paid in one place for bills of exchange on another place.

Exchange is *at par, above par,* or *below par,* according to the fluctuations of trade between the two countries or places.

349. In exchange, the following are corresponding *terms:*

1. The **face** of the draft is the *base.*
2. The **rate %** of exchange is the *rate.*
3. The **premium** or **discount** is the *percentage.*
4. The **cost** of the draft is the *amount* or *difference.*

WRITTEN EXERCISES.

350. Find the *cost* of the following *sight draft,* at 101¼.

$1750. SPRINGFIELD, July 1, 1884.

At sight, pay to the order of L. A. GRAY, Seventeen hundred fifty dollars, and charge the same to the account of

O. M. BAKER.

TO BAILEY & NOYES,
Portland, Me.

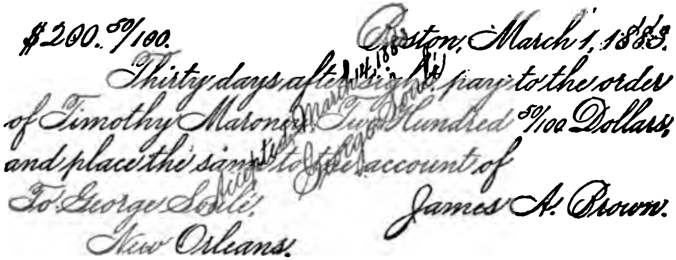
SOLUTION.—$1750 × 1.01¼ = $1771.87½, *cost.*

FORMULAS: $$\textit{Face} \times \left\{ \begin{array}{l} 1.\ 1 + \textit{rate of premium} \\ 2.\ 1 - \textit{rate of discount} \end{array} \right\} = \textit{cost.}$$

Find the *cost of sight drafts*

2. For $1590, premium ¾%.
3. For $840, premium 2%.
4. For $484.50, prem. 1⅓%.
5. For $1120, premium 1⅜%.
6. For $1200, discount ⅛%.
7. For $2480, discount 1¼%.
8. For $560.80, discount ⅞%.
9. For $784.50, discount 1⅛%.

10. Find the *cost* of the following *time draft,* at 1¾% premium, interest 6%:



$200.50/100 Boston, March 1, 1883.

Thirty days after sight pay to the order of Timothy Maloney Two Hundred 50/100 Dollars, and place the same to the account of

James A. Brown.

To George Soule,
New Orleans.

Accepted March 4, 188
George Soule

Solution.—$1.0175, course of exch.; $.0055, bank dis. of $1, for 33 da.; $1.012, cost of exch. of $1; $1.012 × 200.50 = $202.91, cost of dft.

Formula: *Face × cost of $1 exchange = cost of draft.*

Find the *cost of time drafts*

11. For $4720, at 30 da., premium 1½%, interest 6%.
12. For $5275, at 90 da., discount ½%, interest 7%.
13. For $6400, at 90 da., premium 1⅜%, interest 6%.
14. What must be paid in Philadelphia for a draft on St. Paul, drawn at 90 da., for $4800, the course of exchange being 101⅜, and interest 6%?

351. Find the *face*

1. Of a *sight* draft, bought for $711.90, discount 1⅛%.

Solution.—$711.90 ÷ .98875 = $720, *face.*

2. Of a draft on St. Louis, at 90 da., bought for $4500, exchange being at 101½%?

Solution.—$1.015 = course of exchange;
$.0155 = bank dis. of $1, for 93 da., at 6%;
$.9995 = cost of exchange of $1; $4500 ÷ .9995 = $4502.25, *face.*

Formula: *Cost of draft ÷ cost of $1 exchange = face.*

Find the *face* of a *sight* draft bought

3. For $840, premium 1¼%.
4. For $1675, premium ⅝%.
5. For $2600, discount 1⅛%.
6. For $972.50, discount ¾%.
7. An agent in Syracuse, N. Y., having $1324.74 due his employer, is instructed to remit the same by a draft drawn at 30 days. Find the face of the draft, exchange being 101¾.
8. What is the face of a 60 da. draft, at ½% discount, that can be bought for $750, money being worth 7%?
9. How large a draft can be bought for $1875, payable in 60 da., exchange being 101⅛, interest 8%?
10. An agent sold 2780 lb. of cotton at 11½ cts. a pound. If his commission is 2½%, and exchange 98½%, how large a draft can he buy to remit to his consignor?

EQUATION OF PAYMENTS.

352. ***Equation of payments*** is the process of finding the average time for the payment of several sums, due at different times, without loss to debtor or creditor.

353. The ***equated time*** is the date at which the several debts may be discharged by one payment.

354. The ***term of credit*** is the time at the expiration of which a debt becomes due.

355. The ***average term of credit*** is the time to elapse before several debts due at different dates may all be paid at once, without loss to either party.

WRITTEN EXERCISES.

356. To find the equated time and the average of terms of credit, beginning at the same date.

1. Of $300 due in cash, $500 due in 3 mo., $750 due in 8 mo., and $950 due in 10 mo.

$$
\begin{array}{rcl}
300 \times \;\;0 & = & 0 \\
500 \times \;\;3 & = & 1500 \\
750 \times \;\;8 & = & 6000 \\
\underline{950} \times 10 & = & \underline{9500} \\
2500 &) & \underline{17000} \\
 & & 6\tfrac{4}{5}\text{ mo.}
\end{array}
$$

Explanation.—On $300, the first payment, there is no interest, since it is due in cash; the int. of $500 for 3 mo. is the same as the int. of $1 for 1500 mo.; the int. of $750 for 8 mo. is the same as that of $1 for 6000 mo.; and the int. of $950 for 10 mo. is the same as the int. of $1 for 9500 mo. Therefore, the whole amt. of interest is that of $1 for 1500 mo. + 6000 mo. + 9500 mo., or 17000 mo.; but the whole debt is $2500; and the interest of $1 for 17000 mo. is equal to the interest of $2500 for $\frac{1}{2500}$ of 17000 mo., or 6$\frac{4}{5}$ mo.

2. Find the average term of credit of $800 due in 1 mo., $750 due in 4 mo., and $1000 due in 6 mo.

Rule.—1. *Multiply each payment by its term of credit, and divide the sum of the products by the sum of the payments; the quotient is the average term of credit.*

2. To find the equated time of payment.—*Add the average term of credit to the date at which the several credits begin.*

3. On the first day of December, 1880, a man gave 3 notes, the first for \$500 payable in 3 mo.; the second for \$750, payable in 6 mo.; and the third for \$1200, payable in 9 mo. What was the average term of credit, and the equated time of payment?

4. Bought merchandise Jan. 1, 1883, as follows: \$350 on 2 mo., \$500 on 3 mo., \$700 on 6 mo. What is the equated time of payment?

5. A person owes a debt of \$1680 due in 8 months, of which he pays $\frac{1}{3}$ in 3 mo., $\frac{1}{4}$ in 5 mo., $\frac{1}{5}$ in 6 mo., and $\frac{1}{6}$ in 7 mo. When is the remainder due?

6. Bought a bill of goods, amounting to \$1500 on 6 mo. credit. At the end of 2 mo., I paid \$300 on account, and 2 mo. afterward, paid \$400 on account, giving my note for the balance. For what time was the note drawn?

Explanation.—\$300 paid 4 mo. before it is due, and \$400, 2 mo. before it is due, are equivalent to the use of \$1 for 2000 mo., or the use of \$800 (the balance) for $2\frac{1}{2}$ mo. beyond the original time. Hence, the note was drawn for $4\frac{1}{2}$ mo. after the second payment.

$$\begin{array}{rcr} 300 \times 4 & = & 1200 \\ \underline{400} \times 2 & = & \underline{800} \\ 800 &) & \underline{2000} \\ & & 2\frac{1}{2} \end{array}$$

$$(6\text{ mo.} - 4\text{ mo.}) + 2\tfrac{1}{2}\text{ mo.} = 4\tfrac{1}{2}\text{ mo.}$$

7. On a debt of \$2500 due in 8 mo. from Feb. 1, the following payments were made: May 1, \$250; July 1, \$300; and Sept. 1, \$500. When is the balance due?

8. Find the average term of credit, and the equated time of payment from Dec. 15, of \$225 due in 35 da., \$350 due in 60 da., and \$750 due in 90 da.

9. Dec. 1, 1884, bought goods to the amount of $1200, on terms as follows: 25% in cash, 30% in 3 mo., 20% in 4 mo., and the balance in 6 mo. Find the equated time of payment, and the cash value of the goods, computing discount at 7%.

357. To find the equated time and the average of terms of credit beginning at different dates.

1. L. C. Hill bought goods of John Beach as follows: June 1, 1882, amounting to $350 on 2 mo. credit; July 15, 1882, $400, on 3 mo.; Aug. 10, $450, on 4 mo.; and Sept. 12, $600, on 6 mo. What is the equated time of payment?

$350	due	Aug. 1,	350, cash.		
400	"	Oct. 15,	400 × 75	=	30000
450	"	Dec. 10,	450 × 131	=	58950
600	"	Mar. 12,	600 × 223	=	133800
			1800		1800) 222750
					123¾

Hence the equated time is 124 da. from Aug. 1, or *Dec. 3.*

EXPLANATION.—Computing the terms of credit from Aug. 1, the *earliest date* at which any of the debts become due, we find the terms of credit to be from Aug. 1 to Oct. 15, 75 da.; to Dec. 10, 131 da., and to March 12, 223 da. The average term of credit is therefore 124 da. from Aug. 1, and the equated time Dec. 3.

PROOF.—Assume as the standard time the latest date, March 12. The operation will then be as follows:

350 × 223	=	78050
400 × 148	=	59200
450 × 92	=	41400
600, cash.		
		1800) 178650
		99¼

Hence, the equated time is 99 da. previous to March 12, or *Dec. 3.*

2. Tefft, Weller, & Co. sell to H. P. Griffing the following bills of goods: March 1, 1885, on 60 da., $800; April 15, on 30 da., $350; May 20, on 4 mo., $3800.

What is the equated time for settlement?

RULE.—1. *Find the date at which each debt becomes due.*

2. *From the earliest of these dates as a standard, compute the time to each of the others.*

3. *Then find the average term of credit and equated time as in* **(356)**.

PROOF.—*Compute the terms of credit backward from the latest date, and subtract the average time from that date for the equated time.*

If the earliest date is not the *first* of the month, it is more convenient to assume the *first* of the month as the standard date.

3. Bought mdse. as follows: Jan. 15, 1884, on 4 mo., $375; Feb. 3, on 60 da., $550; March 25, on 4 mo., $1100; April 2, on 30 da., $250. Find the equated time.

4. M. H. Decker bought of Halstead & Haines the following bills of goods on 4 months' credit:

Jan. 1, 1882, $650; Feb. 10, $380; Mar. 12, $900; Mar. 18, $350; April 3, $600.

April 5, he discounted his bills at 2% per month. Find the equated time of payment, and the discount.

5. JAMES MATTHEWS *to* THOMAS BARR, *Dr.*

March	10,	1884.	To	mdse.	$835.
"	18,	"	"	"	320.
"	26,	"	"	"	475.
April	5,	"	"	"	600.
"	12,	"	"	"	250.

Allowing 30 days' credit on each of the bills, what is the equated time of payment?

RATIO AND PROPORTION.

358. ***Ratio*** is the relation between two numbers of the same denomination, expressed by the quotient of the first divided by the second.

Thus, the ratio of 12 to 4 is 12÷4 = 3; of 6 to 9 is 6÷9 = $\frac{2}{3}$.

359. The ***sign*** of ratio is the colon (:).

Thus, the ratio of 9 to 3 is expressed 9 : 3, or 9÷3, or in the form of a fraction $\frac{9}{3}$, and is read, the ratio of 9 to 3, or 9 divided by 3.

The **terms** of a ratio are the numbers compared.

The **antecedent** is the first term, or the dividend.

The **consequent** is the second term, or the divisor.

The two terms together form a **couplet.**

360. A ***simple ratio*** is the ratio of two numbers.

Thus, 6 : 8, $12 : $3, and 9 mi. : 27 mi. are *simple ratios.*

361. A ***compound ratio*** is the ratio of the product of the corresponding terms of two or more simple ratios.

Thus, the ratio compounded of the simple ratios,

$$\left.\begin{matrix} 8 : 4 \\ 9 : 12 \end{matrix}\right\} \text{may be expressed} \left\{\begin{matrix} (8 : 4)\times(9 : 12) \\ \text{or, } 8\times9 : 4\times12 \end{matrix}\right\} = 72 : 48;$$

$$\text{Or,} \quad \tfrac{8}{4}\times\tfrac{9}{12} = \tfrac{3}{2} = 3 : 2.$$

When the multiplication is performed, the *result* is a *simple* ratio.

362. The ***reciprocal*** of a ratio is 1 divided by the ratio.

Thus, the ratio of 4 to 5 is 4 : 5, or $\frac{4}{5}$, and its *reciprocal* is $1 \div \frac{4}{5} = \frac{5}{4}$

Reciprocal ratio is sometimes called **inverse ratio.**

The ratio of two *fractions* is obtained by reducing them to a common denominator, when they are to each other as their numerators (**125**).

If the terms of a ratio are denominate numbers, they must be reduced to the same unit value.

363. Since the antecedent is a *dividend,* and the consequent a *divisor,* any change in either or both of the terms of a ratio will affect its value according to the laws of division, or of fractions (**72, 102**). Hence, the following

PRINCIPLES.

I. *Multiplying the antecedent,* or *Dividing the consequent,* } *MULTIPLIES the ratio.*

II. *Dividing the antecedent,* or *Multiplying the consequent,* } *DIVIDES the ratio.*

III. *Multiplying or dividing both antecedent and consequent by the same number,* } *DOES NOT CHANGE the value of the ratio.*

The relations of ratio, antecedent, and consequent to each other are such, that if any two are given, the other may be found. Hence, the

FORMULAS:
1. *The antecedent* ÷ *consequent* = *ratio.*
2. *The antecedent* ÷ *ratio* = *consequent.*
3. *The consequent* × *ratio* = *antecedent.*

WRITTEN EXERCISES.

364. Express in the lowest terms the following ratios:

1. 6 to 42.
2. 80 to 120.
3. 21 to 147.
4. $8\frac{4}{7}$ to 60.
5. $6\frac{1}{2}$ to 78.
6. $3\frac{1}{2}$ to $16\frac{2}{3}$.
7. $\frac{3}{4}$ to $\frac{3}{5}$.
8. $\frac{2}{7}$ to $\frac{4}{9}$.
9. $\frac{1}{6}$ to $\frac{7}{10}$.

10. The consequent is 16 and the ratio $2\frac{3}{7}$; find the antecedent.

11. Antecedent 14.5, ratio 3; find the consequent.

12. Consequent $\frac{7}{8}$, ratio $\frac{3}{4}$; find the antecedent.

13. Antecedent $\frac{1}{2}$ of $\frac{5}{8}$, consequent .75; find the ratio.

Find the *ratio* in lowest terms of

14. 1 mi. to 120 rd.
15. 3 gal. to 2 qt. 1 pt.
16. 8 yd. to 9 inches.
17. $1.68 to $.24.
18. 1 wk. 3 da. 12 hr. to 9 wk.
19. 10 A. 60 sq. rd. to 6 A. 110 sq. rd.
20. 25 bu. 2 pk. 6 qt. to 40 bu. 4.5 pk.
21. 7 cwt. 40 lb. to 5 T. 8 cwt.

22. Find the *reciprocal* of the ratio of 75 to 15.

23. Find the *reciprocal* of the ratio of 2 qt. 1 pt. to 4 gal 1 qt. 1 pt.

PROPORTION.

365. A ***proportion*** is an equation in which each member is a ratio; or it is an equality of ratios.

The equality of two ratios may be indicated by the sign =, or by the double colon (: :).

Thus, we may indicate that the ratio of 8 to 4 is equal to that of 6 to 3, in any of the following ways:

$$8:4=6:3, \qquad 8:4::6:3, \qquad \frac{8}{4}=\frac{6}{3}, \qquad 8\div 4=6\div 3.$$

This proportion, in any of its forms, is read, *The ratio of 8 to 4 is equal to the ratio of 6 to 3*, or, *8 is to 4 as 6 is to 3.*

366. Since each ratio consists of two terms, every proportion must consist of at least *four terms.* Each ratio is called a ***couplet,*** and each term is called a ***proportional.***

367. The ***antecedents*** of a proportion are the *first* and *third* terms; the ***consequents*** are the *second* and *fourth* terms.

368. The ***extremes*** are the *first* and *fourth* terms; the ***means*** are the *second* and *third* terms.

In the proportion 8 : 4 : : 10 : 5, 8, 4, 10, and 5 are the *proportionals;* 8 : 4 is the *first couplet,* 10 : 5 the *second couplet;* 8 and 10 are the *antecedents,* 4 and 5 are the *consequents;* 8 and 5 are the *extremes,* 4 and 10 are the *means.*

When the two *means* are the same number, that number is said to be a **mean proportional** between the two extremes.

369. A ***simple proportion*** is an expression of equality between two *simple* ratios.

Thus, $4 : 8 = 5\frac{1}{2} : 11$, and $8 : 12 = 10 : 15$ are *simple proportions.*

370. An expression of equality between two ratios, one or both of which are *compound,* is sometimes called a ***compound proportion.***

Thus, $\left.\begin{array}{c} 8 : 4 \\ 6 : 9 \end{array}\right\} = 14 : 28$ is a proportion, composed of a compound and a simple ratio, and may be read, 8×6 is to 4×9 as 14 is to 28, equivalent to the simple proportion $18 : 36 = 14 : 28$.

371. The solution of problems in proportion depends upon the following

PRINCIPLES.—I. *The product of the extremes of a proportion is equal to the product of the means.*

II. *The product of the extremes, divided by either mean, will give the other mean.*

III. *The product of the means divided by either extreme will give the other extreme.*

WRITTEN EXERCISES.

372. Find the term omitted, and represented by x, in each of the following proportions:

1. $8 : 52 = 20 : x$.
2. $12 : x = 1 : 144$.
3. $x : 20 = 120 : 50$.
4. $\$80 : \$4 = x : 8$.
5. $2.5 : 62.5 = 5 : x$.
6. $\$175.35 : \$x = \frac{1}{8} : \frac{3}{7}$.
7. $4\frac{1}{2}$ yd. : x yd. $= \$9\frac{3}{4} : \$27\frac{1}{4}$.
8. $x : 9.01 = 16.05 : 5.35$.
9. $\dfrac{x}{9.01} = \dfrac{16.05}{5.35}$.
10. $\frac{3}{8}$ yd. : x yd. $= \$\frac{7}{8} : \$5$.
11. $\left.\begin{array}{c} 8 : 9 \\ 3 : 4 \end{array}\right\} = 40 : x$.
12. $\left.\begin{array}{r} 480 : x \\ 30 : 15 \end{array}\right\} = 84 : 21$.
13. $\left.\begin{array}{r} 6 : x \\ 14 : 12 \end{array}\right\} = \left\{\begin{array}{r} 36 : 48, \\ 56 : 54. \end{array}\right.$
14. $\left.\begin{array}{r} 7 : 28 \\ 12 : x \end{array}\right\} = \left\{\begin{array}{r} 60 : 80, \\ 8 : 3, \\ 6 : 8. \end{array}\right.$

373. The proper arrangement of the terms in the form of a proportion is called the ***statement*** of the question.

1. If 4 tons of coal cost $24, what is the cost of 12 tons?

EXPLANATION.—Since 4 T. and 12 T. have the same unit value, they can be compared, and form one couplet of the proportion. For the same reason, $24, the cost of 4 tons, and $x, the cost of 12 tons, form the other couplet.

STATEMENT.

$$4 \text{ T.} : 12 \text{ T.} :: \$24 : \$x$$

$$\$x = \frac{\overset{6}{\cancel{\$24}} \times 12}{\cancel{4}} = \$72, \text{ PRIN. III}$$

2. If 8 men earn $320 in 8 da., what will 12 men earn in 4 da.?

EXPLANATION.—All the terms in this example appear in couplets, except $320, which has the same unit value as the required term, and is made the *third* term of the proportion, and $x the *fourth*.

STATEMENT.

$$\left.\begin{array}{lcl} 8 \text{ men} & : & 12 \text{ men} \\ 8 \text{ da.} & : & 4 \text{ da.} \end{array}\right\} = \$320 : \$x$$

$$\$x = \frac{\overset{\$5}{\cancel{\$320}} \times 12 \times 4}{\cancel{8} \times \cancel{8}} = \$240, \text{ PRIN. III.}$$

The value of the required term in this example depends upon *two* conditions: 1st, the number of *men* at work; 2d, the number of *days* they work.

Consider each condition *separately*, and arrange the terms of the same unit value in *couplets*. Then find the required term by PRIN. III.

3. If 96 cords of wood cost $240, what will 40 cords cost?

4. If 20 horses consume 36 tons of hay in 9 mo., how many tons will 12 horses consume in 18 months?

RULE.—1. *Of the given numbers, make that the third term which is of the same kind as the answer, or number sought.*

2. *Arrange the remaining numbers, two of a kind, into couplets, in such a manner that the terms shall have the same ratio to each other as the third term has to the required term.*

3. *Divide the product of the means by the given extreme, and the quotient is the fourth* or *required term.*

1. If the terms of any couplet are of different denominations, they must be reduced to the same denomination.

2. If the odd term is a compound number, it must be reduced to its lowest unit.

3. If the divisor and dividend contain factors common to both, cancel them.

5. If 12 gallons of wine cost $30, what will 63 gallons cost?

6. If 9 bu. of wheat make 2 bbl. of flour, how many barrels of flour will 100 bu. make?

7. If 6½ bu. of oats cost $3, what will 9¼ bu. cost?

8. What will 87.5 yd. of cloth cost, if 1¾ yd. cost $.42?

9. If by selling $1500 worth of dry goods I gain $275.40, what amount must I sell to gain $1000?

10. What will 11¼ lb. of tea cost, if 3 lb. 12 oz. cost $3.50?

11. If a speculator in grain gain $26.32 by investing $325, how much would he gain by investing $2275?

12. If 16 horses consume 128 bu. of oats in 50 da., how many bushels will 5 horses consume in 90 da.?

13. If 18 men can build 42 rd. of wall in 16 da., how many men can build 28 rd. in 8 da.?

14. If a man clears $750 by his business in 1 yr. 6 mo., how much would he gain in 3 yr. 9 mo. at the same rate?

15. If a certain business yield $350 net profits in 10 mo., in what time would the same business yield $1050 profits?

16. If $480 gain $84 interest in 30 mo., what sum will gain $21 in 15 mo.?

17. If 2½ yd. of cloth 1⅔ yd. wide cost $3.37½, what cost 36½ yd., 1½ yd. wide?

18. How much land worth $16.50 an acre should be given in exchange for 140 acres, worth $24.75 an acre?

19. If I gain $155.52 on $1728 in 1 yr. 6 mo., how much will I gain on $750 in 4 yr. 6 mo.?

20. If 450 tiles, each 12 in. square, will pave a cellar, how many tiles that are 9 in. by 8 in. will pave the same?

CAUSE AND EFFECT.

374. The *terms* of a proportion have not only the relations of *magnitude,* but also the relations of *cause* and *effect.*

375. ***Causes,*** in computation, are things that produce something: as, men at work, money lent, horses, time, etc.

376. ***Effects*** are the results of causes; as, work done, interest drawn, cost, distance traveled, etc.

377. Every problem in proportion may be considered as a comparison of *two causes* and *two effects.*

Thus, if 4 tons as a *cause* will bring, when sold, $24 as an *effect,* 12 tons as a *cause* will bring $72 as an *effect.* Or, if 6 horses as a *cause* draw 10 tons as an *effect,* 9 horses as a *cause* will draw 15 tons as an *effect.*

378. Since *like causes* produce *like effects,* the ratio of two like causes equals the ratio of two like effects produced by these causes. Hence,

$$\text{1st cause} : \text{2d cause} = \text{1st effect} : \text{2d effect.}$$

1. If 8 men earn $32 in one week, what will 15 men earn at the same rate, in the same time?

EXPLANATION.—In this example an *effect* is required. The first cause is 8 men, the second cause 15 men, and since they are like causes, they can be compared.

The effect of the first cause is $32 earned, the effect of the second cause is $x earned, or the *required* term. Since like effects have the same ratio as their causes (**378**), the *causes* may form the first, and the *effects* the second couplet of the proportion. The required term is found by PRIN. III.

STATEMENT.

$$\begin{array}{ccccccc} \text{1st C.} & & \text{2d C.} & & \text{1st E.} & & \text{2d E.} \\ 8\text{ men} & : & 15\text{ men} & = & \$32 & : & \$x \end{array}$$

$$\$x = \frac{\overset{\$4}{\cancel{\$32}} \times 15}{\cancel{8}} = \$60$$

Or,

$$\begin{array}{rr|ll} \text{2d E.,} & x & 15 & \text{2d C.} \\ \text{1st C.,} & \cancel{8} & \cancel{\$32}^{\,4} & \text{1st E.} \\ \hline \end{array}$$

$$x = 60$$

2. If $240 gain $21 interest in 15 mo., what will $480 gain in 2 yr. 6 mo.?

STATEMENT.

$$\left.\begin{array}{lcl}\text{1st C.} & & \text{2d C.}\\ \$240 & : & \$480\\ 15\text{ mo.} & : & 30\text{ mo.}\end{array}\right\} = \begin{array}{ccc}\text{1st E.} & & \text{2d E.}\\ \$21 & : & \$x.\end{array}$$

$$\$x = \frac{\$480 \times 30 \times 21}{240 \times 15} = \$84.$$

$$\begin{array}{rr|rl}\text{Or, 2d E.,} & x & \$480 & \rule{0pt}{0pt}\\ \text{1st C.,} & \left\{\begin{array}{r}240\\ 15\end{array}\right. & \begin{array}{r}30\\ 21\end{array} & \begin{array}{l}\text{2d C.}\\ \text{1st E.}\end{array}\\ \hline & \multicolumn{2}{c}{x = \$84.} & \end{array}$$

EXPLANATION.—In this example, the causes form a *compound* ratio, and the *required* term is an *extreme*. Hence, the product of the means divided by the product of the given extremes, gives the required term, $84, PRIN. III.

3. If 6 compositors in 14 hr. can set 36 pages of 56 lines each, how many compositors in 12 hr. can set 48 pages of 54 lines each?

STATEMENT.

$$\left.\begin{array}{lcl}\text{1st C.} & & \text{2d C.}\\ 6\text{ cmp.} & : & x\text{ cmp.}\\ 14\text{ hr.} & : & 12\text{ hr.}\end{array}\right\} = \left\{\begin{array}{lcl}\text{1st E.} & & \text{2d E.}\\ 36\text{ pp.} & : & 48\text{ pp.}\\ 56\text{ l.} & : & 54\text{ l.}\end{array}\right.$$

$$\begin{array}{r}\text{2d C.,}\\ \\ \text{1st E.,}\end{array}\left\{\begin{array}{r|l}x & \cancel{48}\\ \cancel{12} & \cancel{54}\ {}^{9}\\ \cancel{36} & \cancel{6}\\ \cancel{56} & \cancel{14}\\ \hline \multicolumn{2}{c}{x = 9\text{ comp.}}\end{array}\right\}\begin{array}{l}\text{2d E.}\\ \\ \text{1st C.}\end{array}$$

EXPLANATION.—In this example, the causes and effects each form a compound ratio, and an *element* of the second cause is required; and the statement is, 6 compositors 14 hours is to x compositors 12 hours as 36 pages of 56 lines each is to 48 pages of 54 lines each. Now, since the *required* term is an *element* of one of the means, we divide the continued product of all the elements of the extremes by the continued product of all the given elements of the means. PRIN. II.

4. If 4 horses consume 24 bushels of oats in 12 days, how many bushels will 20 horses consume in 8 weeks?

5. If 10 men can cut 46 cords of wood in 18 da., working 10 hr. a day, how many cords can 40 men cut in 24 da., working 9 hr. a day?

6. If by selling $5000 worth of dry goods, a merchant gains $456.25, what amount must he sell to gain $1000?

After the statement of problems by cause and effect, the *required* term may be obtained as in the first method, by PRIN. II and III.

But by the use of the *vertical line* and *cancellation*, a method greatly to be preferred for its simplicity and brevity, the *statement* of a problem in the *form* of a proportion may be omitted by observing the following:

Since x represents the *required* quantity, and may be either a cause or an effect, or an *element* of the one or of the other, we have this

RULE.—1. *Write x alone or with its kindred elements, if it has any, first and ALWAYS on the LEFT of a vertical line, and its corresponding cause or effect on the RIGHT.* (**74**, NOTE 2.)

2. *Next, write the other cause and effect, each on the opposite side of the line from that on which the first cause and effect are written.*

3. *Cancelling equal factors on opposite sides of the line, the product of the remaining factors on the right, divided by the remaining factor or product of the factors on the left, will give the required term.*

If any of the terms contain mixed numbers, they should first be changed to improper fractions, or the fractional part to a decimal.

Solve, by either method, the following:

7. If 8 yards of cloth cost $3½, how many yards can be bought for $50?

8. What will 87.5 yd. of cloth cost, if 1¾ yd. cost $1.26?

9. If 1 lb. 4 oz. 10 pwt. of gold may be bought for $260.70, how much may be bought for $39.50?

10. If 5 men reap 52.2 acres in 6 days, how many men will reap 417.6 acres in 12 days?

11. If it cost $95.60 to carpet a room 24 ft. by 18 ft., how much will it cost to carpet a room 38 ft. by 22 ft. with the same material?

12. What sum of money will produce $300 in 8 mo., if $800 produce $70 in 15 months?

13. How many days will 21 men require to dig a ditch 80 ft. long, 3 ft. wide, and 8 ft. deep, if 7 men can dig a ditch 60 ft. long, 8 ft. wide, and 6 ft. deep, in 12 days?

14. What will be the cost of paving an open court 60.5 ft. long and 44 ft. wide, if 14.25 sq. yd. cost $34⅕?

15. What will be the cost of a pile of wood 80 ft. long, 4 ft. wide, 4 ft. high, if a pile 18 ft. long, 4 ft. wide, 6 ft. high cost $30.24?

16. If 36 bu. of wheat are bought for $44.50, and sold for $53.50, what is gained on 480 bu. at the same rate?

17. If it cost $15 to carry 20 tons 1½ miles, what will it cost to carry 400 tons ½ of a mile?

18. How many men will be required to dig a cellar 45 ft. long, 34.6 ft. wide, and 12.3 ft. deep, in 12 da. of 8.2 hr. each, if 6 men can dig a similar one 22.5 ft. long, 17.3 ft. wide, and 10.25 ft. deep, in 3 da. of 10.25 hr. each?

19. If a bin 8 ft. long, 4⅕ ft. wide, and 2½ ft. deep hold 67½ bu., how deep must another bin be made, that is 18 ft. long and 3⅗ ft. wide, to hold 450 bu.?

20. How many days will 12 men require to do a piece of work that 95 men can do in 7⅕ days?

21. If ⅜ of an acre of land cost $60, what will 45¾ A. cost?

22. If $750 gain $202.50 in 4 yr. 6 mo., what sum will gain $155.52 in 1 yr. 6 mo.?

23. If it require 1200 yd. of cloth 5/4 wide to clothe 500 men, how many yards which is ⅞ wide will clothe 960 men?

24. If by traveling 6 hr. a day at the rate of 4½ mi. an hour, a man perform a journey of 540 mi. in 20 da., in how many days, traveling 9 hr. a day at the rate of 4 4/9 mi. an hour, will he travel 600 mi.?

25. If 1 lb. 12 oz. of wool make 2½ yd. of cloth 6 qr. wide, how many lb. of wool will it take for 150 yd. of cloth 4 qr. wide?

PARTNERSHIP.

379. ***Partnership*** is an association of two or more persons under a certain name, for the transaction of business.

The persons associated are called **partners,** and the association is called a **company, firm,** or **house.**

380. The ***capital*** is the money or property invested by the partners, called also *investment,* or *joint-stock.*

Capital may be money, real estate, personal property, time, labor, or skill.

381. The ***resources*** of a firm are the amounts due it, together with the property of all kinds belonging to it; called also *assets* or *effects.*

382. The ***liabilities*** of a firm are its *debts.*

The **net capital** is the excess of the resources over the liabilities.

383. The gains or losses of a firm are apportioned among its members according to the following

PRINCIPLE.—*The greater the capital and the longer it is invested, the greater is each partner's share of the gains or losses.*

WRITTEN EXERCISES.

384. To apportion gains or losses according to capital invested.

1. A and B engage in trade; A furnishes $500, and B $700 as capital; they gain $96; what is each man's share?

SOLUTION BY FRACTIONS.

$\$500 + 700 = \$1200$, whole capital.

A's stock $= \frac{500}{1200} = \frac{5}{12}$; $\$96 \times \frac{5}{12} = \$40$, A's gain.

B's stock $= \frac{700}{1200} = \frac{7}{12}$; $\$96 \times \frac{7}{12} = \$56$, B's gain.

Explanation.—Since $500, A's capital, is $\frac{500}{1200}$ or $\frac{5}{12}$ of the whole capital, he is entitled to $\frac{5}{12}$ of the gain, or $40. For a similar reason, B's share is $\frac{7}{12}$ of the gain, or $56.

2. Three men engage in trade; A puts in $6470, B $3780, and C $9860, and they gain $7890. What is each partner's share of the profit?

SOLUTION BY PROPORTION.

$$\$20110 : \left\{\begin{array}{l}\$6470\\ \$3780\\ \$9860\end{array}\right\} = \$7890 : \left\{\begin{array}{l}\$2538.45,\text{ A's gain;}\\ \$1483.05,\text{ B's gain;}\\ \$3868.49,\text{ C's gain.}\end{array}\right.$$

The ratio of the whole capital invested to each partner's share of the same, is equal to the ratio of the whole gain or loss to each partner's share of the gain or loss.

We may also regard the whole capital as the *first cause*, and each man's share of the capital as the *second cause*, the whole profit or loss as the *first effect*, and each man's share of the profit or loss as the *second effect*, and solve by proportion, thus:

$$\overset{\text{1st C.}}{\$20110} : \overset{\text{2d C.}}{\$6470} = \overset{\text{1st E.}}{\$7890} : \overset{\text{2d E.}}{\$x} = \$2538.45,\text{ A's gain, etc.}$$

3. Three men trade in company; A furnishes $8000, B $12000, and C $20000 of the capital; their gain is $1680. What is each man's share?

4. Three persons purchased a house for $2800, of which A paid $1200, B $1000, and C $600; they rented it for $224 a year. What part of the rent should each receive?

5. A man failed in business for $20000, and his available means amounted to only $13654. What will two of his creditors respectively receive, to one of whom he owes $3060, and to the other $1530?

6. Four persons engage in the lumber trade, and invest jointly $22500; at the expiration of a certain time, A's share of the gain is $2000, B's $2800.75, C's $1685.25, and D's $1014. How much capital did each put in?

7. A cargo of corn, valued at $3475.60, was entirely lost; $\frac{3}{8}$ of it belonged to A, $\frac{1}{4}$ of it to B, and the remainder to C. What was the loss of each, there being an insurance of $2512?

385. To apportion gains or losses according to the amt. of capital invested, and the time it is employed.

1. Three partners, A, B, and C, furnish capital as follows: A, $500 for 2 mo.; B, $400 for 3 mo.; C, $200 for 4 mo. They gain $600; what is each partner's share?

$$\begin{array}{l} 500 \times 2 = 1000 = \frac{1000}{3000} = \frac{1}{3} \times \\ 400 \times 3 = 1200 = \frac{1200}{3000} = \frac{2}{5} \times \\ 200 \times 4 = \underline{800} = \frac{800}{3000} = \frac{4}{15} \times \\ 3000 \end{array} \left.\vphantom{\begin{array}{l}1\\1\\1\end{array}}\right\} \$600 = \left\{\begin{array}{l} \$200\text{, A's share.} \\ \$240\text{, B's } \text{''} \\ \$160\text{, C's } \text{''} \end{array}\right.$$

Explanation.—The gain of each partner depends upon two elements, viz., the *amount* of his capital, and the *time* it is employed. Hence, the use of the whole capital is equal to the use of $3000 for 1 mo.; and A should receive $\frac{1}{3}$, B $\frac{2}{5}$, and C $\frac{4}{15}$ of the gain respectively.

We may also solve by proportion, the *causes* being compounded of the *two* elements, *capital* and *time*. Thus,

$$\$3000 : \$1000 = \$600 : \$200, \text{ A's share, etc.}$$

Rule.—*Multiply each partner's capital by the time it is invested, and consider the products as their capitals, and proceed as in Art.* **384.**

2. Smith, Banks & Co. gain in trade $8000. Smith furnished $12000 for 6 mo., Banks $10000 for 8 mo., and Crane $8000 for 11 mo. Apportion the gain.

3. A, B, and C enter into partnership. A puts in $357 for 5 months, B $371 for 7 months, and C $154 for 11 months, and they gain $347.20. What is each one's share?

4. Three men hire a pasture for $55.50. A put in 5 cows, 12 wk.; B, 4 cows, 10 wk.; and C, 6 cows, 8 wk. What ought each to pay?

5. Three men took a field of grain to harvest and thresh for $\frac{1}{4}$ of the crop; A furnished 4 hands 5 da., B 3 hands 6 da., and C 6 hands 4 da.; the whole crop was 372 bu. What was each man's share?

6. A and B engaged in trade. A put in $4200 at first, and 9 mo. afterward $200 more. B put in at first $1500, and at the end of 6 mo. took out $500. At the end of 16 mo. their gain was $772.20. What was the share of each?

INVOLUTION.

386. A ***power*** of a number is the number itself, or the product of equal factors, each of which is that number.

Thus, 27 is the product of $3 \times 3 \times 3$, or 3 taken 3 times as a factor. Hence, 27 is a *power* of 3.

387. ***Involution*** is the process of finding any power of a number.

388. An ***exponent*** of a power is a number used to denote the *degree* of a power. It is written at the right, and a little above the number. Thus,

3^1 or $3 = 3$, the *first* power of 3.
$3^2 = 3 \times 3 = 9$, the *second* power of 3.
$3^3 = 3 \times 3 \times 3 = 27$, the *third* power of 3.
$3^4 = 3 \times 3 \times 3 \times 3 = 81$, the *fourth* power of 3.

The **square** of a number is its *second* power, or the product of *two* equal factors.

The **cube** of a number is its *third* power, or the product of *three* equal factors.

WRITTEN EXERCISES.

389. To find any power of a given number.

1. Find the third power of 24.

Solution.—$24^3 = 24 \times 24 \times 24 = 13824$.

2. Find the square of 37. Of 42. Of 56. Of 75.

3. Find the cube of 15. Of 18. Of 42. Of 54.

4. What is the value of 63^2? Of 48^3? Of 32^4? Of 12^5?

A common fraction is raised to any power by involving each of its terms.

5. What is the third power of $\frac{5}{8}$?

Solution.—$(\frac{5}{8})^3 = \frac{5}{8} \times \frac{5}{8} \times \frac{5}{8} = \frac{5 \times 5 \times 5}{8 \times 8 \times 8} = \frac{5^3}{8^3} = \frac{125}{512}$.

Rule.—*Take the given number as many times as a factor as there are units in the exponent of the power.*

Involution may sometimes be facilitated by means of the following

Principle.—*The sum of the exponents of two powers of the same number is equal to the exponent of the product of those powers.*

Thus, $2^2 \times 2^3 = 2^5$; for $2^2 = 2 \times 2$, and $2^3 = 2 \times 2 \times 2$; hence $2^2 \times 2^3 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$.

What power of a number is produced by multiplying its square by its cube? Its first power, square, and cube? Its cube by its cube?

6. Find the squares and cubes of the numbers from 4 to 16 inclusive.

7. Find the squares and cubes of $\frac{3}{4}$, $\frac{7}{8}$, $\frac{7}{16}$, $\frac{11}{12}$, and $2\frac{1}{4}$.

8. Find the squares of 1.05, .08, 4.8, 2.36, and .007.

Find the required *power* of the following:

9. 25.4^2.
10. 106^3.
11. $(44\frac{1}{4})^2$.
12. $(\frac{11}{16})^4$.
13. $(\frac{2}{7})^5$.
14. $(2\frac{1}{2})^6$.
15. $.0342^2$.
16. $.05^6$.
17. 36.02^3.
18. $(182\frac{1}{8})^2$.
19. $(4.07\frac{1}{2})^2$.
20. $(1\frac{2}{15})^5$.

390. To find the square of a number in terms of its tens and units.

1. Find the square of 27 in terms of its tens and units.

$$\begin{array}{rcr} 27 & = & 20+7 \\ 27 & = & 20+7 \\ \hline 189 & = & 20\times 7+7^2 \\ 540 & = & 20^2+20\times 7 \quad \\ \hline 729 & = & 20^2+2\times 20\times 7+7^2 \end{array}$$

Explanation.—The product of $20+7$ by 7 is $20\times 7+7^2$, and the product of $20+7$ by 20 is $20^2+20\times 7$; hence, $20^2+2\times 20\times 7+7^2$, which is the sum of these partial products, is the square of $20+7$ or 27.

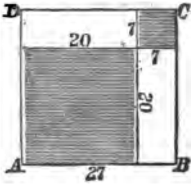
Principle.—*The square of a number consisting of tens and units is equal to the square of the tens, plus twice the product of the tens by the units, plus the square of the units.*

Using t and u respectively to denote the *tens* and *units* of a number, we have the

Formula: $t^2+2\times t\times u+u^2=(t+u)^2$.

Geometrical Illustration.

Let ABCD be a square, each side of which is 27 ft., and draw the lines as represented in the figure. It is evident that in the square ABCD 27^2 is equal to the sum of two squares, one of which is the square of the tens 20^2, the other the square of the units 7^2, together with two rectangles, each of whose areas is 20×7.



2. Find the square of 37.

$$\begin{array}{rcrcr} t^2 & = & 30^2 & = & 900 \\ 2\times t\times u & = & 2\times 30\times 7 & = & 420 \\ u^2 & = & 7^2 & = & 49 \\ \hline \end{array}$$

$$t^2+2\times t\times u+u^2=30^2+2\times 30\times 7+7^2=1369=37^2, \text{ Prin}$$

In like manner, find the square

3. Of 48.
4. Of 56.
5. Of 98.
6. Of 125.
7. Of 105.
8. Of 225.
9. Of 197.
10. Of 342.

391. To find the cube of a number in terms of its tens and units.

1. Find the cube of 25 in terms of its tens and units.

$$\begin{array}{rl}
25^2 = & \qquad\qquad 20^2 + 2\times 20\times 5 + 5^2 \\
25 = & \qquad\qquad\qquad\qquad 20 + 5 \\ \hline
25^2\times 5 = & \qquad 20^2\times 5 + 2\times 20\times 5^2 + 5^3 \\
25^2\times 20 = & 20^3 + 2\times 20^2\times 5 + 20\times 5^2 \\ \hline
25^3 = & 20^3 + 3\times 20^2\times 5 + 3\times 20\times 5^2 + 5^3
\end{array}$$

Explanation.—The *square* of 25 is $20^2+2\times20\times5+5^2$ (**390**, **Prin.**). Multiply this square by $20+5$; the product is $20^3+3\times20^2\times5+3\times20\times5^2+5^3$, which is the *cube* of 25.

Principle.—*The cube of a number composed of tens and units consists of four parts:* 1st, *the cube of the tens;* 2d, *three times the square of the tens multiplied by the units;* 3d, *three times the tens multiplied by the square of the units;* 4th, *the cube of the units.* Hence,

Formula: $(t + u)^3 = t^3 + 3\times t^2\times u + 3\times t\times u^2 + u^3$.

2. Find the cube of 46.

$$\begin{array}{rcrcr}
t^3 = & & 40^3 & = & 64000 \\
3\times t^2\times u = & & 3\times 40^2\times 6 & = & 28800 \\
3\times t\times u^2 = & & 3\times 40\times 6^2 & = & 4320 \\
u^3 = & & 6^3 & = & 216 \\ \hline
 & & 46^3 & = & 97336
\end{array}$$

3. Find the cube of 34 in terms of its tens and units.
4. Find the cube of 45 in terms of its tens and units.

Raise to the third power the following numbers:

5. 64.
6. 72.
7. 125.
8. 182.
9. 236.
10. 1.52.
11. 9.4.
12. .75.
13. $87\frac{1}{2}$.
14. .063.
15. 3.50.
16. $45\frac{3}{4}$.
17. $\frac{7}{8}$.
18. $\frac{25}{32}$.
19. $\frac{30}{45}$.

GEOMETRICAL ILLUSTRATION.

FIG. 1.

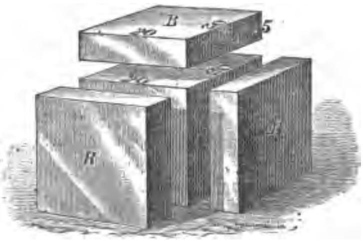
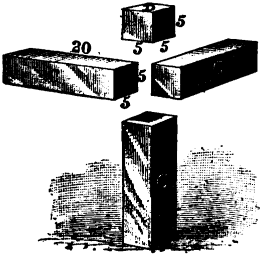
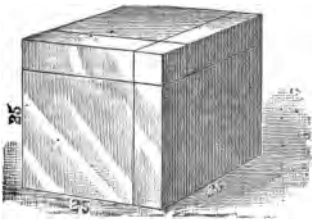


FIG 2.



The volume of the cube marked A, Fig. 1, is 20^3; the volume of each of the rectangular solids marked B is $20 \times 20 \times 5$, or $20^2 \times 5$; the volume of each of the rectangular solids marked C, in Fig. 2, is $20 \times 5 \times 5$, or 20×5^2; and the volume of the small cube marked D is 5^3. It is evident that if all these solids are put together as represented in Fig. 3, a cube will be formed, each edge of which is 25.

FIG. 3.



Find the value of each of the following expressions.

20. $4.6^3 \times 25^3$.
21. $6.75^4 - (7\frac{1}{4})^2$.
22. $\frac{7}{8}$ of $(\frac{4}{5})^3 \times (3\frac{5}{8})^2$.
23. $8^6 \div .4096$.
24. $2.5^3 \times (12\frac{3}{4})^2$.
25. $(7.5)^3 \div (1\frac{1}{2})^3$.

EVOLUTION.

392. The ***root*** of a number is either the number itself, or one of its equal factors.

393. ***Evolution*** is the process of finding a root of a number, and is the converse of *involution.*

394. The ***radical sign*** is $\sqrt{\ }$, or a *fractional exponent.*

A small figure called the **index,** written above the radical sign, denotes the root. A *fractional exponent* is sometimes used. Thus, $16^{\frac{1}{2}}$ is equivalent to $\sqrt{16}$; $64^{\frac{1}{3}}$ to $\sqrt[3]{64}$.

The numerator of the exponent indicates a *power,* and the denominator a *root.* Thus, $8^{\frac{2}{3}}$ is equivalent to $\sqrt[3]{8^2}$.

SQUARE ROOT.

395. The ***square root*** of a number is one of its *two* equal factors. Thus, the square root of 81 is 9, since $9 \times 9 = 81$.

PRINCIPLES.—I. *The square of a number contains twice as many figures as the number, or twice as many less one.* Thus,

$$\begin{array}{rrr}
1^2 = 1, & 10^2 = 100, & .05^2 = .25, \\
9^2 = 81, & 100^2 = 10000, & .05^2 = .0025, \\
99^2 = 9801, & 1000^2 = 1000000, & 2.5^2 = 6.25.
\end{array}$$

Hence,

II. *If any perfect square be separated into periods of two figures each, beginning with units' place, the number of periods will be equal to the number of figures in the square root of that number.*

If the number of figures in the number is *odd,* the left-hand period will contain only one figure.

WRITTEN EXERCISES.

396. To find the square root of a number.

1. Find the square root of 729.

$$\begin{array}{l}
t^2 + 2 \times t \times u + u^2 = 7{,}29\ (\ \overset{t}{20} + \overset{u}{7} = 27 \\
t^2 \qquad\qquad\quad = 20^2 \quad = 400 \\
\hline
\qquad 2 \times t \times u + u^2 = \qquad\qquad\qquad 329 \\
\qquad \underline{2 \times t \times u + u^2 = 2 \times 20 \times 7 + 7^2 = 329}
\end{array}$$

Explanation.—Since 729 has two periods, its square root has two figures (Prin. II). Since 29 cannot be a part of the tens, the tens of the root must be found from the first period 7.

The greatest number of tens whose square is contained in 7 is 2. Subtracting the square of 2 tens from 729, the remainder is 329. This remainder is composed of *twice* the product of the tens of the root by the units, and the square of the units (**390**). Hence, if 329 be divided by 40, which is *twice* the tens of the root, the quotient 7 will be either equal to the units' figure of the root, or greater. Subtracting $2 \times 20 \times 7 + 7^2$, or $(40+7) \times 7$, from 329, nothing remains. Hence, 27 is the required root.

In practice, the work may be abridged as follows:

```
      7,29 ( 27
      4
      ----
47 )  329
      329
      ----
```

In this example, 40 is the *partial* or *trial divisor*, and 47 the *complete divisor*.

Geometrical Explanation of Square Root.

1. Find one side of a square whose area is 729 sq. ft.?

Fig. 1.

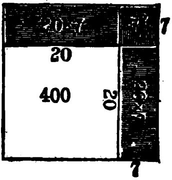
Let Fig. 1 represent a square whose area is 729 square feet. It is required to find the length of one side of this square.

Since the area of a square is equal to the square of one of its sides, a side may be found by extracting the square root of the area.

Since 729 consists of two periods, its square root will consist of two figures. The greatest number of *tens* whose square is contained in 700 is 2. Hence the length of the side of the square is 20 feet plus the units' figure of the root.

Removing the square whose side is 20 ft. and whose area is 400 sq. ft., there remains a surface whose area is 329 sq. ft. (Fig. 2). This remainder consists of two equal rectangles, each of which is 20 ft. long, and a square whose side is equal to the width of each rectangle. The units' figure of the root is equal to the width of one of these rectangles.

Fig. 2.



The area of a rectangle is equal to the product of its length and width (**215**); hence, if the area be divided by the length, the quotient will be the width. Now, since the two rectangles contain the greater portion of the 329 sq. ft., 2 × 20 or 40, the length of the two rectangles, may be used as a trial divisor to find the width. Dividing 329 by 40, the quotient is 8. But this quotient is too large for the width of the rectangles, for if 8 ft. is the width, the area of Fig. 2 will be $40 \times 8 + 8^2$ or 384 sq. ft. Taking 7 ft. for the width of the rectangles, the area of Fig. 2 is $40 \times 7 + 7^2$ or 329 sq. ft. Hence, 20 + 7 or 27 ft. is the length of a side of the square whose area is 729 sq. ft.

If the root contains more than two figures, it may be found by a similar process, as in the following example, where it will be seen that the partial divisor at each step is obtained by doubling that part of the root already found.

2. Find the square root of 186624.

$$\begin{array}{rl} & 18,66,24\ (\,400+30+2 = 432 \\ & \underline{16\ 00\ 00} \\ 400\times 2+30 = 830\ ) & 2\ 66\ 24 \\ & \underline{2\ 49\ 00} \\ 400\times 2+30\times 2+2 = 862\ ) & 1724 \\ & \underline{1724} \end{array}$$

In practice, thus:

$$\begin{array}{rl} & 18,66,24\ (\,432 \\ & \underline{16} \\ 83\ ) & 266, \text{etc.} \end{array}$$

Rule.—1. *Separate the given number into periods of two figures each, beginning at the units' place.*

2. *Find the greatest number whose square is contained in the period on the left; this will be the first figure in the root. Subtract the square of this figure from the period on the left, and to the remainder annex the next period to form a dividend.*

3. *Divide this dividend, omitting the figure on the right, by double the part of the root already found, and annex the quotient to that part, and also to the divisor; then multiply the divisor thus completed by the figure of the root last obtained, and subtract the product from the dividend.*

4. *If there are more periods to be brought down, continue the operation in the same manner as before.*

If a cipher occur in the root, annex a cipher to the trial divisor, and another period to the dividend, and proceed as before.

2. If there is a remainder after the root of the last period is found, annex periods of ciphers and continue the root to as many decimal places as are required.

Find the square root of

3. 1296.
4. 2401.
5. 2916.
6. 5625.
7. 11449.
8. 20736.
9. 125.44.
10. 207.36.

The square root of a fraction may be found by extracting the square root of each of its terms.

If either term of the fraction is an imperfect square, or the number is a mixed number, reduce to the decimal form before extracting the root.

Find the square root of

11. 9.3025.
12. 21025.
13. 11881.
14. 104976.
15. 266256.
16. 10795.21.
17. 196.1369.
18. .001225.
19. $\frac{625}{6561}$.
20. $\frac{7056}{9216}$.
21. $17\frac{3}{4}$.
22. 7.2.
23. $\sqrt{.1369} + \sqrt{1296}$.
24. $(2.8)^3 \div \sqrt{.117649}$.
25. $\frac{\sqrt{9}}{\sqrt{3^2}} \times \frac{3^2}{\sqrt{9^2}}$.

Extending to 4 decimal places, find the

26. $\sqrt{2}$.
27. $\sqrt{3}$.
28. $\sqrt{8}$.
29. $\sqrt{12}$.
30. $\sqrt{26}$.

31. A square field contains 1016064 square feet. What is the length of each side?

32. A field is 208 rd. long and 13 rd. wide. What is the length of the side of a square containing an equal area?

33. If 251 A. 65 sq. rd. of land are laid out in the form of a square, what will be the length of each of its sides?

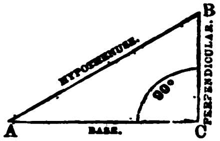
34. A circular island contains 21170.25 sq. rd. of land. What is the length of the side of a square field of equal area?

35. If it cost $312 to enclose a field 216 rd. long and 24 rd. wide, what will it cost to enclose a square field of equal area with the same kind of fence?

36. If an army of 55225 men be drawn up in the form of a square, how many men will there be on a side?

APPLICATIONS.

397. A ***triangle*** is a plane figure having three sides and three angles, as A, B, C.



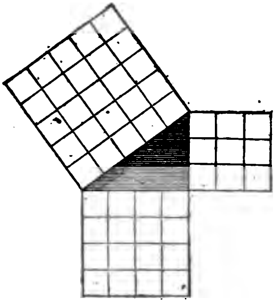
A **right triangle** is a triangle having one right angle, as at C.

The **base** is the side on which it stands, as AC.

The **perpendicular** is the side forming a right angle with the base, as BC.

The **hypothenuse** is the side opposite the right angle, as AB.

398. The following principles relating to *right-angled* triangles have been established by Geometry:



PRINCIPLES.—I. *The square of the hypothenuse of a right-angled triangle is equal to the sum of the squares of the other two sides.*

II. *The square of the base, or of the perpendicular, of a right-angled triangle is equal to the square of the hypothenuse diminished by the square of the other side.*

$$\text{FORMULAS:}\left\{\begin{array}{l} 1.\ \sqrt{Base^2 + perpendicular^2} = hypothenuse. \\ 2.\ \sqrt{Hypothenuse^2 - base^2} = perpendicular. \\ 3.\ \sqrt{Hypothenuse^2 - perpendicular^2} = base. \end{array}\right.$$

1. The base of a right-angled triangle is 12, and the perpendicular 16. What is the length of the hypothenuse?

2. The hypothenuse of a right-angled triangle is 35 ft., and the perpendicular 28 ft. What is the base?

3. The hypothenuse of a right-angled triangle is 53 yd., and the base 84 feet. Find the perpendicular.

4. If the gable end of a house 40 ft. wide is 16 ft. high, what is the length of the rafters?

5. The foot of a ladder is 15 feet from the base of a building, and the top reaches a window 36 ft. above the base. What is the length of the ladder?

6. A line reaching from the top of a precipice 120 ft. high, on the bank of a river, to the opposite side is 380 feet long. How wide is the river?

CUBE ROOT.

399. The ***cube root*** of a number is one of the *three* equal factors of that number.

Thus, the cube root of 125 is 5, since $5 \times 5 \times 5 = 125$.

PRINCIPLES.—I. *The cube of a number contains three times as many figures as the number, or three times as many, less one or two.* Thus,

$$\begin{array}{rlrlrl} 3^3 = & 27 & 10^3 = & 1{,}000 & .5^3 = & .125 \\ 9^3 = & 729 & 100^3 = & 1{,}000{,}000 & .05^3 = & .000125 \\ 99^3 = & 907{,}299 & 1000^3 = & 1{,}000{,}000{,}000 & 2.5^3 = & 15.635 \end{array}$$

Hence,

II. *If any perfect cube be separated into periods of three figures each, beginning with units' place, the number of periods will be equal to the number of figures in the cube root of that number.*

WRITTEN EXERCISES.

400. To find the cube root of a number.

1. Find the cube root of 405224.

$$\begin{array}{rl} t^3 + 3 \times t^2 \times u + 3 \times t \times u^2 + u^3 = & 405{,}224 \;(\; 70 + 4 = 74 \\ t^3 = 70^3 \qquad\qquad\qquad\qquad\qquad = & 343\,000 \\ \hline 3 \times t^2 \times u + 3 \times t \times u^2 + u^3 = & \multicolumn{1}{|l}{62\,224} \\ 3 \times t^2 = 3 \times 70^2 = 14700 \qquad & \multicolumn{1}{|l}{} \\ \cline{2-2} t^3 = 74^3 = & 405\,224 \end{array}$$

The preceding work may be abridged as follows:

$$\begin{array}{lrr|l}
 & & 405{,}224 & \!\!\!\!\!\!\!\!\; (\; 74 \\
 & 7^3 = & 343 & \\
\text{PARTIAL DIVISOR,} & 3\times70^2 = & 14700 & 62\,224 \\
 & 3\times70\times4 = & 840 & \\
 & 4^2 = & \underline{16} & \\
 & \text{COMPLETE DIVISOR,} & \underline{15556} & \underline{62\,224}
\end{array}$$

Explanation.—Since 405224 consists of two periods, its cube root will consist of two figures (**399**, Prin. II). Since 224 cannot be a part of the cube of the tens of the root, the first figure of the root must be found from the first period, 405. The greatest number of *tens* whose cube is contained in 405000 is 7. Subtracting the cube of 7 tens from the given number, the remainder is 62224. This remainder is equal to the product of three times the square of the tens of the root by the units, plus three times the product of the tens by the square of the units, plus the cube of the units (**400**).

If the remainder 62224 be divided by 3×70^2 or 14700, the quotient, which is 4, will be the units' figure of the root or a figure greater than the units' figure. Subtracting 74^3 from the given number, the result is 0; hence 74 is the required root.

Instead of cubing 74, the parts which make up the remainder 62224 may be formed and added thus:

$$\begin{array}{rcrcr}
3\times t^2\times u & = & 3\times70^2\times4 & = & 58800 \\
3\times t\times u^2 & = & 3\times70\times4^2 & = & 3360 \\
u^3 & = & 4^2 & = & \underline{64} \\
 & & & & 62224
\end{array}$$

Or, since 4 is a common factor in the three parts which make up the remainder, these parts may be combined thus:

$$\begin{array}{rcrcr}
3\times t^2 & = & 3\times70^2 & = & 14700 \\
3\times t\times u & = & 3\times70\times4 & = & 840 \\
u^2 & = & 4^2 & = & \underline{16} \\
 & & & & 15556\times4 = 62224.
\end{array}$$

In this example, 14700 is a *partial* or *trial divisor*, and 15556 is a *complete divisor*.

GEOMETRICAL EXPLANATION OF CUBE ROOT.

What is the length of the edge of a cube whose volume is 405224 cubic feet?

Fig. 1.

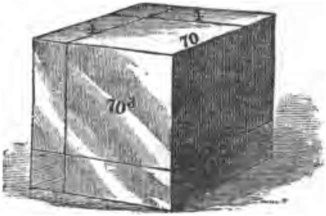
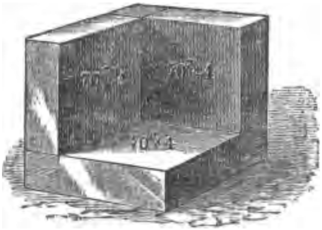


Fig. 2.



Let Fig. 1 represent a cube whose volume is 405224 cu. ft. It is required to find the length of the edge of this cube.

Since the volume of a cube is equal to the cube of one of its edges, an edge may be found by extracting the cube root of the volume.

Since 405224 consists of two periods, its cube root will consist of two figures.

The greatest number of *tens* whose cube is contained in 405224 is 7. Hence, the length of the edge of the cube is 70 ft., plus the units' figure of the root. Removing the cube whose edge is 70 ft. and whose volume is 343000 cu. ft., there remains a solid whose volume is 62224 cu. ft., Fig. 2. This remainder consists of solids similar to those marked B, C, and D, in Fig 1 and Fig 2 of Art. **391**.

The volume of a rectangular solid is equal to the product of the area of its base by its height or thickness (**222**): hence, if the volume be divided by the area of the base, the quotient will be the thickness. Now since the three equal rectangular solids, each of which is 70 ft. square, and whose thickness is the units' figure of the root, contain the greater part of the 62224 cu. ft., 3×70^2, or 14700, may be used as a trial divisor to find the thickness. Dividing 62224 by 14700, the quotient is 4. But this quotient may be too large. To test it, find the volume of Fig. 2, considering 4 ft. as the thickness. Thus, $3 \times 70^2 \times 4 + 3 \times 70 \times 4^2 + 4^3 = 62224$. Hence, 4 ft. is the true thickness, and 74 ft. the length of the edge of a cube whose volume is 405224 cu. ft.

If the cube root contains more than two figures, it may be found by a similar process, as in the following example, where it will be seen that the partial divisor at each step is equal to three times the square of that part of the root already found.

2. Find the cube root of 12812904.

ABRIDGED SOLUTION.

			12,812,904 (234
		$2^3 =$	8
1ST PARTIAL DIVISOR,	$3 \times 20^2 =$	1200	4812
	$3 \times 20 \times 3 =$	180	
	$3^2 =$	9	4167
1ST COMPLETE DIVISOR,		1389	645904
2D PARTIAL DIVISOR,	$3 \times 230^2 =$	158700	
	$3 \times 230 \times 4 =$	2760	
	$4^2 =$	16	645904
2D COMPLETE DIVISOR,		161476	

RULE.—1. *Separate the given number into periods of three figures each, beginning at the units' place.*

2. *Find the greatest number whose cube is contained in the period on the left; this will be the first figure in the root. Subtract the cube of this figure from the period on the left, and to the remainder annex the next period to form a dividend.*

3. *Divide this dividend by the partial divisor, which is 3 times the square of the root already found, considered as tens; the quotient is the second figure of the root.*

4. *To the partial divisor add 3 times the product of the second figure of the root by the first considered as tens, also the square of the second figure, the result will be the complete divisor.*

5. *Multiply the complete divisor by the second figure of the root and subtract the product from the dividend.*

6. *If there are more periods to be brought down, proceed as before, using the part of the root already found, the same as the first figure in the previous process.*

1. If a cipher occur in the root, annex two ciphers to the trial divisor, and another period to the dividend; then proceed as before, annexing both cipher and trial figure to the root.

2. If there is a remainder after the root of the last period is found, annex periods of ciphers and proceed as before. The figures of the root thus obtained will be *decimals.*

Find the cube root

3. Of 15625.
4. Of 166375.
5. Of 389017.
6. Of 571787.
7. Of 44361864.
8. Of 4080659192.

The cube root of a fraction may be found by extracting the cube root of the numerator and the denominator.

In extracting the cube root of *decimal* numbers, begin at the units' place and proceed both toward the left and the right, to separate into periods of *three* figures each.

Extract the cube root

9. Of $\frac{1000}{1331}$.
10. Of $\frac{13824}{15625}$.
11. Of $2\frac{7}{8}$.
12. Of $166\frac{3}{8}$.
13. Of .091125.
14. Of 12.812904.

Find the value to 3 decimal places,

15. Of $\sqrt[3]{2}$.
16. Of $\sqrt[3]{4}$.
17. $\sqrt[3]{6}$.
18. $9^{\frac{1}{3}}$.

19. Of $24.8 + \sqrt[3]{103.823} \times (.125)^{\frac{1}{3}}$.

20. Of $\sqrt[3]{16^6} \div \sqrt[3]{64} - \left(4 \times \sqrt[3]{.512}\right)$.

21. What is the length of the edge of a cubical box that contains 46656 cubic inches?

22. How many square feet in the surface of a cube whose volume is 91125 cubic feet?

23. What is the length of the inner edge of a cubical bin that contains 150 bushels of grain?

GENERAL REVIEW.

TEST EXERCISES.

401. 1. What cost 5 T. 14¾ cwt. of hay, at $16½ per ton?

2. What decimal of a rod is 1 ft. 7.8 in.?

3. The difference between 6/7 and 7/8 of a number is 10; what is the number?

4. Find the value of a farm, 3/8 of which is valued at $1575.

5. Find the cost of 1 Cd. of wood, if ⅓ of 5⅓ Cd. cost ¼ of $32.

6. What is the least number of acres in a farm that can be exactly divided into lots of 10 A., 14 A., 16 A., or 20 A. each?

7. What will 115 lb. of hay cost, at $20 per ton?

8. A farmer sold 2 loads of corn, weighing 2352 lb. each, at $.98 per bushel. What did he receive?

9. How many rods of fence will enclose a farm 1 mile sq.?

10. The roof of a house is 42 ft. long, and each side 20 ft. 6 in. wide. Find the cost of roofing at $4.62½ a square.

11. If a bushel of barley cost $.80, what will 20 bu. 3 pk. 6 qt. cost?

12. How many acres in a field 56⅛ rd. long, and 24.6 rd. wide?

13. If 154 bu. 1 pk. 6 qt. cost $173.74, what will 1.5 bu. cost?

14. How many perches of masonry in the wall of a cellar 20 ft. square on the inside, 8 ft. high, and 1½ ft. thick?

15. If 1⅛ cwt. of sugar cost $12¾, how many pounds can be bought for $93½, at the same rate?

16. A dealer bought 240000 ft. of lumber, at $15.90 per M., and retailed it out at $2¼ per C. What was his whole gain?

17. What would a man lose, who owned .15 of $8000 worth *of dry goods*, .375 of which were destroyed by fire?

18. What is the interval of time between March 20, 21 min. past 3 o'clock P. M., and April 11, 5 min. past 7 o'clock A. M.?

19. Find the dimensions of a cubical cistern holding 2000 gallons.

20. A cistern measures 4 ft. 6 in. square, and 6 ft. deep; how many hogsheads of water will it hold?

21. Reduce $\left(\frac{1\frac{3}{4}}{4\frac{1}{2}} \div \frac{2\frac{1}{3}}{2\frac{1}{4}}\right) \times \frac{4}{5}$ of $\frac{1}{2}$ to the form of a decimal.

22. If I lend a neighbor $200, 6 mo., how long ought he lend me $1000 to balance the favor?

23. Bought R. R. stock to the amount of $2356.80, and found that the sum invested was 40% of what I had left. What had I at first?

24. 20% of $\frac{3}{8}$ of a number is what per cent. of $\frac{3}{5}$ of it?

25. Bought merchandise for $2250, cash; for what sum must I draw my note at 3 mo., so as to obtain that sum at the bank, interest at 7%?

26. What part of any sum equals the interest of it for 5 yr. 4 mo. at 9%?

27. A house that is 50 ft. long and 40 ft. wide has a square or pyramidal roof, whose height is 15 ft. Find the length of a rafter reaching from a corner of the building to the vertex of the roof.

28. Find the length of a rafter reaching from the middle of one side to the vertex.

29. Bought merchandise as follows: July 3, $35.26; July 4, $48.65, on 30 da.; Aug. 17, $6.48; Sept. 12, $50. What is due on the account Oct. 12, interest at 9%?

30. What sum must be invested in stocks bearing 6½%, at 105%, to produce an income of $1000?

31. Find the face of a note drawn at 4 mo., the proceeds of which, when discounted at bank at 7%, is $875.50?

32. If a piece of land 40 rd. long and 4 rd. wide make an acre, how wide must it be to contain the same, if 25 rd. long?

33. The shadow of a tree measures 42 ft.; a staff 40 in. in length casts a shadow 18 in. at the same time. What is the height of the tree?

34. If $500 gain $50 in 1 yr., in what time will $960 gain $60?

35. A person exchanges 250 shares of 6% stock, at 70%, for stock bearing 8%, at 120%; what is the difference in his income?

36. Received $33.25 interest on a sum of money loaned 5 yr. previous, at 7%. What was the sum lent?

37. How many sq. ft. of boards will make a box and cover, 14 ft. long, 6 ft. wide, and 4 ft. high, outside measurement.

38. What will be the cost of a pile of wood 30 ft. long, 10 ft. wide, and 7 ft. 6 in. high, at $3.60 a cord?

39. What amount of money must I invest at 6% to receive an annual income of $1290?

40. A bin 20 ft. long, 8 ft. wide, and 3 ft. 6 in. deep is full of wheat. What is its value, at $1.28 a bushel?

41. May 10, 1883, I borrowed $6840, with which I purchased flour at $5.70 a barrel. June 21, 1884, I sold the flour at $6.62½ a barrel cash. What did I gain by the transaction, interest being reckoned at 6%?

42. A floor 52 ft. long by 35 ft. wide is to be covered with carpeting 2 ft. 4 in. wide. Find the length, and the cost, at $1¾ a yard, no allowance being made for waste or matching.

43. Bought tea at $.90 a pound. What must I ask per pound to abate 10% and still make 20%?

44. What should be discounted for the present payment of a note for $430, due in 1 yr. 6 mo., money being worth 5¼ per cent.?

45. What will be the cost of digging a cellar 41 ft. 3 in. long, 33 ft. wide, and 8 ft. deep, at .48 a cu. yard?

46. Bought a hogshead of molasses for $28, and 7 gal. leaked out; at what rate per gallon must the remainder be *sold to gain 20%?*

47. What sum invested in stocks at 105, bearing 6% int., will produce an income of $1050 a year?

48. In 1 yr. 4 mo. $311.50 amounted to $336.42, at simple interest. What was the rate %?

49. B and C trading together, find their stock to be worth $3500, of which C owns $2100; they have gained 40% on their first capital. What did each put in?

50. What is the value of $\left(\frac{31\frac{1}{3}}{8}+\frac{6\frac{3}{4}}{2\frac{1}{12}}\right)\div 4.23$?

51. What is a load of wheat worth, weighing 2430 lb., at $1.20 a bushel?

52. Bought 42 shares Ill. Cent. R. R. stock at 86, and sold the same at a premium of $12\frac{1}{2}$%. What was the gain?

53. What will it cost to insure a factory valued at $21000, at $\frac{4}{5}$%, and the machinery, valued at $15400, at $\frac{5}{8}$%?

54. The pedestal of a certain monument is a cubic block of granite, containing 373248 cu. in. What is the length of one of its sides?

55. What is the amount of $200 for 8 yr. at 6% comp. int.?

56. Paid $148.352 for 9728 ft. of lumber. What was that per M.?

57. Bought goods for $1370, on 3 mo. credit; sold the same at an advance of 25% of the cash value, and took in payment $350 cash, and the balance in two notes of equal amount, one at 3 mo., and the other at 6 mo. Reckoning discount at 6%, what was the net gain?

58. If by working $6\frac{2}{3}$ hr. a day, a man can do a job in $12\frac{1}{2}$ da., how many days will be required if he work $8\frac{1}{3}$ hr. per day?

59. An open court contains 40 sq. yd. How many stones, 9 in. square, will be required to pave it?

60. A merchant bought goods to the amount of $1000, and gave his note, dated Jan. 1, 1882, on int. after 3 mo.; 6 mo. after the note was given, he paid $560, and 5 mo. after the first payment, he paid $406. What was due Aug. 23, 1884?

61. Bought a horse for $250, and sold him for $300. What part of the cost was the gain? What per cent.?

62. Find the value of a piece of land 50.6 rd. long and 30.4 rd. wide, at $45.75 an acre?

63. Sold 60 bu. of oats at $.42; 40 bbl. of flour at $8.50; and 56 bu. of corn at $.58. Make out the bill, adding 1% for cartage, and deduct 3% for cash.

64. How many bushels of wheat in a car-load of 18 tons? How many barrels of beef or pork? How many barrels of flour?

65. What is the value of a pile of wood 200 ft. long, 40 ft. wide, and 10 ft. high, at $3.25 a cord?

66. What is the capacity of a cistern 4.5 ft. square, and 10.5 ft. deep?

67. Bought a farm for $2964.12 ready money, and sold it again for $3665.20, payable in 1 year, 6 mo. What would be gained in ready money, discounting at the rate of 8%?

68. How many tons of ice can be stored in a building 60 ft. by 70 ft. by 20 ft., a cubic foot weighing 55 lb.?

69. What is the value of a piece of land 40⅔ rd. long, and 16¼ rd. wide, at $32.50 an acre?

70. For what sum must a bank note be drawn, at 60 da., to net $1200, when discounted at 5%?

71. What will 48 bu. 45 lb. of wheat cost, at $1.65 a bu.?

72. What is the present worth of a debt for $1500, on which $575 is to be paid in 10 mo., and the remainder in 1 yr 3 mo., the rate of interest being 6%?

73. How large a draft can be purchased for $436.60, exchange being at 1½% premium?

74. If 30 men can do a piece of work in 11 da., how many men can do another piece of work 4 times as large, in ⅙ of the time?

75. How many rods of fence will be required to inclose *10 acres of land* in the form of a square?

76. What is the length of one edge of a cistern in cubical form, the capacity of which is 2744 cubic feet?

77. Jones bought in Boston, merchandise to the amount of $3864 cash. Not having the money, he gave his note, payable in 60 days, with grace. Discount being at 6%, what was the face of the note?

78. How many cedar rails 9 ft. long will be required to enclose a field one-half of a mile square with a fence 6 rails high, the rails lapping 6 in., and what will be the cost, at $50 per thousand?

79. In what time will $627.50 loaned at 7% produce as much interest as $2510 will produce, at 3½% in 1 yr. 8 mo.?

80. If $10 worth of provision serve 7 men 4 da., how many days will the same provision serve 12 men?

81. In what time will $1600, at 6% interest, amount to $2000?

82. If a farm of 160 acres rents for $450, for how much should a farm of 840 acres rent?

83. If I buy bank stock at 20% discount, and sell it at 10% premium, what per cent. do I gain?

84. A farmer sold 4 loads of corn, weighing respectively 2564 lb., 2723 lb., 3000 lb., and 3109 lb., for $.62 a bushel. What did he receive for the whole?

85. What is the depth of a lot that has 120 ft. front, and contains 18720 sq. ft.?

86. What is the value of 24640 lb. of wheat, at $2.50 per cental?

87. At $14.50 per M., how many feet of boards can be purchased for $222.75?

88. In settling with a creditor Sept. 3, 1883, I gave my note for $365.80, at 8%. What must be paid July 3, 1884?

89. What is the cost of 9648 lb. of hay, at $16.50 a ton?

90. What per cent. profit does a merchant make, who buys at a discount of 20, 10, and 5%, and sells at the list price?

91. A gas company declares a dividend of 5%, and divides among its stockholders $12500. What is its capital stock?

92. What is the length of one side of a square piece of land, containing 40 acres?

93. A lawyer collects 75% of an account of $2860, charging 5% commission. What amount should he pay over?

94. A man bought 500 bu. of wheat for $637.50. He finds 15% of it worthless. For what must he sell the remainder per bushel to gain 20% on the cost?

95. How many tiles 8 in. square will cover an open court, 36 ft. long and 24 ft. wide?

96. Paid $4048 for stock, at 12% discount, and sold it at 115. What was the whole gain, and the gain %?

97. If $1249.50 rent is paid in 2 yr. 3 mo. 18 da. for a house that cost $3500, what is the rate of interest?

98. What will be the cost of filling in a space 320 ft. long and 72 ft. wide, averaging 3.5 ft. below grade, at $.56 a cubic yard?

99. Three persons engage in trade. A puts in $250, 6 mo., C $275, 8 mo., and D $450, 4 mo., and they gain $825. What is each man's share of the gain?

100. How many bushels of grain will a bin 8 ft. long, 6 ft. wide, and 4½ ft. deep contain?

101. How many yards of carpeting 30 in. wide will cover a floor 18 ft. by 15 ft., if the strips run lengthwise, and the matching of the figures requires 12 in. to be turned under; and what will be its cost, at $1.87½ a yard?

102. Find the area of a square field, the distance around which is 3 miles.

103. Insurance was effected on a store to the amount of $4500, at the rate of $8.50 on the $1000. Find the cost of insurance, including $1.50 for the policy.

104. A rectangular bin will hold 300 bu. of grain. What *is its volume* in cubic feet?

105. Find the difference between the present worth of $1250, due in 1 yr. 9 mo., and the bank proceeds of the same for the same time, money being worth 8%.

106. What is the cost of sawing a pile of wood 28 ft. long, 4 ft. wide, and 6 ft. high, at $1.12½ a cord?

107. A speculator in Chicago bought 40000 bu. of wheat, at $1.20 a bushel; he paid 6 cents a bushel freight, and ½ cent a bushel insurance to Buffalo, where he sold the wheat at $1.62½ a bushel. What was his profit on the transaction?

108. There is a park 500 ft. square. If a walk 3 ft. wide is made in it along the edge, how many square yards in the walk?

109. If a person breathe 80 cu. ft. of air in a minute, how long would he be in breathing the air of a room 24 ft. by 18 ft. by 11 ft. 6 in.?

110. C bought a farm of 120 A., at $28 an acre, paid $480 for fencing, then sold it for 12½% advance on the whole cost. Find his whole gain, and what he received an acre?

111. A owes B $321, payable in 1 year. What is the present worth of the debt, money being worth 7%?

112. A man sold a piece of property for $11320; the terms were $3200 in cash on delivery, $3500 in 6 mo., $2500 in 10 mo., and remainder in 1 yr. 3 mo., with 7% interest. What was the whole amount paid?

113. A contractor employs 45 men to complete a work in 3 months. What additional number of men must he employ to complete the work in 2½ months?

114. If a piece of silk cost $1.20 a yard, at what price must it be marked that it may be sold at 10% less than the marked price, and still make a profit of 20%?

115. A man bought 150 acres of land at $87.50 per acre. He sold at one time 60 acres at $75 per acre, and at another time 50 acres at $95.60 per acre. What must be the selling price of the remainder to make a gain of $1155 on the cost?

116. If the ridge of a building is 8 ft. above the beams, and the building is 32 ft. wide, what is the length of the rafters?

117. A commission merchant in Brooklyn wishes to remit to his employer in Memphis, $512.36 by draft at 60 da.; what is the face of the draft that he can purchase with this sum, exchange being at 2½% discount, interest 7%?

118. Find the cost of the flooring for a two-story house, at $30 per M., it being 1¼ in. thick, each floor being 48 ft. by 25 ft., no allowance made for waste.

119. What is the simple interest and the amount, the compound interest and amount, the present worth and the true discount, the bank discount and the proceeds of $1920, for 2 yr. 5 mo. 12 da. at 6%? Also the face of the note, which when discounted at a bank for the same time, and at the same rate, will produce the same sum?

120. Bought 2500 bu. of wheat in Chicago, at $1.40 a bu.; paid 4½% insurance, 3% freight, and 2½% commission for buying. At what price per bushel must it be sold in New York to realize 40% net profit on the cost?

121. A mining corporation, consisting of 30 persons are taxed $4342.75; their property is assessed for $188000, and each poll is assessed $.62½. What % is their tax, and what must he pay who is assessed for $2500, and who pays for 1 poll?

122. A speculator wishing to purchase a tract of land containing 450 acres at $27.50 an acre, borrows the money at 5½ per cent. At the end of 4 yr. 11 mo. 20 da. he sells ⅜ of the land at $34 an acre, and the remainder at $32.55 an acre. What does he lose by the transaction?

123. Shipped a car load of fat cattle to Boston, and offered them for sale at 25% advance on the cost; but the market being dull I sold for 14% less than my asking price, and gained thereby $170. What did the cattle cost; for what did they sell; and what was my asking price?

ADDENDA.*

402. DRILL TABLE No. 4.

480	48	4800	48000
840	120	1200	12000
720	72	7200	72000
864	96	9600	96000
648	24	2400	24000
240	240	1920	64800
120	192	3840	19200

403. Fix the following well in mind,—

16⅔% of anything is ⅙ of that thing; 12½% is ⅛.
25% is ¼; 33⅓% is ⅓; 50% is ½.

Next read, filling the blanks,—

½ of anything is — % of that thing; ¼ is — %.
⅓ is — %; ⅛ is — %; ⅙ is — %; ⅒ is — %.

404. Then practice as directed on page 174, for 16⅔%, 12½%, 25%, 33⅓%, 50%, etc. This table will give additional drill on 1%, 10%, 100%, etc.

To drill on other rates % make factors of the above, as 7%; read rapidly 7% is 7 times 1% of 480, or 7 times 4.8; 6% is 6 times 1% of 480, or 6 times 4.8; etc.

For drill in Decimals, read the above as divided by 10, 100, 1000. Thus, by column divided by 10 read 48, 84, 72, 86.4, 64.8, etc.

Read as tenths, etc., of miles, rods, acres, days, etc.

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* For the nature and design of this Addenda, see Preface.

ROMAN NOTATION.

405. ***Roman Notation*** employs seven capital letters to express numbers. Thus,

Letters.	I,	V,	X,	L,	C,	D,	M.
Values.	1,	5,	10,	50,	100,	500,	1000.

All other numbers may be expressed by combining these letters according to the following

PRINCIPLES.

I. Repeating a letter repeats its value.

Thus, XX represents 20, CCC, 300; DD, 1000.

II. When a letter is placed *after* one of greater value, its value is to be *added* to that of the greater.

Thus, VI represents 6; XV, 15; LXX, 70; DC, 600.

III. When a letter is placed *before* one of greater value, its value is to be *taken from* that of the greater.

Thus, IV represents 4, IX, 9; XL, 40; XC, 90.

IV. When two or more letters, each of a less value, are placed after one of greater value, their values are to be *added* to that of the *greater*.

Thus, XIV represents 14; LIX, 59; CXL, 140.

V. A bar or dash placed over a letter increases its value *one thousand times.*

Thus, $\overline{\text{X}}$ represents 10000; $\overline{\text{XC}}$, 90000; $\overline{\text{DL}}$, 550000.

Express by Roman notation:

1. Twelve.
2. Seventeen.
3. Twenty-four.
4. Thirty-nine.
5. Forty-six.
6. Fifty-eight.
7. Sixty-five.
8. Seventy-seven.
9. Eighty-two.
10. Ninety-nine.
11. 124.
12. 245.
13. 597.
14. 620.
15. 1674.

CIRCULATING DECIMALS.

406. When the numerator of a fraction, with ciphers annexed, is exactly divisible by the denominator, the result is called a ***perfect decimal.***

Thus, $\frac{1}{4}$ equals .25, and $\frac{7}{8}$ equals .875 are *perfect decimals.*

407. A ***circulating decimal*** is a decimal in which a figure, or set of figures, is repeated in the same order; and the figure, or set of figures repeated, is called the ***repetend.***

Thus, $\frac{1}{3}$ equals .333 + and $\frac{8}{11}$ equals .7272 are *circulating decimals,* and the 3 and 72 are the *repetends.*

408. A repetend is written but once, and a point (.) is placed over the single figure, or over the first and last of the set of figures.

Thus, .555 and .135135 are written $.\dot{5}$ and $.\dot{1}3\dot{5}$.

PRINCIPLE.—*Every fraction whose denominator, in its lowest terms, contains other prime factors than 2 or 5 is equivalent to a circulating decimal.*

A **pure circulating decimal** is a decimal which commences with a repetend; as $.\dot{7}$, or $.\dot{2}7\dot{9}$.

A **mixed circulating decimal** is a decimal in which the repetend is preceded by one or more decimal places called the *finite part* of the decimal; as, $.2\dot{7}$, or $.04\dot{6}4\dot{8}$, in which .2 or .04 is called the *finite part.*

409. To change a fraction to a perfect or circulating decimal.

Change

1. To perfect decimals, $\frac{7}{8}$, $\frac{9}{25}$, $\frac{13}{16}$, $\frac{23}{40}$, $\frac{11}{125}$, and $\frac{77}{200}$.
2. To pure circulating decimals, $\frac{6}{7}$, $\frac{7}{9}$, $\frac{9}{11}$, $\frac{12}{37}$, $\frac{10}{21}$, and $\frac{7}{41}$.
3. To mixed circulating decimals, $\frac{5}{12}$, $\frac{8}{15}$, $\frac{13}{22}$, $\frac{11}{30}$, and $\frac{7}{36}$.

The *law* for the formation of repetends will be apparent from the following:

1. $\frac{1}{9} = .1111+ = .\dot{1}$.
2. $\frac{1}{99} = .01010+ = .\dot{0}\dot{1}$.
3. $\frac{1}{999} = .001001+ = .\dot{0}0\dot{1}$.
4. $\frac{1}{9999} = .00010001+ = .\dot{0}00\dot{1}$.
5. $\frac{4}{9} = .4444+ = .\dot{4}$.
6. $\frac{23}{99} = .2323+ = .\dot{2}\dot{3}$.
7. $\frac{135}{999} = .135135+ = .\dot{1}3\dot{5}$.
8. $\frac{1728}{9999} = .17281728+ = .\dot{1}72\dot{8}$.

PRINCIPLE.—*A repetend is changed to the form of a fraction by writing for a denominator as many* NINES *as there are figures in the repetend.*

410. To change a circulating decimal to the form of a fraction.

1. Change $.\dot{2}1\dot{6}$ to the form of a fraction.

SOLUTION.—$.\dot{2}1\dot{6} = \frac{216}{999} = \frac{8}{37}$.

In like manner, change to the form of fractions:

2. $.\dot{4}\dot{5}$.
3. $.\dot{6}\dot{6}$.
4. $.\dot{2}9\dot{7}$.
5. $.\dot{6}7\dot{5}$.
6. $.\dot{3}2\dot{4}$.
7. $.\dot{4}15\dot{8}$.
8. $.\dot{6}43\dot{5}$.
9. $.\dot{9}512\dot{1}$.

10. Change $2.\dot{2}9\dot{7}$ and $12.\dot{0}8\dot{1}$ to the form of improper fractions.

11. Change $.2\dot{2}\dot{7}$ to the form of a fraction.

SOLUTION.—$.2\dot{2}\dot{7} = .2\frac{27}{99} = \frac{2\frac{27}{99}}{10} = \frac{225}{990} = \frac{5}{22}$.

In like manner, change to the form of fractions or mixed numbers:

$.5\dot{7}$.
$.04\dot{8}$.
$.100\dot{4}$.
$.64\dot{7}\dot{2}$.
$.\dot{7}85\dot{2}$.
$.01\dot{2}\dot{6}$.
$7.5\dot{4}\dot{3}$.
$2.5\dot{6}\dot{4}$.
$5.2\dot{7}$.
$.04\dot{6}4\dot{8}$.
$.35\dot{1}3\dot{5}$.
$7.01\dot{2}\dot{6}$.

To *add, subtract, multiply,* or *divide* circulating decimals, reduce them *to fractions,* and then perform the required operation.

DENOMINATE TABLES.

411. ***Roads*** and ***boundaries*** of land.

7.92 Inches = 1 Link, . . *l.*
25 Links = 1 Rod, . . *rd.*
4 Rods = 1 Chain, . *ch.*
66 Feet = 1 Chain, . *ch.*
80 Chains = 1 Mile, . . *mi.*

412. For ***mariners'*** use.

9 Inches = 1 Span.
8 Spans or 6 Ft. = 1 Fathom.
120 Fathoms = 1 C. Length.
7½ C. Length = 1 Nau. Mile.
3 Miles = 1 League.

413. ***Area*** or ***contents*** of land.

625 Sq. Links = 1 Pole, . *P.*
16 Poles = 1 Sq. Ch., *sq. ch.*
10 Sq. Chains = 1 Acre, . *A.*
640 Acres = 1 Sq. Mile, *sq. mi.*
36 Sq. Miles = 2 Townsh., *Tp.*

414. For ***geographical*** calculations, etc.

1 Geog. Mile = $1\frac{3}{20}$ Stat. Mi.
3 " " = 1 League.
60 " " or
69.16 Stat. Mi. } = 1 Degree.
360 Degrees = Circum. of Earth.

A *knot*, used in measuring distances at sea, is equivalent to a *nautical* mile.

415. ***Apothecaries' fluid measure*** is used in prescribing and in compounding *liquid* medicines.

TABLE.

60 Minims, or drops (♏) = 1 Fluidrachm, . . *f*ʒ.
8 Fluidrachms = 1 Fluidounce, . . *f*℥.
16 Fluidounces = 1 Pint, ... *O.*
8 Pints = 1 Gallon . . . . *Cong.*

1 *Cong.* = 8 *O.* = 128 *f*℥ = 1024 *f*ʒ = 61440 ♏.

1. *O* stands for the Latin *Octarius*, a Pint; *C*, for *Congius*, a Gallon.

A common teaspoonful, or 45 drops, makes about one fluidrachm. A common tablespoon holds about half a fluid ounce.

℞ is an abbreviation for *recipe*, or take; ā, aa, for equal quantities; j. for 1; ij. for 2; *ss.* for *semi*, or half; gr. for grain; P. for a little part; P. æq. for equal parts; q. p., as much as you please; gtt. for drops.

416. ***Apothecaries' weight*** is used by apothecaries and physicians in compounding *dry* medicines.

TABLE.

20 Grains (*gr.* xx) = 1 Scruple, . *sc.* or ℈.
3 Scruples (℈iij) = 1 Dram, . . *dr.* or ʒ.
8 Drams (ʒ viij) = 1 Ounce, . . *oz.* or ℥.
12 Ounces (℥ xij) = 1 Pound, . . *lb.* or ℔.

1 ℔ = ℥ 12 = ʒ 96 = ℈ 288 = *gr.* 5760.

1. *Medicines* are bought and sold in quantities by Avoirdupois weight.

2. The pound, ounce, and grain are the same as those of Troy Weight, the ounce being *differently divided.*

417. ***Diamond weight.***

16 Parts = 1 Grain.
4 Grains = 1 Carat.
1 Carat = 3⅙ Troy gr., nearly.

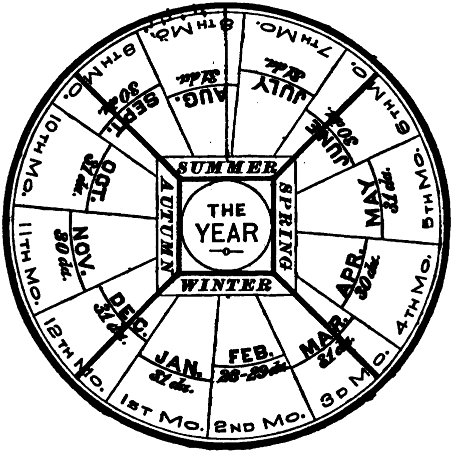
418. ***Assayers' weight.***

1 Carat = 10 pwts.
1 Carat gr. = 60 Troy gr.
24 Carats = 1 Troy lb.

419. ***Time*** is a measured portion of duration. The *calendar year* is divided as shown in the diagram.

The *solar year* is exactly 365 da. 5 hr. 48 min. 49.7 sec.

The *common year* consists of 365 da. for three successive years, every *fourth* year containing 366 da., one day being added for the excess of the solar year over 365 da. This day is added to the month of February, which then has 29 da., and the year is called *leap year.*



420. ***Circular*** or ***angular measures*** are used in measuring angles, determining latitude and longitude, etc.

The **unit** for measuring angles is the *degree.*

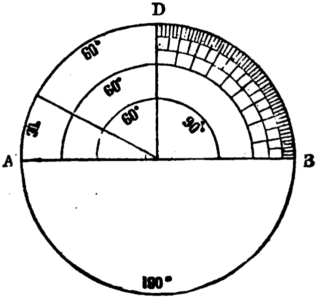
421. A ***Circle*** is a plane figure bounded by a curved line, every point of which is equally distant from a point within called the ***centre.***

The **circumference** of a circle is the line that bounds it.

An **arc** is any part of the circumference, as AD, or DB.

The **measure** of an angle is the *arc* of the circle included between its two sides; as the arc AD, or DE.

A **degree** is the angle measured by the arc of $\frac{1}{360}$ part of the *circumference* of a circle.



422. TABLE OF UNITS. EQUIVALENTS.

60 Seconds (″) = 1 Minute, . . ′.
60 Minutes = 1 Degree, . . °.
30 Degrees = 1 Sign, . . . *s.*
12 Signs, or 360° = 1 Circumference, *c.*

1 *C.* = 1296000″. 21600′. 360°. 12 *s.*

1. A **semi-circumference** is *one-half* of a circumference, or 180°.
2. A **quadrant** is *one-fourth* of a circumference, or 90°.
3. A **sextant** is *one-sixth* of a circumference, or 60°.
4. A **sign** is *one-twelfth* of a circumference, or 30°.
5. A **degree** (1°) is *one-ninetieth* of a *right angle.*
6. The length of a degree varies with the size of the circle; thus, a degree of longitude at the Equator is 69.16 statute miles, at 30° of latitude it is 59.81 miles, at 60° of latitude it is 34.53 miles, and at 90°, or the poles, it is nothing.
7. A *minute* of the earth's circumference is called a geographic, or nautical mile, and is about $1\frac{1}{8}$ common miles.

WRITTEN EXERCISES.

423. To change higher to lower denominations.

Reduce	Change
1. 3 mi. 51 ch. 6 l. to links.	5. 10 S. 10° 10′ 10″ to ″.
2. ℔ 16 ℥ 7 ʒ 3 to ℈.	6. 75 A. 4 sq. ch. 18 P. to sq. l.
3. Cong. 4 O. 5 f ℥ 8 to f ʒ.	7. 2 common years to hours.
4. 4 fath. 3 ft. 8 in. to in.	8. 16° 24′ 26″ to seconds.

Find the value, in *integers* of lower denominations, of

9. .1934 S.	12. f ℥ .7.	15. $\frac{3}{8}$ of ℔ $2\frac{1}{2}$.	18. .625 fath.
10. ℔ $\frac{15}{22}$.	13. .715°.	16. $\frac{9}{16}$ of a cir.	19. $\frac{3}{7}$ S.
11. .67 lea.	14. Cong. $\frac{7}{8}$.	17. f ℥ $\frac{43}{64}$.	20. .765 lb. Troy.

21. A field 17 ch. 31 l. long, 12 ch. 87 l. wide, is enclosed by a board fence. What is the length of the fence in feet?

23. How many acres in a piece of land 105 ch. 85 l. long, and 40 ch. 15 l. wide?

424. To change lower to higher denominations.

How many	Reduce
1. Miles are 8470 links?	4. f ʒ 6748 to Cong.
2. Fath. are 1435 feet?	5. 5270 naut. mi. to deg.
3. Degrees are 2007200″?	6. ℈ 90562 to ℔.

To change a compound number to a *fraction* of a higher denomination:

What part of	What decimal of
7. f ℥ 1 is f ʒ 5 ♏ 36?	10. 1 lea. is 2 mi. 3 rd. 1 yd.?
8. 1 mi. is 18 fathoms?	11. 1 mi. is 16.02 chains?
9. 1 chain is $4\frac{1}{8}$ feet.	12. 25° 42′ 40″ is 7° 42′ 48″?

13. In what time will a heavenly body move through a quadrant, if it move at the rate of 3′ 12″ per minute?

14. What will be the cost of a farm containing 4550000 sq. l, *at $50 an acre?*

15. How many rods of fence will enclose a farm 60 chains square?

16. In a storm at sea, a ship changed her longitude 423 geographic miles. How many degrees and minutes?

GOVERNMENT LANDS.

425. *Government Lands* are usually surveyed into rectangular tracts, bounded by lines conforming to the cardinal points of the compass.

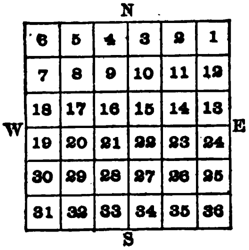
A **base line** on a parallel of latitude, and a **principal meridian** intersecting it, are first established. Other parallel lines are then run *six miles* apart, each way.

The tracts thus formed are called **townships**, and are designated by their number N. or S. of the *base line*.

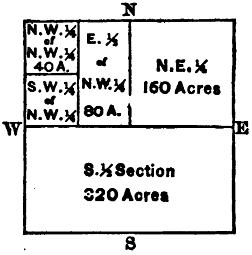
A **range** is a line of *townships* extending north and south of the *base line*, and is designated by its number E. or W. of the *principal meridian*.

A *township* is divided into 36 equal squares, called **sections**, each 1 mile square, and containing 640 acres.

A Township.



A Section.



The U. S. Land Office recognizes the following divisions of a *section* for purposes of sale:

A half-section,	1 mi. × $\frac{1}{2}$ mi. = $\frac{1}{2}$ sq. mi. = 320 A.
A quarter-section,	$\frac{1}{2}$ " × $\frac{1}{2}$ " = $\frac{1}{4}$ " " = 160 "
A half-quarter-section,	$\frac{1}{2}$ " × $\frac{1}{4}$ " = $\frac{1}{8}$ " " = 80 "
A quarter-quarter-section,	$\frac{1}{4}$ " × $\frac{1}{4}$ " = $\frac{1}{16}$ " " = 40 "

WRITTEN EXERCISES.

426. 1. If a township of land is equally divided among 288 families, what does each receive? What part of a section?

2. What number of rails will enclose a quarter section of land with a fence 6 rails high, and 3 lengths for every 2 rods: and what will be the cost of the rails, at $40 per thousand?

3. A man bought the S. ½ of a section of land at $2¼ an acre, and afterward sold the E. ½ of what he bought at $4.37½ an acre. What was his gain? on the part sold?

4. If I buy the N. E. ¼ and the E. ½ of N. W. ¼ of a section of land, how many acres do I purchase? What part of a whole section? Locate the parts in a diagram.

5. A man bought of the Ill. Cent. R. R. Co., the S. ½ of Sect. 4, Tp. 10 N., range 6 E., at $2 an A. He sold the E. ½ of S. E. ¼ at $2.75 an A.; the N.W. ¼ of S. E. ¼ at $3½ an A.; and the N. ½ of S. W. ¼ at $3.84 an A. How many A. has he left? What was his gain on the cost of the whole? Draw diagram.

6. From a section of land bought of the U. S. Government at $1.25 an A., was sold the S. ½ of S.W. ¼ at $2.50 an A.; the N. E. ¼ of N.W. ¼ at $1.75 an A.; the W. ½ of S. E. ¼ at $2 an A.; and the W. ½ of S. W. ¼ of N. E. ¼ at $3 an A. How many A. remained, and what was the gain, if the remainder sold at $2¼ an acre? Draw diagram and explain.

FOREIGN MONEYS.

427. ***English*** or ***Sterling money*** is the legal currency of Great Britain.

The **unit** of English money is the *sovereign* or *pound sterling*, equal to $4.8665 U. S. money.

TABLE OF UNITS.				U. S. VALUE.
4 Farthings (*far.*)	=	1 Penny, ...	*d.*	.02
12 Pence	=	1 Shilling.	*s.*	.2433+
2 Shillings	=	1 Florin, ...	*fl.*	.48665
5 Shillings	=	1 Crown, ...	*cr.*	$1.2166+
20 Shillings	=	1 Sovereign, or . . . 1 Pound, ...	*sov.* *£.*	4.86+
21 Shillings	=	1 Guinea, ...	*G.*	5.11

The **coin** of Great Britain in general use is as follows: **Gold,** the sovereign and half-sovereign; **silver,** the crown, half-crown, florin, shilling, six-penny, and three-penny pieces; **copper,** the penny, half-penny, and farthing.

428. ***Canada money*** is the legal currency of the Dominion of Canada.

The denominations are *dollars, cents*, and *mills*, and have the same *nominal* value as the corresponding denominations of U. S. Money.

The **coin** of the Dominion of Canada in use is as follows: **Gold,** the soveregn and half-sovereign; **silver,** the fifty-cent, twenty-five cent, ten-cent, and five-cent pieces.

429. ***French money*** is the legal currency of France, and is *decimal*.

The legal **unit** of French money is the *silver franc*, equal to $.193 U. S. money.

10 Millimes (*m.*)	=	1 Centime, . .	*ct.*	$0.00193.
10 Centimes	=	1 Decime, . .	*dc.*	.0193.
10 Decimes	=	1 Franc, . . .	*fr.*	.193.

The **coin** of France is: **Gold,** 100, 40, 20, 10, and 5 franc pieces; **silver,** 5, 2, and 1 franc; **bronze,** 10, 5, 2, and 1 franc pieces.

430. The ***Empire of Germany*** has adopted a new and uniform system of coinage.

The **unit** of this new German system of coinage is the *Reichsmark* (*Mark*), equal to \$.238 U. S. money.

The *Mark* is divided into 100 equal parts, one of which is called a **Pfennig.**

The **coins** of the *New Empire* are as follows: **Gold,** 20, 10, and 5 marks; **silver,** 2 and 1 mark; **nickel,** 10 and 5 pfennig.

431. ***Japan*** has a new and *decimal system* of coinage.

The **unit** of Japan money is the gold *yen*, valued at \$.997 U. S. money.

The **coin** of Japan embraces *five gold* coins, valued at \$20, \$10, \$5, \$2, and \$1. Also *five silver* coins, valued at 5, 10, 20, 50, and 100 cents, respectively.

LONGITUDE AND TIME.

432. The ***Longitude*** of a place is its distance east or west from some assumed meridian.

Since the earth turns on its axis once in 24 hr., $\frac{1}{24}$ of 360°, or 15°, passes under the sun in 1 hr., and $\frac{1}{60}$ of 15°, or 15′, passes under the sun in 1 min. of time, and $\frac{1}{60}$ of 15′, or 15″, passes under the sun in 1 sec. of time. Hence, the following

433. COMPARISON OF LONGITUDE AND TIME.

A diff. of 15° in longitude makes a diff. of 1 hr. in time.
" 15′ " " " 1 min. "
" 15″ " " " 1 sec. "

1. Since the earth revolves from west to east, time is earlier to all places west, and later to all places east of any given meridian. Thus, when it is 12 o'clock at one place, it will be *past* 12 o'clock at all places east, and *before* 12 at all places west.

2. If one place is in east and the other in west longitude, the difference of longitude is found by *adding* them, and if the sum is greater than 180°, by *subtracting* it from 360°.

434. TABLE OF LONGITUDE FROM GREENWICH,

as compiled from the records of the U. S. Coast Survey.

Albany	73° 44′ 50″ W.	New Orleans	90° 2′ 30″ W.
Ann Arbor	83° 43′ W.	Paris	2° 20′ E.
Boston	71° 3′ 30″ W.	Rome	12° 27′ E.
Berlin	13° 23′ 45″ E.	Richmond, Va.	77° 25′ 45″ W.
Cincinnati	84° 29′ 31″ W.	San Francisco	122° 26′ 45″ W.
Chicago	87° 37′ 45″ W.	St. Paul, Minn	95° 4′ 55″ W.
Jefferson City, Mo.	92° 8′ W.	St. Louis, Mo.	90° 15′ 15″ W.
Mexico	99° 5′ W.	Univ. of Virginia	78° 31′ 30″ W.
New York	74° 3′ W.	Washington, D. C.	77° 0′ 15″ W.

435. STANDARD TIME.

Time in this country is now mostly reckoned from four meridians, known as the *Eastern, Central, Mountain,* and *Pacific* meridians, and the standard time is named from them, as Eastern time, Central time, Mountain time, and Pacific time.

Eastern time is that of the meridian of 75° W. from Greenwich, and therefore is just 5 hours slower than London time; Central time is that of the 90th meridian, 6 hours slower than London; Mountain time, that of the 105th meridian, 7 hours slower than London; and Pacific time, that of the 120th meridian, 8 hours slower than London.

All the leading Railroads of the country have adopted these standards in operating their lines.

WRITTEN EXERCISES.

436. To find the difference of longitude between two places, when the difference of time is known.

1. When it is 9 o'clock at Washington, it is 7 min. 4 sec. past 8 o'clock at St. Louis. Find the diff. of longitude.

RULE.—*Multiply the difference of time expressed in hours, minutes, and seconds by 15; the product will be the difference of longitude in degrees, minutes, and seconds.*

2. The diff. in the time of Wash. and of St. Petersburgh is 7 hr. 9 min. 19¼ sec. Find the diff. in their longitudes?

3. When it is 12 o'clock M. at Rochester, N. Y., it is 9 hr. 1 min. 37 sec. A. M. at San Francisco. The long. of Rochester being 77° 51′ W., what is the long. of the latter?

437. To find the difference of time between two places, when their longitudes are given.

1. Find the diff. in the time between Cincinnati and St. Paul.

RULE.—*Divide the difference of longitude expressed in degrees, minutes, and seconds, by 15; the quotient will be the difference of time in hours, minutes, and seconds.*

Find the difference in the time of*

2. Washington, and Rome.
3. Chicago, and Paris.
4. New Orleans and N. York.
5. Albany, and Jefferson City.
6. Richmond, and St. Louis.
7. New York, and Mexico.
8. Ann Arbor, and Berlin.
9. Mexico, and San Fran.

10. When it is 6 A. M. at Boston, what time is it at Cincinnati? At Chicago? At St. Louis?

11. When it is 6 P. M. at the University of Va., what time is it at Berlin? At St. Paul? At Astoria, Or.?

12. How much later does the sun rise in N. Y. than in Paris?

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* Take from the Table the required longitude of the different places.

EXACT OR ACCURATE INTEREST.

438. ***Exact*** or ***accurate interest*** is computed by allowing 365 da. to the year, instead of 360 da., the latter being 5 da. or $\frac{5}{365}$ equal to $\frac{1}{73}$ *less* than a common year, and 6 da or $\frac{6}{366}$ equal to $\frac{1}{61}$ *less* than a leap year.

For example, the *exact* interest of any sum for 63 da. is $\frac{63}{365}$ of the interest for 1 yr. ; but this is $\frac{1}{73}$ *less* than that obtained by ordinary methods. Hence,

RULE.—*Find the interest for years by the common method, and for days, take such part of 1 year's interest as the number of days is of* 365. Or,

When the time is in months and days, and less than 1 year, find the interest by the common method, and then subtract $\frac{1}{73}$ part of itself for the common year, or $\frac{1}{61}$, if it be a leap year.

Find the accurate interest of $1560 for 93 da., at 6%.

SOLUTION.

$$\begin{array}{r|l} & \$\cancel{1560}^{\$312} \\ & .06 \\ {}^{73}\ \cancel{365} & 93 \\ \hline 73 & \$1740.96 \\ \cline{2-2} & \$23.848 \end{array}$$

Exact int.

$$\text{Or, } \$24.18 - \frac{\$24.18 \times 1}{73} = \$23.848.$$

2. Find the exact int. of $1600 for 1 yr. 3 mo. at 6%.

3. Find the exact interest of $875.60 at 7% for 63 da.

4. What is the difference between the *exact* interest of $648.40 at 8% for 1 yr. 3 mo. 20 da. and the interest reckoned by the 6% method?

5. Required the exact interest on three U. S. Bonds of $1000 each, at 6%, from May 1 to Oct. 15.

6. What is the exact interest on a $500 U. S. Bond, at 5%, from Nov. 1 to April 10 following?

ANNUAL INTEREST.

439. ***Annual interest*** is interest on the principal, and on each year's interest remaining unpaid.

Annual interest is allowed on promissory notes, and other contracts which contain the words, "with interest payable annually."

1. Find the annual int. and amt. of $8000 for 5 yr., at 6%.

Int. of $8000 for 5 yr. at 6% = $2400.
" $480 for 10 yr. at 6% = $288.
$2400 + $288 = $2688, annual int.
$8000 + $2688 = $10688, amount.

EXPLANATION.—The int. on $8000 for 1 yr. at 6% is $480, and for 5 yr. is $2400.

The int. for the first year, remaining unpaid, draws int. for 4 yr.; that for the second year, for 3 yr.; that for the third year, for 2 yr.; and that for the fourth year, for 1 yr., the sum of which is equal to the int. of $480 for 4 yr. + 3 yr. + 2 yr. + 1 yr. = 10 yr.; and the int. of $480 at 6% for 10 yr. is $288. Hence the total amount of int. is $2400 + $288, or $2688, and the amt. is $10688.

2. What is the annual interest of $1500 for 4 yr. at 7%?

RULE.—*Compute the interest on the principal for the given time and rate, to which add the interest on each year's interest for the time it has remained unpaid.*

3. What will $3500 amt. to in 10 yr., annual int., at 8%?

4. What is the difference between the annual interest and the compound interest of $2500 for 6 yr. at 6%?

5. Find the amt. of $575, at 8% annual int., for 9½ yr.

6. $600. NEW YORK, June 15, 1882.

On demand, I promise to pay L. H. BLAKEMAN, or order, six hundred dollars, with annual int., at 6%. Value rec'd.

HENRY BRIDGEMAN.

What amount will be due on this note June 15, 1884, no interest having been paid?

VERMONT RULE FOR PARTIAL PAYMENTS.

440. *On all notes, bills, or other similar obligations, whether made payable on demand or at a specified time,* WITH INTEREST, *when payments are made, such payments shall be applied: first, to cancel the interest that has accrued at the time of such payments; and, second, to the extinguishment of the principal.*

If payable WITH INTEREST ANNUALLY, *the annual interests that remain unpaid shall be subject to simple interest, from the time they become due to the time of final settlement; but if in any year, reckoning from the time such annual interest began to accrue, payments have been made, the amount of such payments at the end of such year, with interest thereon from the date of payment, shall be applied: first, to cancel the interest upon the annual interest; second, to cancel the annual interest; and third, to the extinguishment of the principal.*

$872. ST. JOHNSBURY, VT., Nov. 22, 1878.

1. *For value received, I promise to pay* JAMES FERGUSON, *or order, eight hundred seventy-two dollars, on demand, with interest annually.* SYLVANUS E. BOYLE.

Indorsed as follows: April 4, 1879, $28; July 10, 1882, $94.40; Dec. 10, 1884, $6.72; Jan. 14, 1887, $396.

What will be due Dec. 28, 1888?

441. The NEW HAMPSHIRE RULE is the same as that of Vermont, with this provision, viz.:

If at the time of any payment, no interest is due, except what is accruing for the year, and the payment or payments are less than the interest due at the end of the year, then no interest is allowed on such payment or payments.

442. The CONNECTICUT RULE is the same as the UNITED STATES RULE (**322**), with this exception, viz.:

When LESS than a year's interest has accrued at the time of a payment, except it be the last payment, the difference between the amount of the principal for an ENTIRE year, and the amount of the payments made that year, will form the new principal.

If the interest which has accrued at the time of a payment exceeds the payment, compute interest upon the principal only.

At the option of the teacher, the examples under the U. S. Rule (**322**) may be solved by the preceding rules.

FOREIGN EXCHANGE.

443. ***Foreign exchange*** relates to remittances made between different countries.

The drafts or bills of exchange are expressed in the money of the country in which they are made payable.

Exchanges with Europe are effected chiefly through prominent financial centres, as London, Paris, Amsterdam, Berlin, Frankfort, etc.

Sterling exchange consists of bills on any part of Great Britain.

444. ***Quotations*** are the published rates at which bills of exchange, stocks, bonds, etc., are bought and sold in the money market.

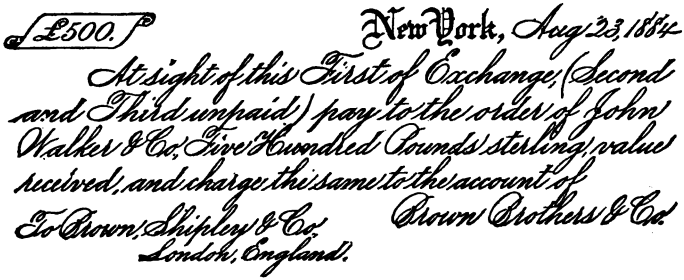
Thus, quotations of exchange on London give the value of £1 sterling in *dollars* and *cents;* on Paris, Antwerp, and Geneva, the value of $1 in *francs;* on Amsterdam, the value of 1 *guilder* or *florin* in *cents;* on Hamburg, Frankfort, and Berlin, the value of 4 marks in *cents.*

445. A ***set of exchange*** is a bill drawn in triplicate, named, FIRST, SECOND, and THIRD of exchange, each copy being valid until the amount of the bill is paid. These copies are sent by different mails, to provide against miscarriage. *When one is* paid, the others are *void.*

WRITTEN EXERCISES.

446. To find the cost of a foreign bill of exchange.

1. Find the cost in New York, of the following bill on London, at 3 da. sight, exchange quoted at $4.87½.



£500. New York, Aug. 23, 1884.
At sight of this First of Exchange, (Second and Third unpaid) pay to the order of John Walker & Co. Five Hundred Pounds sterling, value received, and charge the same to the account of
To Brown, Shipley & Co.
London, England.
Brown Brothers & Co.

SOLUTION.—$4.875 × 500 = $2437.50, *cost* of the bill, *Ans.*

2. Find the cost of a bill on Paris, for 495 francs, at 5.15.

3. Find the cost of a bill of exchange on Berlin for 1750 marks, quoted at 96¼.

SOLUTION.—$.9625 ÷ 4 × 1750 = $421.09, *cost* of the bill.

FORMULAS:

$$\left.\begin{array}{l}\text{1. } \textit{Face} \times \textit{value of monetary unit in U. S. money; } \text{or} \\ \text{2. } \textit{Face} \div \textit{value of \$1 in foreign money,}\end{array}\right\} = \textit{cost.}$$

Find the *cost* of a bill

4. On Liverpool, for £600 15s., at $4.86½, brokerage ½%.

5. On Geneva, Switzerland, for 5460 francs, at 5.21¼, brokerage ¼%.

6. On Hamburg, for 2560 marks, at 95⅝, brokerage ⅛%.

7. On Amsterdam, for 3000 guilders, quoted at 41⅛, brokerage ¼%.

8. On Dublin, for £384 10s. 6d., exchange quoted at $4.88¼.

9. On Bremen, for 2872 marks, exchange at 96⅝.

10. On Antwerp, for 2750 francs, exchange being 5.19¾.

11. Bought exchange on Amsterdam, at 41¼, for 3750 guilders; on Hamburg, at 95½ for 1000 marks; and on London for £500, at $4.85. What was the cost of the whole?

447. To find the face of a bill of exchange.

What will be the *face* of a bill

1. On London, that can be bought for $5488.26, exchange selling at $4.85?

Solution.—$5488.26÷$4.85 = 1131.6, or £1131.6 = £1131 12s.

2. On Paris, bought for $325, exchange at 5.17?

Solution.—5.17 fr. × 325 = 1680.25 *francs.*

3. On Hamburg, bought for $4000, exchange quoted at 96?

Solution.—($4000÷$.96) × 4 = 16666⅔ *marks.*

Formulas:

1. *Cost ÷ value of monetary unit in U. S. money;* or } = *face.*
2. *Cost* × *value of* $1 *in foreign money,*

Find the *face* of a bill

4. On Manchester, Eng., bought for $7500, exchange 4.86.

5. On Frankfort, bought for $395.75, exchange at 95⅛.

6. On Geneva, Switzerland, bought for $4856, at 5.22¼.

7. On Amsterdam, bought for $3750.67, exchange at 42½.

8. On Berlin, bought for $4000, exchange being 93¾.

9. A merchant in New Orleans gave $6186, currency, for a bill on Paris, at 5.15½. What was its face?

10. What is the face of a bill on Antwerp, that may be purchased in New York for $2500, exchange at 5.16¾?

11. A draft on Philadelphia cost £125 in Birmingham, Eng., *exchange* selling for 4.855; required the face of the draft.

PROGRESSIONS.

448. A ***progression*** is a series of numbers increasing or decreasing according to some fixed law.

449. An ***arithmetical progression*** is a series of numbers which increase or decrease by a common or constant *difference.*

Thus, 5, 7. 9, 11, 13, 15, is an *ascending* series,
15, 13, 11, 9, 7, 5, is a *descending* series,
in each of which 2 is the *common difference.*

450. A ***geometrical progression*** is a series of numbers, which increase or decrease by a common or constant *ratio.*

Thus, 1, 3, 9, 27, 81, 243, is an *ascending* series,
243, 81, 27, 9, 3, 1, is a *descending* series.
In each series the *ratio* is 3.

451. The ***terms*** of a progression are the numbers of which it consists. The *first* and *last* terms are called the ***extremes,*** and the other terms the ***means.***

452. The following are the ***elements*** considered in *arithmetical* progression and the symbols used:

1. The first term, (a).
2. The last term, (l).
3. The common difference, (d).
4. The number of terms, (n).
5. The sum of all the terms, (s).

453. Any *three* of these quantities being given, the *other two* may be found.

$$1. \quad a = \begin{cases} l - \overline{(n-1) \times d}. & \text{Ascending series.} \\ l + \overline{(n-1) \times d}. & \text{Descending series.} \end{cases}$$

The first term is equal to the last term minus or plus the product of the number of terms less 1, by the common difference, according as the series is ascending or descending.

2. $$l = \begin{cases} a + \overline{d \times (n - 1)}. & \text{Ascending series.} \\ a - \overline{d \times (n - 1)}. & \text{Descending series.} \end{cases}$$

The last term is equal to the first term, plus or minus the product of the common difference by 1 less than the number of terms according as the series is ascending or descending.

3. $$d = \frac{l - a}{n - 1}; \quad \text{or,} \quad d = \frac{a - l}{n - 1}.$$

The common difference is equal to the difference of the extremes divided by the number of terms less 1.

4. $$n = \frac{l - a}{d} + 1; \quad \text{or,} \quad n = \frac{a - l}{d} + 1.$$

The number of terms is equal to the difference of the extremes divided by the common difference, plus 1.

5. $$s = \frac{a + l}{2} \times n.$$

The sum of the terms of an arithmetical series is equal to one half the sum of the extremes, multiplied by the number of terms.

WRITTEN EXERCISES.

454. 1. The first term of an increasing progression is 8, the common difference 5, and the number of terms 20. What is the last term?

2. The first term of a decreasing progression is 203, the common difference 5, and the number of terms 40. What is the last term?

3. What is the 13th term of a descending series whose first term is 75, and common difference 5?

4. What is the amount of $100, at 7%, for 45 years?

5. The first term is 2, the last term is 17, and the number of terms is 6. What is the common difference?

6. The amount of $800 for 60 years, at simple interest, is $4160. What is the rate per cent.?

7. The extremes are 7 and 43, and the common difference is 4. What is the number of terms?

8. The first term is 2½, the last term is 40, and the common difference is 7½. What is the number of terms?

9. In what time will $500, at 7% int., amount to $885?

10. The extremes are 5 and 32, and the number of terms 12. What is the sum of all the terms?

11. How many strokes does a common clock make in 12 hours?

12. What debt can be discharged in a year by weekly payments in arithmetical progression, the first being $24, and the last $1224?

455. The following are the ***elements*** considered in *geometrical* progression:

1. The first term (a).
2. The last term (l).
3. The ratio (r).
4. The number of terms (n).
5. The sum of all the terms (s).

456. Any *three* of these quantities being given, the *other two* may be found.

The following formulas or rules cover all the cases:

$$1.\quad a = \begin{cases} l \div r^{n-1}. & \text{Ascending series.} \\ l \times r^{n-1}. & \text{Descending series.} \end{cases}$$

In an ascending series divide, and in a descending series multiply the last term by that power of the ratio whose exponent is 1 less than the number of terms.

$$2.\quad l = a \times r^{n-1}; \quad \text{or} \quad l = a \div r^{n-1}.$$

In an ascending series multiply, and in a descending series divide the first term by that power of the ratio whose exponent is 1 less than the number of terms.

3. $r = \sqrt[n-1]{\frac{l}{a}}$.

The ratio is equal to that root of the quotient of the last term divided by the first, whose index is 1 less than the number of terms.

4. $r^{n-1} = \frac{l}{a}$.

The number of terms less 1 is equal to the exponent of the power to which the ratio must be raised to be equal to the quotient of the last term divided by the first.

5. $s = \frac{l \times r - a}{r - 1}$.

The sum of a geometrical series is equal to the difference between the product of the last term by the ratio and the first term, divided by the ratio less 1.

WRITTEN EXERCISES.

457. 1. The last term is 192, the ratio 2, and the number of terms 7. What is the first term?

2. The first term of a geometrical series is 6, the ratio 4, and the number of terms 6. What is the last term.

3. The extremes are 2 and 512, and the number of terms is 5. What is the ratio?

4. The extremes are 2 and 1458, and the ratio is 3. What is the number of terms?

5. The extremes are 3 and 384, and the ratio is 2. What is the sum of the series?

6. If a man were to buy 12 horses, paying 2 cents for the first horse, 6 cents for the second, and so on, what would they cost him?

7. A drover bought 7 oxen, agreeing to pay $3 for the first ox, $9 for the second, $27 for the third, and so on. What did the last ox cost him?

MENSURATION.

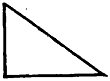
458. ***Mensuration*** treats of the measurement of lines, surfaces, and solids or volumes.

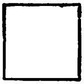
PLANE FIGURES.*

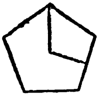
459. A ***polygon*** is a plane figure bounded by straight lines.

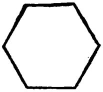
The **area** of a plane figure is the surface included within the lines which bound it.

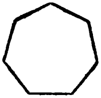
A polygon takes its name from the number of sides which bound it, as follows:

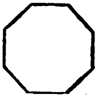

Trigon.


Tetragon.


Pentagon.


Hexagon.

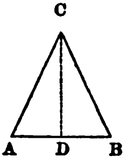

Heptagon.


Octagon.

The **perimeter** of a polygon is the sum of its sides; the **base** is the side on which it stands; and the **altitude** is the perpendicular distance between its base and a side or angle opposite.

460. A ***trigon*** or ***triangle*** is a polygon bounded by three sides, having three angles.

The **base** of a triangle is the side on which it is supposed to stand, as AB; the **vertical angle** is the angle opposite the base, as C; and the **altitude** is the perpendicular line drawn from the vertical angle to the base, as CD.



461. A ***tetragon*** or ***quadrilateral*** is a polygon bounded by four straight lines.

Parallel lines are lines which are equally distant from each other at every point.

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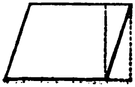
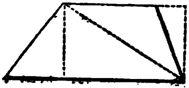
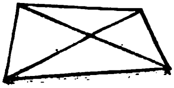
* Lines, angles, and plane figures, in part, are defined on pages 158, 159.

462. *Quadrilaterals* are of three kinds, as follows :


Parallelogram.

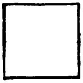

Trapezoid.

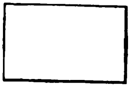

Trapezium.

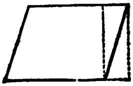
A **parallelogram** has its *opposite sides* parallel; a **trapezoid** has only *two sides* parallel; and a **trapezium** has no two sides parallel.

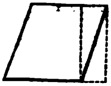
The **diagonal** of a quadrilateral, as shown in the figures, is a line joining any two angles not adjacent.

463. *Parallelograms* are of four kinds, as follows:


Square.


Rectangle.


Rhomboid.


Rhombus.

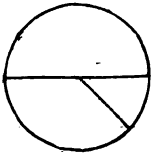
A **square** has *all* its sides equal, and *all* its angles *right angles;* a **rectangle** has its *opposite* sides equal, and all its angles *right angles;* a **rhomboid** has its *opposite* sides equal, and its angles *acute* and *obtuse;* a **rhombus** has *all* its sides equal, and its angles *acute* and *obtuse.*

An **acute angle** is *less* than a right angle; an **obtuse angle** is *greater* than a right angle.

The **altitude** of a parallelogram or a trapezoid is the perpendicular distance between its *parallel* sides.

THE CIRCLE.

464. A ***circle*** is a plane figure bounded by a curved line, called the ***circumference,*** every point of which is equally distant from a point within called the ***center.***



The **diameter** of a circle is a line passing through its center, and terminated at both ends by the circumference.

The **radius** of a circle is a line extending from its center to any point in the circumference. It is *one half the diameter.*

PROBLEMS.

465. The base and altitude of a triangle given, to find the area.

1. Find the area of a triangle whose base is 26 ft. and altitude 14.5 feet.

SOLUTION. $\frac{14.5 \times 26}{2} = 188\frac{1}{2}$; hence, the *area* is $188\frac{1}{2}$ sq. ft.

FORMULA: $\frac{1}{2}$ (*base* × *altitude*) = *area of triangle.*

Find the *area* of a triangle,

2. Whose base is 12 ft. 6 in. and altitude 6 ft. 9 in.
3. Whose base is 25.01 chains and altitude 18.14 chains.
4. What is the cost of a triangular piece of land whose base is 15.48 ch. and altitude 9.67 ch., at $60 an acre?
5. At $.40 a square yard, find the cost of paving a triangular court, its base being 105 feet, and its altitude 21 yards?

466. The area and one dimension given, to find the other dimension.

1. Find the base of a triangle whose area is 189 sq. ft., altitude 14 ft.

SOLUTION. $\frac{189 \times 2}{14} = 27$; hence, the *base* is 27 ft.

2. Find the altitude of a triangle whose area is $20\frac{1}{4}$ sq. ft., base 9 yards.

RULE.—*Divide twice the area by the given dimension.*

Find the *other dimension* of the triangle,

3. When the area is 65 sq. in. and the altitude 10 in.
4. When the base is 42 rods and the area 588 sq. rods.
5. When the area is $6\frac{1}{2}$ acres and the altitude 17 yards.
6. When the base is 12.25 chains and the area 5 A. 33 P.

467. The three sides of a triangle given, to find its area.

1. Find the area of a triangle whose sides are 30, 40, and 50 ft.

SOLUTION. $(30 + 40 + 50) \div 2 = 60$; $60 - 30 = 30$; $60 - 40 = 20$; $60 - 50 = 10$. $\sqrt{60 \times 30 \times 20 \times 10} = 600$ ft., *area.*

2. Find the area of a triangle whose base is 20 ft. and each of the other sides 15 ft.

RULE.—*From half the sum of the three sides subtract each side separately; multiply the half-sum and the three remainders together; the square root of the product is the area.*

3. Find the area of a triangle whose sides are 25, 36, and 49 in.

4. How many acres in a field in the form of an equilateral triangle whose sides measure 70 rods?

468. To find the area of a parallelogram.

1. Find the area of a parallelogram 16.25 ft. by 7.5 ft. wide.

SOLUTION. $16.25 \times 7.5 = 121.875$; hence, 121.875 sq. ft. is the *area.*

FORMULA: *Base* × *altitude* = *area of parallelogram.*

2. The base of a rhombus is 10 ft. 6 in., and its altitude 8 ft. What is its area?

3. How many acres in a piece of land in the form of a rhomboid, the base being 8.75 ch. and altitude 6 ch.?

4. A man bought a farm 198 rods long and 150 rods wide, and agreed to give $32 an acre. What did the farm cost?

5. A certain rectangular piece of land measures 1000 links by 100. How many acres does it contain?

6. How many square feet in a board 16 ft. long, 18 inches wide at one end and 25 inches wide at the other end?

469. To find the area of a trapezoid.

1. Find the area of a trapezoid whose parallel sides are 23 and 11 ft., the altitude 9 ft.

Solution. $\frac{23+11}{2}\times 9 = 153$; hence, 153 sq. ft. is the *area.*

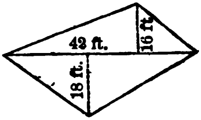
Formula: ½ (*sum of the bases* × *altitude*) = *area of trapezoid*

2. Required the area of a trapezoid whose parallel sides are 178 and 146 feet, and the altitude 69 feet.

3. One side of a quadrilateral field measures 38 rods; the side opposite and parallel to it measures 26 rods, and the distance between the two sides is 10 rods. Find the area.

470. To find the area of a trapezium.

1. Find the area of a trapezium whose diagonal is 42 ft. and perpendiculars to this diagonal, as in the diagram, are 16 ft. and 18 ft.



Solution. $\frac{18+16}{2}\times 42 = 714$; hence, 714 sq. ft. is the *area.*

Formula: ½ (*sum perpendic.* × *diag.*) = *area of trapezium.*

2. Find the area of a trapezium whose diagonal is 35 ft. 6 in., and the perpendiculars to this diagonal 9 ft. and 12½ ft.

3. How many acres in a quadrilateral field whose diagonal is 80 rd. and the perpendiculars to this diagonal 20.453 and 50.832 rd.?

To find the area of any *regular* polygon, multiply its perimeter, or the sum of its sides, by the perpendicular falling from its centre to one of its sides.

To find the area of an *irregular* polygon, divide the figure into triangles and trapeziums, and find the area of each separately. The sum of these areas will be the area of the whole polygon.

471. The diameter or circumference of a circle given, to find the other dimension.

1. Find the circum. of a circle whose diameter is 20 in.

SOLUTION.—20 in. × 3.1416 = 62.832 in., the *circumference.*

2. Find the diameter of a circle whose circumference is 62.832 ft.

SOLUTION.—62.832 ft. ÷ 3.1416 = 20 ft., the *diameter.*

FORMULAS: $\begin{cases} \text{1. } \textit{Diameter} \times 3.1416 = \textit{circumference.} \\ \text{2. } \textit{Circumference} \div 3.1416 = \textit{diameter.} \end{cases}$

3. Find the diameter of a wheel whose circum. is 50 ft.

4. What is the diameter of a tree whose girt is 18 ft. 6 in.?

5. What is the radius of a circle whose circum. is 31.416 ft.?

6. Find the circumference of the greatest circle that can be drawn with a string 14 inches long, used as a radius.

472. To find the area of a circle, when both its diameter and circumference are given, or when either is given.

1. Find the area of a circle whose diameter is 10 ft. and circumference 31.416 feet.

SOLUTION.—31.416 ft. × $\overline{10 \div 4}$ = 78.54 sq. ft., *area.*

2. Find the area of a circle whose diameter is 10 ft.

SOLUTION.—10 ft.^{2} × .7854 = 78.54 sq. ft., *area.*

3. Find the area of a circle whose circum. is 31.416 ft.

SOLUTION.—31.416 ft. ÷ 3.1416 = 10 ft., *diam.*;
(10 ft.)^2 × .7854 = 78.54 sq. ft., *area.*

FORMULAS: $\begin{cases} \text{1. } \tfrac{1}{4}(\textit{diameter} \times \textit{circumference}) = \textit{area.} \\ \text{2. } \textit{Square of diameter} \times .7854 = \textit{area.} \end{cases}$

4. Find the area of a circular pond, its circum. being 200 ch.?

5. The distance around a circular park is 1¼ miles. How many acres does it contain?

6. How much land in a circular garden, that requires 84 rd. of fencing to inclose it?

473. To find the diameter or circumference of a circle, when the area is given.

1. What is the diameter of a circle whose area is 1319.472?

SOLUTION. $\frac{1319.472}{.7854} = 1680$; $\sqrt{1680} = 40.987+$, the *diameter*.

2. What is the circumference of a circle whose area is 19.635?

SOLUTION. $\frac{19.635}{3.1416} = 6.25$; $\sqrt{6.25} = 2.5$, *radius*;
$2.5 \times 2 \times 3.1416 = 15.708$, the *circumference*.

$$\text{FORMULAS:} \begin{cases} 1.\ \sqrt{\dfrac{area}{.7854}} = diameter. \\ 2.\ \sqrt{\dfrac{area}{3.1416}} \times 2 \times 3.1416 = circumference. \end{cases}$$

3. The area of a circular lot is 38.4846 sq. rd. Find its diameter.

4. The area of a circle is 78.75 sq. yd. Find its diameter and its circumference.

5. The area of a circle is 286.488 square feet. Required the diameter and the circumference.

SOLIDS.*

474. A ***prism*** is a solid whose bases are equal and parallel polygons, and whose sides are parallelograms.

The planes which bound a solid are called *faces*, and their intersections *edges*.

The **altitude** of a prism is the perpendicular distance between its bases.

A prism is *triangular*, *quadrangalar*, *pentagonal*, etc., according as its bases have *three* sides, *four* sides, *five* sides, etc.

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* For some other definitions, see page 162.

Triangular Prism.

Quadrangular Prism.

Pentagonal Prism.

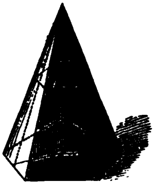
Hexagonal Prism.

475. A ***cylinder*** is a body bounded by a uniformly curved surface, its ends being equal and parallel circles.

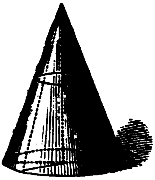
The **altitude** or **axis** of a cylinder is the line joining the centers of the bases or ends.

476. A ***pyramid*** is a body having for its base a polygon, and for its other faces three or more triangles, which terminate in a common point called the *vertex.*

Cylinder.



Pyramid.



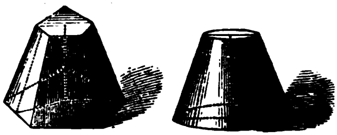
Cone.

477. A ***cone*** is a body having a circular base, and whose convex surface tapers uniformly to the *vertex.*

The **altitude** of a pyramid or of a cone is the perpendicular distance from its vertex to the plane of its base.

The **slant height** of a *pyramid* is the perpendicular distance from its vertex to one of the sides of the base; of a *cone,* is a straight line from the vertex to the circumf. of the base.

The **frustum** of a pyramid or cone is that part which remains after cutting off the top by a plane parallel to the base.

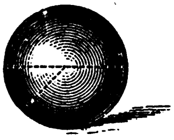


Frustums.

478. A ***sphere*** is a body bounded by a uniformly curved face, all the points of which are equally distant from a point within called the *center*.

The **diameter** of a sphere is a straight line passing through the center of the sphere, and terminated at both ends by its surface.

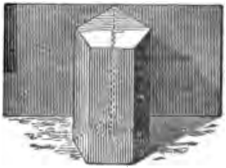
The **radius** of a sphere is a straight line drawn from the center to any point in the surface.



PROBLEMS.

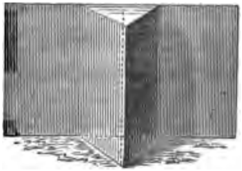
479. To find the convex surf. of a prism or cylinder.

1. Find the area of the convex surface of a prism whose altitude is 7 ft., and its base a pentagon, each side of which is 4 feet.



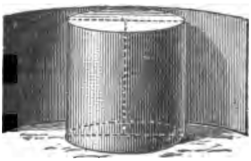
SOLUTION.—4 ft. × 5 = 20 ft., *perimeter*.
20 ft. × 7 = 140 sq. ft., *convex surface*.

2. Find the area of the convex surface of a triangular prism, whose altitude is 8½ feet, and the sides of its base 4, 5, and 6 feet, respectively.



SOLUTION.—4 ft. + 5 ft. + 6 ft. = 15 ft., *perimeter*.
15 ft. × 8½ = 127½ sq. ft., *convex surface*.

3. Find the area of the convex surface of a cylinder whose altitude is 2 ft. 5 in., and the circumference of its base 4 ft. 9 in.



SOLUTION.—2 ft. 5 in. = 29 in.; 4 ft. 9 in. = 57 in.
57 in. × 29 = 1653 sq. in. = 11 sq. ft. 69 sq. inches, *convex surface*.

FORMULA: *Perimeter of base* × *altitude* = *convex surface.*

To find the *entire* surface, add the area of the bases or ends.

4. If a gate 8 ft. high and 6 ft. wide revolve upon a point in its center, what is the entire surface of the cylinder described by it?

5. Find the superficial contents, or entire surface of a prism 8 ft. 9 in. long, 4 ft. 8 in. wide, and 3 ft. 3 in. high.

6. Find the entire surface of a cylinder formed by the revolution about one of its sides of a rectangle that is 6 ft. 6 in. long and 4 ft. wide?

7. Find the entire surface of a prism whose base is an equilateral triangle, the perimeter being 18 ft., and the altitude 15 feet?

480. To find the volume of any prism or cylinder.

1. Find the volume of a triangular prism, whose altitude is 20 ft., and each side of the base 4 ft.

SOLUTION.—The area of the base is 6.928 sq. ft. (**465**).
6.928 sq. ft. × 20 = 138.56 cu. ft., *volume.*

2. Find the volume of a cylinder whose altitude is 8 ft. 6 in., and the diameter of its base 3 ft.

SOLUTION.—$3^2 \times .7854 = 7.0686$ sq. ft., *area of base* (**472**).
7.0686 sq. ft. × 8.5 = 60.083 cu. ft., *volume.*

FORMULA: *Area of base* × *altitude* = *volume.*

3. What is the volume of a log 18 ft. long and $1\frac{1}{2}$ ft. in diameter?

4. Find the solid contents of a cube whose edges are 6 ft. 6 in.?

5. Find the cost of a piece of timber 18 in. square and 40 ft. long, at $.30 a cubic foot.

6. What is the value of a log 24 ft. long, of the average *circumference* of 7.9 ft., at $.45 a cubic foot?

481. To find the convex surface of a pyramid or cone.

1. Find the convex surface of a triangular pyramid, the slant height being 16 ft., and each side of the base 5 feet.

SOLUTION.—(5 ft. + 5 ft. + 5 ft.) × $\overline{16 \div 2}$ = 120 sq. ft., *convex surface.*

2. Find the convex surface of a cone whose diameter is 17 ft. 6 in., and the slant height 30 ft.

SOLUTION.—17.5 ft. × 3.1416 = 54.978 ft., *circumference*
54.978 ft. × $\overline{30 \div 2}$ = 324.67 sq. ft., *convex surface.*

FORMULA: *Perim. of base* × ½ *slant height* = *convex surf.*

To find the *entire* surface, add to this product the area of the base.

3. Find the entire surface of a pyramid whose base is 8 ft. 6 in. square, and its slant height 21 feet.

4. Find the entire surface of a cone the diameter of whose base is 6 ft. 9 in. and the slant height 45 feet.

482. To find the volume of any pyramid or cone.

1. What is the volume, or solid contents, of a square pyramid whose base is 6 ft. on each side, and its altitude 12 ft.

SOLUTION.—6 × 6 × $\overline{12 \div 3}$ = 144 cu. ft., *volume.*

2. Find the volume of a cone, the diameter of whose base is 5 ft. and its altitude 10½ ft.

SOLUTION.—5^2 ft. × .7854 × $\overline{10\frac{1}{2} \div 3}$ = 68.72¼ cu. ft., *volume.*

FORMULA: *Area of base* × ⅓ *altitude* = *volume.*

3. Find the solid contents of a cone whose altitude is 24 ft., and the diameter of its base 30 inches.

4. What is the cost of a triangular pyramid of marble, whose altitude is 9 feet, each side of the base being 3 feet, at $2½ per cubic foot?

5. Find the volume and the entire surface of a pyramid whose base is a rectangle 80 ft. by 60 ft., and the edges which meet at the vertex are 130 feet.

483. To find the convex surface of a frustum of a pyramid or of a cone.

1. What is the convex surface of a frustum of a square pyramid, whose slant height is 7 ft., each side of the greater base 4 ft., and of the less base 18 inches?

Solution.—The *perimeter* of the greater base is 16 ft., of the less, 6 ft. $\overline{16\text{ ft.}+6\text{ ft.}} \times \overline{7\div 2} = 77$ sq. ft., *convex surface.*

2. Find the convex surface of a frustum of a cone whose slant height is 15 ft., the circumference of the lower base 30 ft., and of the upper base 16 ft.

Formula: *Sum of perimeters* × ½ *slant height* = *conv. surf.*

To find the *entire* surface, add to this product the areas of both ends.

3. How many square yards in the convex surface of a frustum of a pyramid, whose bases are heptagons, each side of the lower base being 8 ft., and of the upper base 4 ft., and the slant height 55 ft.?

484. To find the volume of a frustum of a pyramid or cone.

1. Find the volume of the frustum of a sq. pyramid, whose altitude is 10 ft., each side of the lower base 12 ft., and of the upper base 9 ft.

Solution.—$12^2+9^2=225$; $(225+\sqrt{144\times 81})\times\overline{10\div 3}=1110$; hence, 1110 cu. ft. is the *volume.*

2. How many cubic feet in the frustum of a cone whose altitude is 6 ft. and the diameters of its bases 4 ft. and 3 ft.?

Rule.—*To the sum of the areas of both bases add the square root of the product, and multiply this sum by one-third of the altitude.*

3. How many cu. ft. in a piece of timber 30 ft. long, the greater end being 15 in. square, and that of the less 12 in.?

4. How many cu. ft. in the mast of a ship, its height being 50 ft., the circumference at one end 5 ft. and at the other 3 ft.?

485. To find the surface of a sphere.

1. Find the surface of a sphere whose diameter is 9 in.

Solution.—$9^2 \times 3.1416 = 254.4696$; hence, 246.4696 sq. in., *surface.*

Formula: *Diameter*$^2 \times 3.1416 =$ *surface of sphere.*

2. What is the surface of a globe 3 feet in diameter?

3. Find the surface of a globe whose radius is 1 foot.

486. To find the volume of a sphere.

1. Find the volume of a globe whose diameter is 30 in.

Solution.—$18^3 \times .5236 = 3053.6352$; hence, 3053.6352 cu. ft., *volume.*

Formula: *Diameter*$^3 \times .5236 =$ *volume of sphere.*

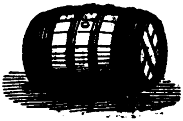
2. Find the volume of a globe whose diameter is 30 in.

3. Find the solid contents of a globe whose radius is 5 yd.

487. ***Gauging*** is the process of finding the capacity or volume of casks and other vessels.

For ordinary purposes the diagonal rod is used, which gives only approximate results.

A cask is equivalent to a cylinder having the same length and a diameter equal to the *mean diameter* of the cask.



To find the *mean diameter* of a cask (*nearly*).

Rule.—*Add to the head diameter ⅔, or, if the staves are but little curved, .6, of the difference between the head and bung diameters.*

To find the *volume* of a cask in gallons:

Rule.—*Multiply the square of the mean diameter by the length (both in inches) and this product by .0034.*

1. How many gallons in a cask whose head diameter is 24 in., bung diameter 30 in., and its length 34 inches?

Solution.—$24 + (\overline{30 - 24} \times \frac{2}{3}) = 28$ in., *mean diameter.*

$28^2 \times 34 \times .0034 = 90.63$ gal., *capacity.*

2. What is the volume of a cask whose length is 40 in., the diameters 21 and 30 in. respectively?

3. How many gallons in a cask of slight curvature, 3 ft. 6 in. long, the head diameter being 26 in., the bung diam. 31 in.?

METRIC SYSTEM.

488. The ***metric system*** of weights and measures is based upon the *decimal notation*.

489. The ***meter*** is the *base* of the system, and is *one ten-millionth* part of the distance on the earth's surface from the equator to the north pole, and is equal to 39.37079 inches.

490. The metric system has three *standard* units; viz., the ***me'ter*** (*mee'ter*) or *unit* of *length*, the ***li'ter*** (*lee'ter*), or *unit* of *capacity* or *volume*, and the ***gram***, or *unit* of *weight*. The ***ar*** and ***ster*** are the *units* of *square* and *cubic* measures.

Each of these units has its *multiples* and *sub-multiples*.

491. The ***multiple units***, or higher denominations, are formed by prefixing to the standard unit the Greek numerals, *dĕk'a* (10), *hĕk'to* (100), *kĭl'o* (1000), *my̆r'ia* (10000).

492. The ***sub-multiple units***, or lower denominations, are formed by prefixing to the standard unit the Latin ordinals, *dĕc'i* ($\frac{1}{10}$), *cĕnt'i* ($\frac{1}{100}$), *mĭl'li* ($\frac{1}{1000}$).

Hence, it is apparent from the *name* of a unit, whether it is *greater* or *less* than the standard unit, and also *how many times*.

Thus, 1 deka-meter (*Dm.*) denotes 10 meters; 1 hekto-meter (*Hm.*), 100 meters; 1 kilo-meter (*Km.*), 1000 meters; and 1 myria-meter (*Mm.*) 10000 meters. Also, 1 deci-meter (*dm.*) denotes $\frac{1}{10}$ (.1) of a meter; 1 centimeter (*cm.*), $\frac{1}{100}$ (.01) of a meter; 1 millimeter (*mm.*), $\frac{1}{1000}$ (.001) of a meter.

493. The following diagram shows how the tables are formed:

Milli	METER.	AR.	STER.	LITER.	GRAM.
Centi					
Deci					
——					
Deka					
Hecto					
Kilo					
Myria					

494. The *metric system* being based upon the *decimal scale*, the *denominations* correspond to the *orders* of the Arabic notation, and hence are written like United States money, the lowest denomination at the right. Thus,

Tens of thou.	Thousands.	Hundreds.	Tens.	Units.	*Dec. point.*	Tenths.	Hundredths.	Thousandths.
6	7	0	1	5	.	6	3	8
Mm.	*Km.*	*Hm.*	*Dm.*	*m.*		*dm.*	*cm.*	*mm.*

The number is read, 67015.638 *me'ters.* It may be expressed in different denominations by placing the decimal point at the right of the required denomination, and writing the name or abbreviation after the figures.

Thus, the above may be read, 670.15638 Hm.; or 67.015638 Km.; or 670156.38 dm.; or 6701563.8 cm.; or it may be read,

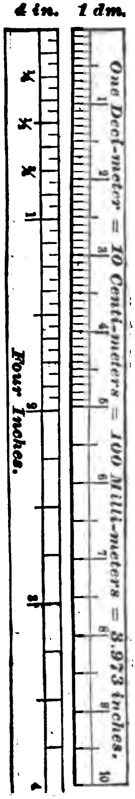
6 Mm. 7 Km. 0 Hm. 1 Dm. 5 m. 6 dm. 3 cm. 8 mm.

Write 3672.045 meters, and read it in the several orders; read it in kilometers; in hektometers; in dekameters; in decimeters; in centimeters.

The names *mill*, *cent*, and *dime*, used in U. S. money, correspond to *milli*, *centi*, and *deci* in the metric system. Hence, the eagle might be called a deka-dollar, since it is 10 dollars; the dime, a deci-dollar, since it is $\frac{1}{10}$ of a dollar, etc.

MEASURES OF LENGTH.

495. The *me′ter* is the *unit of length,* and is equal to 39.37 in., or 1.0936 yd. +.


4 in.
1 dm.
Four Inches.
One Deci-meter = 10 Centi-meters = 100 Milli-meters = 3.973 inches.
1
2
3
4
5
6
7
8
9
10

Metric Denominations.			Symbols.	U.S. Values.
		1 Mĭl′li-mē′ter.	*mm.*	.03937 in.
10 Mĭl′li-mē′ters	=	1 Cĕn′ti-mē′ter.	*cm.*	.3937 "
10 Cĕn′ti-mē′ters	=	1 Dĕc′i-mē′ter.	*dm.*	3.937 "
10 Dĕc′i-mē′ters	=	1 *Me′ter.*	*m.*	39.37 "
10 Me′ters	=	1 Dĕk′a-mē′ter.	*Dm.*	32.81 ft.
10 Dĕk′a-mē′ters	=	1 Hĕk′to-mē′ter.	*Hm.*	19.88 rd.
10 Hĕk′to-mē′ters	=	1 Kĭl′o-mē′ter.	*Km.*	.6214 mi.
10 Kĭl′o-mē′ters	=	1 Mȳr′ia-mē′ter.	*Mm.*	6.214 "

Units of length form a scale of *tens;* hence, in writing numbers expressing *length, one* decimal place must be allowed for each denomination.

Thus, 9652 mm. may be written 965.2 cm., or 96.52 dm., or 9.652 m., or .9652 Dm.

1. The *me′ter* is used in measuring cloths and short distances.

2. The *kĭl′o-mē′ter* is commonly used for measuring *long* distances, and is about ⅝ of a common mile.

3. The *cĕn′ti-mē′ter* and *mĭl′li-mē′ter* are used by mechanics and others for *minute* lengths.

4. In business, *dĕc′i-mē′ters* are usually expressed in *cĕn′ti-mē′ters.*

5. The *dĕk′a-mē′ter, hĕk′to-mē′ter,* and *mȳr′ia-mē′ter* are seldom used, but their values are expressed as *kĭl′o-mē′ters.*

Read the following:

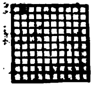
3.9 m.	346 Dm.	451 Hm.	
36 dm.	57.9 Hm.	593.7 Km.	13.043 Km.
428 cm.	479.6 m.	105.6 Dm.	500.032 m.

Write the following, expressing each in *three* denominations:

2. 24379 dm.; 15032036 cm.; 2475064 mm.; 30471 Dm.
3. 6704 Hm.; 85 Km.; 120000 m.; 780109 cm.; 75 m.

MEASURES OF SURFACES AND SOLIDS.

496. The ***square meter*** is the *unit* of ordinary *surface* measures, and is 1.196 sq. yd., or 10.764 sq. ft.



1 sq. cm., exact size.

		1 Sq. Mĭl′li-mē′ter.	*sq. mm.*	.00155 sq. in.
100 Sq. Mĭl′li-mē′ters	=	1 Sq. Cĕn′ti-mē′ter.	*sq. cm.*	.155 sq. in.
100 Sq. Cĕn′ti-mē′ters	=	1 Sq. Dĕc′i-mē′ter.	*sq. dm.*	15.5 sq. in.
100 Sq. Dĕc′i-mē′ters	=	1 ***Sq. Mē′ter.***	*sq. m.*	1.196 sq. yd.

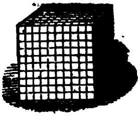
497. The ***ar*** is the *unit* of land measures, and is a square whose side is 10 meters, and contains 100 sq. meters.

1 Cĕn′tar (*ca.*)	=	1 Sq. Mē′ter.	*sq. m.*	1.196 sq. yd.
100 Cĕn′tars	=	1 ***Ar.***	*a.*	119.603 sq. yd.
100 Ars	=	1 Hĕk′tar.	*Ha.*	2.471 acres.

1. The *meter* is used in measuring cloths and *short* distances.

2. The *kilo-meter*, equal to ⅝ of a mile, nearly, is used for measuring *long* distances.

3. The *dĕc′i-är* is $\frac{1}{10}$ of an *är;* the *dĕk′är* is 10 a.; an acre is 40 ärs, nearly.



1 cu. cm., exact size.

498. The ***cubic meter*** is the *unit* of ordinary *solid* measures.

		1 Cu. Mĭl′li-mē′ter.	*cu. mm.*	.000061 cu. in.
1000 Cu. Mĭl′li-mē′ters	=	1 Cu. Cĕn′ti-mē′ter.	*cu. cm.*	.061 cu. in.
1000 Cu. Cĕn′ti-mē′ters	=	1 Cu. Dĕc′i-me′ter.	*cu. dm.*	.0353 cu. ft.
1000 Cu. Dĕc′i-mē′ters	=	1 ***Cu. Me′ter.***	*cu. m.*	35.3166 cu. ft.

499. The ***ster*** is the *unit* of *wood* or *stone* measures, and is a *cubic meter*, equal to 35.3166 cu. ft., or .2759 cord.

		1 Dĕc′i-stēr.	*ds.*	3.53 cu. ft.
10 Dĕc′i-stērs	=	1 ***Ster.***	*s.*	35.316 cu. ft.
10 Sters	=	1 Dĕk′a-stēr.	*Ds.*	2.759 cord.

1. The square and cubic measures are only the *squares* and *cubes* of the measures of *length.*

2. The *cubic deci-meter* is called a *lī′ter* when used as a unit of *liquid* or of *dry* measures.

MEASURES OF CAPACITY.

500. The ***li'ter*** is the *unit of capacity*, both of liquid and of dry measures, and is equal in volume to 1 *cu. dec'i-me'ter*, equal to 1.0567 qt. liquid measure, or .908 qt. dry meas.

Dry M.

Liquid M.

		1 Mĭl'li-lĭ'ter.	*ml.*	.061 cu. in.	.27 fl. dr.
10 Mĭl'li-lĭ'ters	=	1 Cĕn'ti-lĭ'ter.	*cl.*	.61 “	.338 fl. oz.
10 Cĕn'ti-lĭ'ters	=	1 Dĕc'i-lĭ'ter.	*dl.*	6.10 “	.845 gi.
10 Dĕc'i-lĭ'ters	=	1 ***Li'ter.***	*l.*	.908 qt.	1.0567 qt.
10 LI'TERS	=	1 Dĕk'a-lĭ'ter.	*Dl.*	9.081 “	2.64175 gal.
10 Dĕk'a-lĭ'ters	=	Hĕk'to-lĭ'ter.	*Hl.*	2.837 bu.	26.4175 “
10 Hĕk'to-lĭ'ters	=	1 Kĭl'o-lĭ'ter or Ster.	*Kl.*	28.37 bu. 1.308 cu. yd.	264.175 “
10 Kĭl'o-lĭ'ters	=	1 Mÿr'ia-lĭ'ter.	*Ml.*	283.72 bu.	2641.75 “

1. The *li'ter* is used in measuring liquids in moderate quantities.

2. The *hek'to-li'ter* is used for measuring grain, fruit, roots, etc., in large quantities, also wine in casks.

MEASURES OF WEIGHT.

501. The ***gram*** is the *unit of weight*, and is equal to the weight of a *cu. cen'ti-me'ter* of distilled water.

A *gram* is equal to 15.432 gr. Troy, or .03527 oz. Avoir.

		1 Mĭl'li-gram.	*mg.*	.01543 gr. Tr.
10 Mĭl'li-grams	=	1 Cĕn'ti-gram.	*cg.*	.1543 + “
10 Cĕn'ti-grams	=	1 Dĕc'i-gram.	*dg.*	1.5432 + “
10 Dĕc'i-grams	=	1 ***Gram.***	*g.*	15.432 + “ .03527 + oz. Av.
10 GRAMS	=	1 Dĕk'a-gram.	*Dg.*	.3527 + “
10 Dĕk'a-grams	=	1 Hĕk'to-gram.	*Hg.*	3.5274 + “
10 Hĕk'to-grams	=	1 Kĭl'o-gram, or ***Kil'o.***	*Kg.*	2.6792 lb. Tr. 2.2046 + lb. Av.
10 Kĭl'o-grams	=	1 Mÿr'ia-gram.	*Mg.*	22.046 + “
100 KIL'OS	=	1 Quĭn'tal.	*Q.*	220.46 + “
10 Quintals	=	1 Tonneau, or Ton.	T.	2204.62 + “

1. The ***gram*** is used for weighing letters, gold, silver, medicines, and all small, or costly articles.

2. The ***kil′o-gram*** or ***kil′o*** is the unit of common weight in trade, being a trifle less than 2¼ lb. Avoir.

3. 1 ***kil′o-gram*** = 1 ***cu. dec′i-me′ter*** = 1 ***li′ter*** = $\begin{cases} 2.6792 \text{ lb. Troy.} \\ 2.2046 \text{ lb. Avoir.} \end{cases}$

4. 15 ***grams*** are taken as ½ an ***ounce*** at the U. S. Post Office.

5. The ***ton*** is the weight of a ***cu. m.*** of water, and is used for weighing very heavy articles, being about 204½ lb. more than a common ton.

502. Units of *length*, of *capacity*, and of *weight*, form a scale of *tens* (10); hence, each order of units will occupy ***one place.***

503. Units of ***square measures*** form a scale of ***hundreds***, 100 (10 × 10); hence, ***two places*** must be allowed to each order.

504. Units of ***solid measures*** form a scale of ***thousands***, 1000 (10 × 10 × 10); hence, ***three places*** must be allowed to each order.

505. To add, subtract, multiply, and divide metric denominations.

1. What is the sum of 314.217 m., 53.062 Hm., and 225 cm.?

Solution. 314.217 m. + 5306.2 m. + 2.25 m. = 5622.667 m.

2. Find the difference between 4.37 Km. and 1246 m.

Solution. 4.37 Km. − 1.242 Km. = 3.128 Km.

3. How much cloth in 8¼ pieces, each containing 43.65 m.?

Solution. 43.65 m. × 8.25 = 360.1125 m.

4. How many garments, each containing 3.5 m., can be made from a piece of cloth containing 43.75 Dm.?

Solution. 437.5 m. ÷ 3.5 m. = 125 times; hence, 125 garments.

All computations in METRIC NUMBERS *are similar to those in integers and decimals, or in U. S. money.*

5. Add 7.6 m., 36.07 m., 125.8 m., and 9.127 m.

6. Express as meters and add 475 dm., 3241 cm., and 725 mm.

7. Add 56.07 m., 1058.2 dm., 430765 cm., 6034.58 m., and express the result in kilometers.

8. From 8.125 Km. take 3276.4 m.

9. How many meters of ribbon will be required to make 32 badges, each containing 40 centimeters?

10. What will be its cost, at 15 cents a meter?

11. Find the difference between 25.3 Km. and 425.25 m.

12. If an engine runs 36.8 Km. in an hour, how far does it run between 8 o'clock and 12 o'clock?

13. In 15 sq. Hm. how many square meters?

14. What is the surface of a floor 12 m. long and 7 m. wide?

15. Add 8 times 4 Ha., 7 times 9 a., and 12 times 14 ca.

16. What is the area of a piece of land 42 Dm. long and 36 Dm. wide?

17. How many meters of carpeting .6 m. wide will cover a floor 8 m. long and 5.4 m. wide?

18. From 36 cu. m. subtract 8 times 42 cu. dm.

19. How many cubic meters of brick in a wall 16 m. long, 3 m. high, and 8 dm. thick?

20. How many cu. meters of earth must be removed in digging a cellar 16.5 m. long, 8.2 m. wide, and 3.2 m. deep?

21. In a pile of wood 9.3 m. long, 2.8 m. high, and 1.5 m. wide, how many sters, and what is its value, at $2.75 a ster?

22. Find the sum of 175 l., 25 Hl., 42 cl., and 16 dl.

23. From 6 times 25 Hl. take 15 times 36 l.

24. Divide 5 Hl. of corn equally among 25 persons.

25. From a cask of wine containing 2 Hl. of wine, 125 l were drawn out. How much remained?

26. How many Hl. of wheat can be put into a bin 3 m long, 2 m. wide, and 1.5 m. deep?

27. Change 16.5 T. to kilos; to grams; to decigrams.

28. If coffee is $.80 a kilo, what will 5 quintals cost?

506. Units of the *metric* system are changed to units of the *common* system by using the equivalents in the *preceding* tables.

507. Units of the *common* system are changed to units of the *metric* system by using the equivalents in the following

TABLE OF EQUIVALENTS.

Linear Measures.

1 Inch = 2.54 Centimeters.
1 Foot = .3048 Meter.
1 Yard = .9144 Meter.
1 Rod = 5.029 Meters.
1 Mile = 1.6093 Kilometers.

Square Measures.

1 Sq. Inch = 6.4528 Sq. Centim.
1 Sq. Foot = .0929 Sq. Meter.
1 Sq. Yard = .8361 Sq. Meter.
1 Sq. Rod = 25.293 Centars.
1 Acre = 40.47 Ars.
1 Sq. Mile = 259 Hektars.

Solid Measure.

1 Cu. Inch = 16.387 Cu. Centim
1 Cu. Foot = 28.317 Cu. Decim.
1 Cu. Yard = .7645 Cu. Meter.
1 Cord = 3.624 Sters.

Measures of Capacity.

1 Fl. Ounce = .02958 Liter.
1 Liq. Qt. = .9463 Liter.
1 Gallon = 3.785 Liters.
1 Dry Qt. = 1.101 Liters.
1 Bushel = .35243 Hektoliter.

Measures of Weight.

1 Grain Tr. = .0648 Gram.
1 Ounce Tr. = 31.103 Grams.
1 Pound Tr. = .37324 Kilogram.
1 Ounce Av. = 28.35 Grams.
1 Pound Av. = .4536 Kilogram
1 Ton = .907 Tonneau.

WRITTEN EXERCISES.

508. 1. How many feet in 24.5 m.? In 472 cm.?

2. How many miles in 6000 m.? In 400 Km.?

3. How many meters in 75 yd.? In 18 yd. 1 ft. 6 in.?

4. In 436 ars, how many acres? How many sq. rods?

5. In 3588 sq. yd. how many sq. m.? How many ars?

6. In 13 gal. 3 qt. 1 pt. 3 gi. how many liters? How many Hl.?

7. How many grams in 6 lb. Troy? In 6 lb. Avoir.?

8. How many square metres of plastering in a room 18 ft. 6 in. long, 14 ft. wide, and 9 ft. 6 in. high?

9. Bought 454 bu. of wheat, at $3 a bushel, and sold the same at $8.75 per hektoliter; how many hektoliters did I sell? Did I gain or lose, and how much?

10. Find the weight of a barrel of flour (196 lb.) in Kg.

11. Find the cost of a carpet for a room 10.5 m. long and 8.4 m. wide, if the carpeting is 84 cm. wide and costs $2.75 a meter.

12. What is the cost of a building lot 75 m. long and 62 m. wide, at $40 an ar?

13. What will be the cost of a pile of wood 15.7 m. long, 3 m. high, and 7.52 m. wide, at $1.50 a ster?

14. How many meters of shirting, at $.18 per m., must be given in exchange for 250 Hl. of oats, at $1.20 per Hl.?

15. A lady bought 40.5 m. of silk in Paris. What would be its value in Boston, at $4.75 per yard?

16. How many sters of wood can be piled in a shed 8.5 m. long, 5.8 m. wide, and 4.2 m. high? Find its value, at $3.25 a cord.

17. A farmer sold 540 Hl. of wheat, at $2 a bushel, and invested the proceeds in coal at $7 a ton. How many tons did he buy?

18. Find the cost of excavating a cellar 18.3 m. long, 10.73 m. wide, and 3.4 m. deep, at 20 cents a ster.

19. If a field produces 40 Hl. of oats to the hektar, how many bushels is that to the acre?

20. How many liters will a cistern hold that measures on the inside 5.5 ft. long, 4 ft. 6 in. wide, and 4 ft. deep?

21. A dray is loaded with 60 bags of grain, each bag holding 8 Dl.; allowing 75 K. of grain to the hektoliter, what is the weight of the load in metric tons?

FRENCH AND SPANISH MEASURES.

509. The old ***French linear*** and ***land measures*** are used quite generally in Louisiana and in other French settlements in the United States.

TABLE.

12 Lines = 1 Inch.
12 Inches = 1 Foot.
6 Feet = 1 Toise.
32 Toises = 1 Arpent.
1024 Square Toises = 1 Square Arpent.

The *French foot* equals 12.79 inches, American meas., nearly.

The *Arpent* is the old French name for *Acre*, and contains very nearly ⅞ of an American acre.

EXAMPLE.—A sugar plantation on Bayou Teche, La., has a frontage on the bayou of 17 arpents 11 toises 4 ft. 8 lines, and a depth of 184 arpents 22 toises 3 feet between parallel lines. What is the area in French measure? What is the equivalent in American measure?

Ans. { 3237 sq. arpents 354 sq. toises 20 sq. ft. 24 sq. in.
Equiv. in Am. meas., 3083 A. 73 P. 7¾ sq. yd.

510. In Texas, New Mexico, and in other Spanish settlements of the United States, the following denominations of ***Spanish land measure*** are almost universally used, outside of incorporated cities, for measuring lands:

TABLE.

12 Dots = 1 Line.
12 Lines = 1 Inch.
12 Inches = 1 Foot.
3 Feet = 1 Vara.
5000 Varas = 1 League.

Also, 1000000 Sq. Varas = 1 La-bor′ = 177.136 acres (Am. meas.)
25 La-bors′ = 1 Sq. League = 4428.4 " "

Also, 5000 Varas Square = 1 Square League.
1000 Varas Square = 1 La-bor′ = $\frac{1}{25}$ Sq. League.
5645.376 Square Varas = 4840 Sq. Yd. = 1 Acre (Eng.)
564.5376 Square Varas = 1 Sq. Ch. = $\frac{1}{10}$ Acre "
1900.8 Varas Square = 1 Section = 640 Acres "

The Spanish foot = 11.11 + in. (Am.); 1 Vara = 33⅓ in.; 108 Varas = 100 Yards; and 1900.8 Varas = 1 mile.

"La-bor′" in above tables is accented on the last syllable.

EXAMPLE.—A Spanish grant in Texas has a front on the Nueces River of 3½ lea., with a depth between parallel lines of 12 lea. on the north boundary line, and of 12 lea. 350 varas on the south boundary line. What is the area in Spanish measure? What is the equivalent in American measure?

Ans. { 42 leagues 3 la-bors′ 62500 sq. varas.
Equiv. Amer. meas., 291 sq. mi. 295 A. 44 sq. rd. 19 sq. yd. 3 sq. ft. 34.56 sq. in.

SUPPLEMENTARY PROBLEMS.

511. The following practical problems are given for those who desire further application of the principles and rules previously taught.

1. Sold 20900 feet of timber for $339.62½, and gained thereby $78.37½. What did it cost per C.?

2. What cost 3 T. 17 cwt. 20 lb. of hay, at $22¾ a ton?

3. A farmer exchanged 28½ bu. of oats worth $.75 per bushel, and 453 lb. of middlings worth $1½ per hundred, for 12520 lb. of plaster. What was the plaster worth per ton?

4. Bought 27½ bbl. of sugar for $453.75, and sold it at a profit of $4.62½ a barrel. At what price was it sold?

5. In £6 10s. 10d. how many dollars U. S. currency?

6. Find the value in Troy weight of 9 lb. 10 oz. Avoir.

7. Find the value in integers of ⅜ of 3⅔ A.

8. A physician bought 1℔ 10℥ of quinine at $2.25 an ounce, and dealt it out in doses of 10 gr. at $.12½ each. How much more than cost did he receive?

9. How large an edition of an octavo book can be printed from 4 bales 4 bundles 1 ream 10 quires of paper, allowing 8 sheets to the volume?

10. A grocer bought 10 bu. of chestnuts at $3.75 a bushel, and retailed them at $.06¼ a pint. How much was his whole gain?

11. What length of time will elapse from 40 min. 25 sec. past 12 o'clock, noon, April 21, 1886, to 4 min. 36 sec. before 5 o'clock, A. M., Jan. 1, 1888?

12. How many years, months, and days from your birthday to this date?

13. An excavation 58 ft. long, 37 ft. wide, and 6 ft. deep is to be made for a cellar; after 471 cu. yd. 16 cu. ft. 972 cu. in. of earth have been removed, how much more still remains to be taken out?

14. The longitude of Pekin is 118° east, and of Sacramento City 120° west; what is the difference in time?

15. The longitude of Jerusalem is 35° 32′ east, and that of Baltimore 76° 37′ west; when it is 40 minutes past 6 o'clock A. M. at Baltimore, what is the time at Jerusalem?

16. Bought 7640 bricks, at $4.75 per 1000; 26789 feet of lumber, at $2.75 per 100; and 986 feet of pine boards, at $20 per 1000; I paid $.35 per 1000 for planing 23463 feet of boards. What was the whole bill?

17. At 20 cents a cubic yard, what will it cost to dig a cellar 32 ft. long, 24 ft. wide, and 6 ft. deep?

18. If the wall of the same cellar be laid 1½ feet thick, what will it cost at $1.25 a perch?

19. A man owns a piece of land which is 105 ch. 85 l. long, and 40 ch. 15 l. wide; how many acres does it contain?

20. At \$3.40 per square, what is the cost of tinning both sides of a roof 40 ft. in length, the rafters being 20 ft. 6 in. long ?

21. What is the value of a farm 189.5 rd. long and 150 rd. wide, at \$31¾ per acre ?

22. If a load of wood is 12 ft. long, and 3 ft. 6 in. wide, how high must it be to make a cord ?

23. A bin 7 ft. long, 6 ft. wide, and 5 ft. deep is ¾ full of rye. What is its value at \$1.37½ a bushel ?

24. Find the cost of 2400 ft. of boards, at \$7 per M.; 865 ft. of scantling, at \$5.40 per M.; and 1256 ft. of lath, at \$.80 per C.

25. Bought 288 bbl. of flour for \$1728, and sold one-half the quantity for the same price I gave for it, and the other half for \$8 a bbl. What did I receive for the whole ?

26. If 4¾ bbl. of flour cost \$32.30, what will 7½ bbl. cost ?

27. If .875 of a ton of coal cost \$5.635, what will 9¼ tons cost ?

28. A gardener sold, from his garden, 120 bunches of onions at \$.12½ a bunch, 18 bushels of potatoes at \$.62½ per bushel, 47 heads of cabbage at \$.07 a head, 6 doz. cucumbers at \$.18 a doz.; he expended \$1.50 in spading, \$1.27 for fertilizers, \$1.87 for seeds, \$2.30 in planting and hoeing. What were the profits of his garden ?

29. Reduce $\frac{3}{32}$, .62½, .37$\frac{1}{16}$, ⅜, to decimals, and find their sum.

30. If 8 be added to both terms of the fraction $\frac{11}{12}$, will its value be increased, or diminished, and how much ?

31. If 8 be added to both terms of the fraction $\frac{10}{7}$, will its value be increased, or diminished, and how much ?

32. A, B, and C can do a piece of work in 5 da.; B and C can do it in 8 da.; in what time can A do it alone ?

33. How many bushels of oats at \$⅜ a bushel, will pay for ⅝ of a barrel of flour at \$9½ a barrel ?

34. $(\overline{7\frac{1}{2}+1\frac{17}{22}}\div\overline{8\frac{2}{3}+3\frac{22}{33}})-(\overline{3\frac{4}{5}+\frac{29}{45}}\div\overline{3\frac{3}{4}+14\frac{1}{36}})=?$

35. A man sold a horse for \$125, and received in payment $12\frac{1}{2}$ yd. of cloth at \$$3\frac{1}{4}$ a yard, and the balance in tea at \$.$62\frac{1}{2}$ a pound. How many pounds of tea did he receive?

36. A dealer bought 240000 ft. of lumber at \$15.90 a M., and retailed it out at \$$2\frac{1}{4}$ per C. What was his whole gain?

37. Bought 150 bbl. of flour @ \$$6\frac{3}{8}$, and 350 bu. of wheat @ \$1.44. Having sold 105 bbl. of the flour @ \$$8\frac{1}{4}$, and all the wheat @ \$2.06, at what price per barrel must the remainder of the flour be sold, to gain \$$245\frac{1}{8}$ on the whole investment?

Complete the following equations:

38. $(34\times.193+2.7\times.4\frac{1}{6})\div(4.81-\frac{2}{3}\text{ of }1.662)=?$

39. $(\$262.90\div\$.56)\times\overline{.0084+.02\frac{7}{8}}\times100=?$

40. $(\$1260\times3.49)\div\$10.47-\$850\div\$6.80=?$

41. How many rods of fence will inclose a farm $\frac{3}{4}$ of a mile square?

42. If a grocer's scales give $\frac{1}{2}$ oz. short of true weight on every pound, of how much money does he defraud his customers, in the sale of 3 bbl. of sugar, each weighing 2 cwt. 10 lb., at $12\frac{1}{2}$ cents a pound?

43. Reduce £10 18s. 6d. to United States Money.

44. Paid \$425.75 for $2\frac{1}{4}$ tons of cheese, and retailed it at $12\frac{1}{2}$ cents a pound. What was the whole gain?

45. Reduce 580 francs to United States Money.

46. Change \$291.99 to Sterling Money.

47. How many feet of boards will it require to inclose a building 60 ft. 6′ long, 40 ft. 3′ wide, 22 ft. high, and each side of the roof 24 ft. 2′, allowing 523 ft. 3′ for the gables, and making no deduction for doors and windows?

48. How many pickets, each 3 in. wide, placed 3 in. apart, will be required to fence in a lot 6 rd. by 14 rd., and what will be their cost at \$2.75 per C.?

49. If a man's property is worth $1500, and he owes $1800, how many cents can he pay on the dollar?

50. What is the value of a field 15 ch. 75 l. long, and 12 ch. 50 l. wide, at $64 per acre?

51. How many boards 12 ft. long will be required to inclose a field 48 rd. by 36 rd. with a fence 5 boards high?

52. There is a park 300 ft. square; if a walk 4 ft. wide be made around it, within the inclosure, how many square yards will it contain?

53. Make a bill for lumber bought by John Mills of Geo. Arnold & Co., of Duluth, Sept. 20, 1885, as follows:

124	boards,	10 in.	by	16 ft.	@	$15	per M.
120	"	16 "	"	14 "	"	$16½	"
40	planks,	2½ × 12	"	15 "	"	$18.75	"
96	joists,	3 × 10	"	18 "	"	$14	"
60	scantling,	3 × 4	"	12 "	"	$12½	"

What is the amount?

54. What is the value, at $5 a cord, of a pile of wood 200 ft. long, 30 ft. wide, and 5 ft. 4 in. high?

55. If 24 men, in 9 da. of 12 hr. each, build a wall 200 ft. long, 6 ft. high, and 2 ft. thick, how many men, in 72 da. of 10 hr. each, can build a wall 950 ft. long, 8 ft. high, and 5 ft. thick?

56. A merchant pays $10050 for a stock of goods; he sells them at an advance of 33⅓%; the expenses connected with the business are $1750. How much does he gain?

57. A grocer bought 2 T. 14½ cwt. of cheese at $240 per ton, and retailed the same at 16 cents a pound. What per cent. was his profit?

58. The capital of an insurance company is $250000; its receipts for one year are $58760; its losses and expenses are $40010. What rate of dividend can it declare?

59. If 360 bu. of oats will last 12 horses 15 da., how long will 636 bu. last 18 horses?

60. What per cent. is gained by buying oil at 80 cents a gallon, and selling it at 12 cents a pint?

61. A man bought a farm of 160 A. at $32 an acre; he paid $200 for fencing, $150 for repair of buildings, and $18 for improving the grounds; at what price per acre must it be sold to gain 25% on the entire cost?

62. The taxable property of a School District is $756000; a Union School-house is to be built, worth $24956.16; the number of taxable polls is 520, each of which is assessed $1.50. Allowing 3% for collecting, what will be the tax upon $1.00? and how much will be the tax of S. P. Norton, who pays for 2 polls, and is worth $15420?

63. Engaging in business, I borrowed $750 of a friend at 6%, and kept it until it amounted to $942. How long did I retain it?

64. The triangular gable of a certain building has a base of 44 ft. and an altitude of 18 ft. How many square feet of boards will cover the two gables?

65. H. Jones & Co. failed in business. They owed A $4000, B $5000, and C $7000. Their assets amounted to $11536, and the charges of the assignees were 3% on the amount distributed. What % did they pay? and how much did each creditor receive?

66. A bridge company, whose stock is $28000, pays 5% dividends semi-annually. The annual expenses of the company are $2950. What are the gross earnings?

67. A having a farm of 109 A., which rents for $681.25, sells the same for $125 an acre, and invests the proceeds in Pacific R. R. 6's at 108¾%, brokerage ¼% for purchasing; will his yearly income be increased or diminished, and how much?

68. What will be the difference in the expense of fencing two fields of 25 acres each, one square, and the other in the form of a rectangle, whose length is twice its breadth, the fence costing $.62½ a rod?

69. The list price of a certain size and quality of slates is $12 a gross. On 15 gross, Mr. B. will give a discount of 25% and 10%. Mr. H. will give a discount on the same of 20%, 10%, and 5%. In buying of Mr. B. will I gain or lose, and how much?

70. If my rate of profit is 33⅓%, how must I mark goods for sale that cost me $1.50? $2¼? $7.20? 18¾ cts. a yd.? 27 cts. a pound? 67½ cts. each?

71. What sum must I invest in U. S. 6's, selling at 2½% premium, to secure an annual income of $840?

72. What is the rate of income upon money invested in 6% bonds, purchased at a discount of 10%?

73. How many cans, 8 in. by 6 in. by 3 in., can be packed in a box 32 in. by 24 in. by 15 in. in the clear?

74. A tank in the attic of a house is 6 ft. 6 in. long, 4 ft. wide, and 3 ft. 6 in. deep. How many gallons of water will it hold, and what will be its weight?

75. A man purchased goods for $10500, to be paid in three equal installments, without interest; the first in 3 months, the second in 4 months, the third in 8 months; how much ready money will pay the debt, money being worth 7%?

76. On the first day of January, 1880, a man gave 3 notes, the first for $500 payable in 30 da.; the second for $400 payable in 60 da.; the third for $600 payable in 90 da. What was the average term of credit, and what the equated time of payment?

Ans. Term of credit, 62 da.; time of payment, Mar. 3, 1880.

77. A young man, receiving a legacy of $48000, invested one-half in 5% stock at 95½%, and the other half in 6% stock at 112%, paying brokerage at ½%. What annual income did he secure from his legacy?

78. I sent $12300 to my agent in New York, with which to purchase flour at $5 per barrel, after deducting his commission of 2½%. How many barrels of flour did I receive?

79. If a merchant wishes to draw $1275 at bank, for what sum must he give his note at 60 days, discounting at 6% ?

80. If 8 men can perform a piece of work in 32 days, how many men must be added to the number to perform the work in 8 days ?

81. For what sum must a vessel, valued at $25000, be insured, so that in case of its loss, the owners may recover both the value of the vessel and the premium of 2¼% ?

82. If a piece of silk cost $.80 per yard, at what price shall it be marked, that the merchant may sell it at 10% less than the marked price, and still make 20% profit ?

(83.)

575\frac{63}{100}$. CHICAGO, March 2, 1885.

Six months after date, I promise to pay to the order of Jones and Ellis, five hundred seventy-five $\frac{63}{100}$ dollars, value received. JAMES MILLER.

Discounted at a bank, Apr. 10, at 6%. What are the proceeds ?

84. A house that is 50 ft. long and 40 ft. wide has a square or pyramidal roof, whose height is 15 ft. Find the length of a rafter reaching from a corner of the building to the vertex of the roof.

85. Find the length of a rafter reaching from the middle of one side.

86. Find the length of a rafter reaching from the middle of one end.

87. Divide $1500 among 3 persons, so that the share of the second may be ½ greater than that of the first, and the share of the third ½ greater than that of the second.

88. A merchant owes for three bills of goods as follows: $500 due March 1, $800 due June 1, and $600 due Aug. 1. He wishes to give two notes for the amount, one for $1000, payable April 1; what must be the face, and when the maturity, of the other ?

89. A man in New York purchased a draft on Chicago for $10640, drawn at 60 da., $10283.56. What was the course of exchange?

90. A speculator bought 420 bales of cotton, each bale containing 488 pounds, at 19 cents a pound, on a credit of 9 months for the amount. He immediately sold the cotton for $42122.23 cash, and paid the debt at 8% discount. What was his gain?

91. A note for $800, dated Jan. 15, 1883, on interest after 6 mo., at 7%, has the following indorsements: Apr. 18, 1884, $100; Jan. 1, 1886, $70; June 15, 1887, $62.50. Required the balance due July 15, 1888.

92. A merchant bought 240 M. of silk at $2, and sold it at $1.95 per yard. Did he gain or lose, and how much?

93. Bought 800 bu. of wheat at $3, and sold the same at $9 per hectoliter. How many Hl. did I sell, and how much did I gain or lose?

94. What is the least common multiple of $6\frac{1}{2}$, $9\frac{3}{4}$, $10\frac{5}{8}$, and $7\frac{5}{16}$?

95. A merchant bought 500 bbl. of flour at $6.50 a barrel, and sold it immediately at $7.25 a barrel, receiving in payment a note due 3 mo. hence, which he had discounted at a bank at 6%. What did he gain on the flour?

96. If a staff 2 ft. 9 in. long, cast a shadow 1 ft. $1\frac{1}{2}$ in., what is the height of a pole which casts a shadow 63 ft.?

97. How many shares of mining stock at 80% must be sold in order that the proceeds invested in California 8's at 108% may yield a profit of $960?

98. I sent $9020 to my agent in Chicago, who purchased grain at an average price of $1.10 per bushel, and charged $2\frac{1}{2}$% commission. How many bushels did he buy?

99. An estate worth $10927.60 is divided between two heirs so that one receives $\frac{1}{8}$ more than the other. What does each receive?

100. A merchant in St. Louis orders goods from New York, to the amount of $530, which amount he remits by draft, exchange being at 2¾% premium. If he pays $20 for transportation, what will the goods cost him in St. Louis?

101. What will be the cost, in Detroit, of a draft on Boston for $800, payable 60 days after sight, exchange being at a premium of 2%?

102. A note was given for $500, with interest payable annually, at 7%. Nothing having been paid for 10 years, how much did the total amount of interest due exceed the simple interest of the principal?

103. A merchant bought goods in Savannah to the amount of $850, and gave his note, July 1, 1885, on interest after 2 mo.; 4 mo. after the note was given, he paid $400; and 6 mo. after the first payment, he paid $360. What was there due Sept. 19, 1887?

104. An importer sold cloth to a wholesale dealer at 10% advance; the wholesale dealer sold it to a clothier at 12½% advance; the clothier sold it at a further advance of 25% and received $1452. What did it cost the importer?

105. A nurseryman sold trees at $18 per hundred, and cleared ⅓ of his receipts. What per cent. profit did he make?

106. In how many years will a man, paying interest at 7% on a debt for land, pay the face of the debt in interest?

107. What must be discounted for the present payment of a debt of $2637.72, of which $517.50 is to be paid in 6 mo., $793.75 in 10 mo., and the remainder in 1 yr. 6 mo., the use of money being worth 7%?

108. Wishing to borrow $500 at a bank, for what sum must my note be drawn, at 30 days, to obtain the required amount, discount being at 7%?

109. When N. Y. 5% are quoted at 108¼, what sum must I invest to secure an annual income of $1080, brokerage ¼%?

110. James Hayes bought merchandise in New York, at cash price, to the amount of $1256. Not having money, he gave his note in payment, drawn at 6 mo. at 6%. What was the face of the note?

111. William Green began trade January 1, 1886, with a capital of $3000, and, succeeding in business, took in M. H. Dodge as a partner on the first day of March following, with a capital of $2000; four months after they admitted C. H. Childs as third partner, who put in $1800 capital; they continued their partnership until April 1, 1888, when they found that $4388.80 had been gained since Jan. 1, 1886. What was each one's share?

112. If 54 men can build a fort in 24½ da., working 12½ hr. each day, in how many days will 75 men do the same, when they work but 10½ hr. each day?

113. A lumber merchant sells a lot of lumber, which he has had on hand 6 mo., on 10 mo. credit, at an advance of 30% on the first cost. If he is paying 5% interest on capital, what are his profits per cent.?

114. If a man clear $750 by his business in 1 yr. 6 mo., how much would he gain in 3 yr. 9 mo. at the same rate?

115. If a certain business yield $350 net profits in 10 mo., in what time would the same business yield $1050 profits?

116. Wishing to borrow $1000 of a Southern bank that is discounting paper at 8%, I give my note for $975, payable in 60 da. How much more will make up the required amount?

117. A man sold his farm containing 195 A. 2 R. 25 P. for $27.50 an acre, and took a note payable in 4 mo. 15 da. at 7% interest. Wishing the money for immediate use, he got the note discounted at a bank. How much did he receive?

118. May 10, 1887, I borrowed $6840, with which I purchased flour at $5.70 a barrel. June 21, 1888, I sold the flour for $6.62½ a barrel, cash. How much did I gain by the transaction, interest being reckoned at 6%?

119. B commenced business with a capital of $15000. Three months afterward C entered into partnership with him, and put in 125 acres of land. At the close of the year their profits were $4500, of which C was entitled to $1800. What was the value of the land per acre?

120. A and B engaged in trade. A put in $4200 at first, and 9 months afterward $200 more. B put in at first $1500, and at the end of 6 mo. took out $500. At the end of 16 mo. their gain was $772.20. How much is the share of each?

121. If 2 bushels of barley are required to sow an acre, how many acres can be sowed with the contents of a bin 4 ft. long, 3 ft. wide, and 18 in. deep?

122. A merchant in Boston receives from his agent 320 barrels of flour, purchased in Chicago at $10 per barrel; in payment for which he remits a draft on Chicago, at 2¼% discount. The transportation of his flour cost $312. What must he sell it for per barrel to gain $400?

123. Owing a man $575, I give him a 60-day note. What should be the face of the note, to pay him the exact debt, if discounted at 1½% a month?

124. What must be the face of a note which, when discounted at a broker's for 110 da. at 1% a month, shall give as its proceeds $187.50?

125. In a square lot containing 1⅗ A., how far is the centre from each corner, and from the middle of each side?

126. I bought a piece of land 16 ch. long and 15 ch. wide, at $100 an acre, and dividing it into lots of 6 rods by 5 rods, sold them at $50 each. What was my gain?

127. At $2.75 a rod, how much less will it cost to fence a piece of land 80 rd. square, than if the same were in the form of a rectangle twice as long and one-half as wide?

128. Find the cost of raising the surface of ⅞ of an acre, 9 in., at $.65 per cubic yard of earth?

129. A bin 9 ft. 6 in. long, 6 ft. wide, and 4 ft. 3 in. deep, is full of wheat. What is its value at $2.05 a bu.?

130. What will be the cost of the following draft, exchange on Boston being in Pittsburgh at 2¼% premium?

$600. PITTSBURGH, June 12, 1884.

Sixty days after sight, pay to William Barnard, or order, six hundred dollars, value received, and charge the same to our account.

To the Suffolk Bank, Boston. THOMAS BAUER & Co.

131. What must be the length of a bin that is 6 ft. wide and 4½ ft. deep, to contain 324 bushels?

132. The area of a map laid down on the scale of an inch to a mile is 4 sq. ft. 4 sq. in. What number of acres are represented?

Find the *date of maturity*, the *term of discount*, and the *proceeds* of the following:

$$957\frac{37}{100}. CHICAGO, July 27, 1885.

133. Three months after date, I promise to pay to the order of D. L. Moody, nine hundred fifty-seven and $\frac{37}{100}$ dollars, for value received.

Discounted Aug. 10, at 8%. JOHN ELLIS.

$$1315\frac{15}{100}. NEW YORK, May 1, 1886.

134. Ninety days after date, I promise to pay to the order of Ivison, Blakeman & Co., one thousand three hundred fifteen and $\frac{15}{100}$ dollars, for value received.

Discounted May 15, at 7%. WILLIAM HEWSON.

$1250. BOSTON, June 12, 1887.

135. Six months after date, I promise to pay W. A. Wilde & Co., or order, twelve hundred fifty dollars, *with interest* at 6%, value received.

Discounted at a broker's, S. R. BROWN.
Nov. 15, at 6%.

136. The triangular gable of a certain building has a base of 45 ft., and the perpendicular height is 16 ft. What will the lumber cost required to cover both gables, at \$1.87½ per hundred feet, board measure?

137. Which is the cheaper, a suit of clothes which costs \$56 and will last 18 mo., or a suit that costs \$35 and will last 10 mo.; and how much will a man save in 20 years who wears the cheaper suit, interest not being reckoned?

138. A commission merchant in Detroit wishes to remit to his employer in St. Louis, \$512.36 by draft at 60 da. What is the face of the draft which he can purchase with this sum, exchange being at 2½% discount?

139. Sold a farm containing 173 A. 95 P., for \$62½ an acre, and received in payment, \$2000 cash, and the balance in a note due in 5 mo. 18 da. at 7% interest, which was discounted at a bank. How much ready money did the farm bring?

140. A laborer received 50 cents the first day, 54 cents the second, 58 cents the third, and so on, until his wages were \$1.54 a day. How many days did he work?

141. If a reservoir 24 ft. 8 in. long by 12 ft. 9 in. wide is full of water, how many cubic feet must be drawn off to lower the surface 9 in.?

142. A merchant holds two notes, one for \$356.25 due Dec. 1, 1885, and the other for \$497.50, due Feb. 1, 1886. What would be due him in cash on both notes Sept. 15, 1885, at 6%?

143. I bought books, worth \$300, at list prices, getting a discount of 33⅓%. I sold them at the retail prices, on 6 mo. Money being worth 6%, what per cent. profit did I make?

144. What is the length of the inner edge of a cubical bin that contains 150 bushels?

145. A ladder 52 ft. long stands close against the side of a building. How many feet must it be drawn out at the bottom, that the top may be lowered 4 ft.?

146. A owes \$1500, of which \$400 is to be paid in 4 mo., \$500 in 5 mo., and \$600 in 6 mo. If he pays the whole at once, at what time must he make the payment?

147. A mechanic is employed for 1 yr. 6 mo., at \$80 a month, payable monthly, but his wages remain unpaid until the end of the whole term of service. What is due, allowing simple interest at 6% per annum?

148. A merchant buys goods for \$4200 on 4 mo. credit, but is offered a discount of 3% for cash. If money is worth ½% a month, what is the difference?

149. Bought a bill of lumber amounting to \$3500, on 6 mo. credit; 2 mo. afterward paid on account \$1500, and 1 month later, \$1000. Find the present worth of the balance, at the time of the second payment, int. at 7%.

150. A merchant sold 238 hhd. of sugar, each containing 963 lb. net weight, at 13 cents a pound, on 4 mo. time, and immediately got the paper discounted at a bank at 7%. How much money did he receive?

151. M. Banks received a legacy of \$3000, \$800 of which was payable in 9 mo., \$800 in 1 yr., and the balance in 2 yr.; but the executor being willing to make present payment, discounting at 7%, what ought he to receive?

152. Four men hire a pasture of 125 A. 48 P. at \$1.75 per acre. A puts in 125, B 150, C 200, and D 225 sheep. How much rent ought each to pay?

153. What must be the length of the edge of a cubical bin that shall contain the same volume as one that is 16 ft. long, 8 ft. wide, and 4 ft. deep?

154. What are the dimensions of a cube that has the same volume as a box 2 ft. 8 in. long, 2 ft. 3 in. wide, and 1 ft. 4 in. deep?

155. How many square feet in the surface of a cube whose volume is 91125 cubic feet?

156. A man's property is insured at $2\frac{1}{2}\%$, payable annually. In how many years will the sum of the premiums equal the policy?

157. Find the first and last terms of an arithmetical progression whose sum is 408, common difference 6, and number of terms 8.

158. $\frac{3}{5}$ of $\frac{10}{9}$ of what number, diminished by $\frac{\frac{1}{2} \text{ of } \frac{3}{10}}{2\frac{1}{6}+\frac{7}{30}}$, leaves $\frac{(4\frac{1}{3})^3-1}{(4\frac{1}{3})^2-1}$?

159. A, B, and C traded in company. A put in $1400; B, $2500; and C, 300 barrels of flour; they gained $2300, of which C took $1000. What were the shares of A and B, and what was the price of C's flour per barrel?

160. If a piece of silk cost $1.20 a yard, at what price must it be marked that it may be sold at 10% less than the marked price, and still make a profit of 20%?

161. My expenditures in building a house, in the year 1884, were as follows: Jan. 16, $536.78; Feb. 20, $425.36; March 4, $259.25; April 24, $786.36. At the last date I sold the house for what it cost, interest at 6% on the money expended added, and took the purchaser's note for the amount. What was the face of the note?

162. I have $32300 to invest, and can buy New York Central 6's at 85%, or New York Central 7's at 95%. How much more profitable will the latter be than the former per year?

163. In a certain town a tax of $5000 is to be assessed. There are 500 polls, each assessed 75 cents, and the valuation of the taxable property is $370000. What will be the rate of property tax, and how much will be A's tax, whose property is valued at $7500, and who pays for 2 polls?

164. My agent in Jacksonville, Fla., sells a house and lot for $7500, on commission of $1\frac{1}{2}\%$, and remits to me the proceeds in a draft purchased at $\frac{1}{2}\%$ premium. What sum do I receive from the sale of my property?

512. TABLE.—Prepared from information received from the Secretaries of the several States and Territories, showing in the first column the legal rate of interest in the different States and Territories.

State or Territory.	Rate.		State or Territory.	Rate.	
Alabama	8%	8%	Mississippi	6%	10%
Alaska (Ter.)	....	....	Missouri	6%	10%
Arkansas	6%	10%	Montana (Ter.)	10%	Any
Arizona (Ter.)	10%	Any	Nebraska	7%	10%
California	10%	Any	Nevada	10%	Any
Colorado	10%	Any	New Hampshire	6%	6%
Connecticut	6%	6%	New Jersey	6%	6%
Dakota (Ter.)	7%	12%	New Mexico (Ter.)	....	Any
Delaware	6%	6%	New York	6%	6%
Florida	8%	Any	North Carolina	6%	8%
Georgia	7%	8%	Ohio	6%	8%
Idaho (Ter.)	10%	18%	Oregon	10%	12%
Illinois	6%	8%	Pennsylvania	6%	6%
Indian (Ter.)	6%	Any	Rhode Island	6%	Any
Indiana	6%	8%	South Carolina	7%	7%
Iowa	6%	10%	Tennessee	6%	6%
Kansas	7%	12%	Texas	8%	12%
Kentucky	6%	6%	Utah (Ter.)	....	Any
Louisiana	5%	8%	Vermont	6%	6%
Maine	6%	Any	Virginia	6%	6%
Maryland	6%	6%	Washington (Ter.)	10%	Any
Massachusetts	6%	Any	West Virginia	6%	8%
Michigan	7%	10%	Wisconsin	7%	10%
Minnesota	7%	10%	Wyoming (Ter.)	12%	Any

1. When the rate is not specified in accounts, notes, mortgages, contracts, etc., the *legal* rate is always understood.

2. In the above table, where *two* rates are specified, any rate above the lower, and not exceeding the higher, is allowed by law, *if stipulated in writing.*

3. The above are liable to be changed at any time by the Legislatures of the respective States.

513. TABLE.—Showing the simple interest of $1, or the number of *hundredths* to be taken of any principal, at 5, 6, 7, and 8%, for years, months, and days:

	5%.	6%.	7%.	8%.		5%.	6%.	7%.	8%.
Yr.					**Yr.**				
1	.05	.06	.07	.08	4	.20	.24	.28	.32
2	.10	.12	.14	.16	5	.25	.30	.35	.40
3	.15	.18	.21	.24	6	.30	.36	.42	.48
Mo.					**Mo.**				
1	.00416	.005	.00583	.00666	7	.02916	.035	.04083	.04666
2	.00833	.01	.01166	.01333	8	.03333	.04	.04666	.05333
3	.01250	.015	.01750	.02000	9	.03750	.045	.05250	.06000
4	.01666	.02	.02333	.02666	10	.04166	.05	.05833	.06666
5	.02083	.025	.02916	.03333	11	.04583	.055	.06416	.07333
6	.02500	.03	.03500	.04000					
Da.					**Da.**				
1	.00013	.00016	.00019	.00022	16	.00222	.00266	.00311	.00355
2	.00027	.00033	.00038	.00044	17	.00236	.00283	.00330	.00377
3	.00041	.00050	.00058	.00066	18	.00250	.00300	.00350	00400
4	.00055	.00066	.00077	.00088	19	.00263	.00316	.00369	.00422
5	.00069	.00083	.00097	.00111	20	.00277	.00333	.00388	00444
6	.00083	.00100	.00116	.00133	21	.00291	.00350	.00408	.00466
7	.00097	.00116	.00136	.00155	22	.00305	.00366	.00427	.00488
8	.00111	.00133	.00155	.00177	23	.00319	.00383	.00447	.00511
9	.00125	.00150	.00175	.00200	24	.00333	.00400	.00466	.00533
10	.00138	.00166	.00194	.00222	25	.00347	.00416	.00486	.00555
11	.00152	.00183	.00213	.00244	26	.00361	.00433	.00505	.00577
12	.00166	.00200	.00233	.00266	27	.00375	.00450	.00525	.00600
13	.00180	.00216	.00252	.00288	28	.00388	.00466	.00544	.00622
14	.00194	.00233	.00272	.00311	29	.00402	.00483	.00563	.00644
15	.00208	.00250	.00291	.00333					

To find the interest of any given principal, at any rate and time given in the above table:

Multiply the given principal by the interest of one dollar as expressed in the table.

514. TABLE. Showing the amount of $1, at different rates, compound interest, from 1 to 20 years.

Year.	2½ per ct.	3 per ct.	3½ per ct.	4 per ct.	4½ per ct.	5 per ct.	6 per ct.	7 per ct.
1	1.0250	1.0300	1.0350	1.0400	1.0450	1.0500	1.0600	1.0700
2	1.0506	1.0609	1.0712	1.0816	1.0920	1.1025	1.1236	1.1449
3	1.0769	1.0927	1.1087	1.1249	1.1412	1.1576	1.1910	1.2250
4	1.1038	1.1255	1.1475	1.1699	1.1925	1.2155	1.2625	1.3108
5	1.1314	1.1593	1.1877	1.2167	1.2462	1.2763	1.3382	1.4026
6	1.1597	1.1941	1.2293	1.2653	1.3023	1.3401	1.4185	1.5007
7	1.1887	1.2299	1.2723	1.3159	1.3609	1.4071	1.5036	1.6058
8	1.2184	1.2668	1.3168	1.3686	1.4221	1.4775	1.5938	1.7182
9	1.2489	1.3048	1.3629	1.4233	1.4861	1.5513	1.6895	1.8385
10	1.2801	1.3439	1.4106	1.4802	1.5530	1.6289	1.7908	1.9672
11	1.3121	1.3842	1.4600	1.5395	1.6229	1.7103	1.8983	2.1049
12	1.3449	1.4258	1.5111	1.6010	1.6959	1.7959	2.0122	2.2522
13	1.3785	1.4685	1.5640	1.6651	1.7722	1.8856	2.1329	2.4098
14	1.4130	1.5126	1.6187	1.7317	1.8519	1.9799	2.2609	2.5785
15	1.4483	1.5580	1.6753	1.8009	1.9353	2.0789	2.3966	2.7590
16	1.4845	1.6047	1.7340	1.8730	2.0224	2.1829	2.5404	2.9522
17	1.5216	1.6528	1.7947	1.9479	2.1134	2.2920	2.6928	3.1588
18	1.5597	1.7024	1.8575	2.0258	2.2085	2.4066	2.8543	3.3799
19	1.5987	1.7535	1.9225	2.1068	2.3079	2.5270	3.0256	3.6165
20	1.6386	1.8061	1.9898	2.1911	2.4117	2.6533	3.2071	3.8697

To find the amount of any given principal, at any rate or time given in the above table:

Multiply the given principal by the amount of one dollar, as expressed in the table.

1. If the interest is compounded semi-annually, take *twice* the number of years at *one-half* the rate. Thus, the amount at 8%, compounded semi-annually, for 5 yr., is equivalent to the amount for 10 periods of 6 mo. each, at 4% for each period, and is the same as the amount for 10 yr. at 4%. If the interest is compounded quarterly, take 4 times the number of years at *one-fourth* the rate.

2. The comp. int. of $1 is $1 less than the amounts in the above table.

TABLE FOR INVESTORS.

515. *The following Table shows the exact, or, the approximate rate per cent. of Annual Income from Bonds bearing 4, 5, 6, and 7 per cent. interest, and costing from 40 to 125.*

Purchase Price.	4%.	5%.	6%.	7%.	Purchase Price.	4%.	5%.	6%.	7%.
40	10.00	12.50	15.00	17.50	83	4.82	6.02	7.22	8.43
41	9.75	12.20	14.64	17.08	84	4.76	5.95	7.14	8.33
42	9.52	11.90	14.28	16.66	85	4.70	5.88	7.05	8.23
43	9.30	11.63	13.95	16.28	86	4.65	5.81	6.97	8.13
44	9.09	11.36	13.63	15.90	87	4.59	5.74	6.89	8.04
45	8.88	11.11	13.32	15.56	88	4.54	5.68	6.81	7.94
46	8.69	10.86	13.04	15 21	89	4.49	5 61	6.74	7.86
47	8.51	10.63	12.77	14.90	90	4.44	5.55	6.66	7.77
48	8.33	10.41	12.50	14.53	91	4.39	5.49	6.59	7.69
49	8.16	10.20	12.25	14.29	92	4.34	5.43	6.52	7.60
50	8.00	10.00	12.00	14.00	93	4.30	5.37	6.45	7.52
51	7.84	9.80	11.76	13.72	94	4.25	5.31	6 38	7.44
52	7.69	9.61	11.53	13.46	95	4.21	5.26	6.31	7.36
53	7.54	9.43	11.32	13.20	96	4.16	5.20	6.25	7.29
54	7.40	9.25	11.11	12.96	97	4.12	5.15	6.18	7.21
55	7.27	9.00	10.90	12.72	98	4.08	5.10	6.12	7.14
56	7.14	8.92	10.70	12.50	99	4.04	5.05	6.06	7.07
57	7.01	8.77	10.52	12.27	100	4.00	5.00	6.00	7.00
58	6.89	8.62	10.34	12.06	101	3.96	4.95	5.94	6.93
59	6.78	8.47	10.16	11.86	102	3.92	4.90	5.88	6.86
60	6.66	8.33	10.00	11.66	103	3.88	4.85	5.82	6.79
61	6.54	8.19	9.83	11.47	104	3.84	4.80	5.76	6.72
62	6.45	8.06	9.67	11.29	105	3.81	4.76	5.71	6.66
63	6.35	7.93	9.52	11.11	106	3.77	4.71	5.66	6.60
64	6.25	7.81	9.37	10.93	107	3.73	4.67	5.60	6.54
65	6.15	7.69	9.23	10.76	108	3.70	4.62	5.55	6.48
66	6.03	7.57	9.09	10.60	109	3.67	4.58	5.50	6.42
67	5.97	7.43	8.95	10.44	110	3.63	4.54	5.45	6.36
68	5.88	7.35	8.82	10.29	111	3.60	4.50	5.40	6.30
69	5.79	7.24	8.69	10.14	112	3.57	4.46	5.35	6.25
70	5.71	7.14	8.57	10.00	113	3.54	4.42	5.30	6.19
71	5.63	7.04	8.45	9.85	114	3.50	4.38	5.26	6.14
72	5.55	6.94	8.33	9.72	115	3.47	4.35	5.21	6.08
73	5.48	6.84	8.21	9.58	116	3.44	4.31	5.17	6.03
74	5.40	6.75	8.10	9.45	117	3.41	4.27	5.12	5.98
75	5.33	6.66	8.00	9.33	118	3.39	4.23	5.08	5.93
76	5.26	6.57	7.89	9.21	119	3.36	4.20	5.04	5.88
77	5.19	6.49	7.79	9.00	120	3.33	4.16	5.00	5.83
78	5.12	6.41	7.69	8.97	121	3.30	4.13	4.95	5.78
79	5.06	6.32	7.59	8.86	122	3.27	4.09	4.91	5.73
80	5.00	6.25	7.50	8.75	123	3.25	4.06	4.87	5.69
81	4.93	6.17	7.40	8.64	124	3.22	4.03	4.83	5.65
82	4.87	6.09	7.31	8.53	125	3.20	4.00	4.80	5.60

ANSWERS.

In answers expressed in U. S. currency, if the mills in the final result were 5 or more than 5, in most cases they have been considered a *cent;* when *less* than 5, they have not been regarded.

Art. 38.

2. 1899 feet.
3. 2011 pounds.
4. 2455 men.
5. $5268.
6. $29.26.
7. $383.90.
8. $65.94.
9. $287.67.
10. $3[illegible]6.71.
11. 7504 pounds.
12. $698.87.
13. 4576.
14. 5040.
15. 4824.
16. 27944.
17. 29600.
18. 61632.
19. 3237.
20. 289142.
21. 415184.
22. $450.56.
23. $3136.81.
24. $5395.61.
25. 4092 yards.
26. $18.12.
27. $80.87.
28. $7393.33.
29. $17.67.
30. $12650.
31. $6692.23.
32. 961 miles.
33. *$149.18.*
34. *105233.*
35. $220.34.
36. 181777.
37. 11965.
38. $944.66.
39. 8227 pounds.
40. $19202.50.
41. 4125 bu.;
$8733.25.
42. $39148.53.
43. $1435.50.
44. $5735.75.
45. $20701.31.
46. $3202.50.
47. $7329.30.
48. $1571.56.
49. $1196.33.
50. $1542.04.
51. $12933.16.
52. $63452.87.

Art. 45.

7. 446 years.
8. 277 "
9. 67 "
10. 60 "
11. 57 "
12. 80 "
13. 48 "
14. 58 "
15. 57 "
16. 183 "
17. 43 "
18. $240.81.
19. $95.58.
20. $38.08.
21. $6.16.
22. 32358.
23. $64.84.
24. $135.28.
25. $14.11.
26. 3231.
27. $51.24.
28. $157.63.
29. 8728 rods.
30. 45736 tons.
31. 2123 tons.
32. 2324 feet.
33. 2324 days.
34. 4175.
35. 51.
36. 5113.
37. $15.21.
38. $22.10.
39. $25.26.
40. 32798.
41. 35555.
42. 30443.
43. 6132.
44. 2517.
45. $3640.
46. 78388.
47. $3015.05.
48. $291.35.
49. $2063 gain.
50. $53945.
52. 3427.
53. 3745.
54. 1152.
56. $6473.
57. $5412.
58. $9505.67.
59. $5044.25.
60. $1807.
61. $.55.
62. $10330.
63. $16948.50.
64. $1500.
65. $2063 gain.

Art. 55.

12. $2079.
13. $17120.
14. $6190.
15. $52.64.
20. $40.50.
21. $30.59.
22. $19.56.
23. $22.74.
24. $47.25.

Art. 56.

2. 12825;
18050.
3. 24408;
31866.
4. $1019.76;
$1675.32.
5. $4579.25;
$5776.90.
6. 482400;
430944;
874752;
$1289.30;

1348.65;
4592.70.
7. 2953216;
5606496;
7083104;
$8977.50;
33345.00;
46312.50.
8. $264958;
$90288.
9. 404914.
10. 186516.
11. 241768.
12. $51188.62.
13. 179029476.
14. $154037.36.
15. 15704325 da.
16. 2082600 cts.
17. 1508741097.
18. 1587862270.
19. 365860576.
20. 8198473608.
21. 982275037.
22. 3363731415.
23. $2715413.50
24. $2529.25.
25. $319192.

Art. 57.

1. $3505.92.
2. 8605472.
3. 3906168.
4. $19789.44.
5. $84338.28.
6. 16810830.
7. 54793296.
8. $133804.80.
9. 73152.
10. 1031875.
11. 0.
12. 14357520.
13. 18899264.
14. 40905480.
15. 0.
16. 538788276.
17. $10904.
18. $5040.
19. 65208.
20. $3594.24.
21. $4101.25.

Art. 58.

3. 4360;
43600;
436000;
4360000.
4. 140400;
1170000;
7488000;
58500000.
5. $80000.
6. $10800.
7. $600.
8. $8232.
9. $48 gain.
10. $9[illegible]0.
11. 90274.
12. 151218.
13. $12.50 gain.
14. $606.32.
15. $858.
16. $933.05.
17. $1700.
18. $466 rec'd.
19. 994 miles.
20. $7242.
21. $163 gain.
22. $7161.20.
23. 77050.
24. 92500.
25. $1714.50.
26. $43187.32.

Art. 69.

2. 5801 tons.
3. 25844 days.
4. 6294 rods.
5. 11966;
13314.
6. 50384$\frac{4}{5}$;
17150;
13338$\frac{2}{3}$.
7. 417911$\frac{2}{9}$;
149510$\frac{3}{8}$;
128151$\frac{6}{7}$.
8. 10421;
9118$\frac{3}{8}$;
14589$\frac{3}{5}$;
12157$\frac{5}{6}$;
24315$\frac{2}{3}$.

9. 53266;
23678$\frac{7}{8}$;
26633;
42612$\frac{4}{5}$;
85510$\frac{4}{8}$.
10. 12515 tons.
11. 6052$\frac{2}{3}$ bu.
12. 7106\frac{5}{8}$.
13. 9375 bu.
20. 1592 bbl;
883$\frac{4}{5}$.
21. $108.50.
22. 93 oranges.
23. 91 yards.
24. 163 pounds;
300 "
25. $172.65.
26. 5801$\frac{1}{3}$ tons.
27. 2584$\frac{1}{3}$ days.
28. $820.50.
29. 976 feet.
30. 2594 men.
31. 32793 pounds.
32. 11750 bu.
33. 71474$\frac{3}{7}$ mi.
34. 8219 men.
35. 20116$\frac{1}{3}$ acres.
36. 63362 rods.

Art. 70.

5. $9.58.
6. $14.89.
7. $25.21.
8. 354 times.
9. 416 "
10. 672 "
11. 1763 "
12. 3300 "
13. 13$\frac{20}{103}$ "
14. 4$\frac{448}{700}$ "
15. 784; 720;
735; 641$\frac{25}{55}$.
16. 35464;
38688;
32736;
30397$\frac{10}{42}$.
17. 840$\frac{2}{114}$;
630$\frac{8}{152}$;
560$\frac{2}{171}$;
469$\frac{86}{264}$.

18. 460 pounds
19. 465 acres;
334$\frac{20}{23}$ "
20. 1607 bbl.
21. 394.
22. 5482.
23. 7198.
24. 31416.
25. 7071.
26. 8723.
27. 1172$\frac{644}{735}$.
28. 4321.
29. 2036.
30. 3645$\frac{2667}{3076}$.
31. 7500.
32. 43785.
33. $97.
34. $76.
35. 1823\frac{86}{313}$.
36. $135.
37. 456 pounds.
38. 296.
39. 122108.
40. 28$\frac{99}{5000}$.
41. 5182.

Art. 71.

3. 1242$\frac{60}{100}$.
4. 304$\frac{631}{1000}$.
5. 7103$\frac{2000}{10000}$.
6. 12.
7. 113$\frac{160}{420}$.
8. 315.
9. 150 bales.
10. 75 horses.
11. $43.
12. 20 lots.

Art. 74.

3. 13.
4. 4.
5. 80.
6. 48.
7. 10 tons.
8. 98 bbl.
9. 8$\frac{1}{3}$ tubs.
10. 38 bu.
11. 100 cents.
12. 24 suits.

13. $4\frac{4}{5}$ tons.
14. $4500.
15. 8 bales.
16. 16.
17. $9\frac{1}{7}$.
18. $1\frac{1}{25}$.
19. 82.
20. 144 bu.
21. 48 days.
22. $2.

Art. 75.

1. 15.
2. 15.
3. 15.
4. 17976.
5. 149.
6. 18.
7. 12933.
8. 75.
9. 21124.
10. 30 tons.
11. $36.
12. $271\frac{5}{13}$ pounds.
13. 5 years.
15. $.76.
16. $107.
17. $\$2123\frac{2}{3}$.
18. 42 weeks.
19. $367.
20. $1806.
21. $\$1442.\frac{14}{33}$;
$6250.
22. 462 pounds.
23. 356 cords;
$4, cost.
24. $1800.
25. $2100.
26. 3000 pounds.
27. $7.50.
28. $109.
29. $68.
30. 7 years.
31. 40 cows.
32. $2.
33. 296.
34. 122108.
35. 159.
36. 5182.
37. 8897.
38. 33099.
39. 50017.
40. 822.

Art. 87.

2. 5, 5, 3, 3.
3. 2, 7, 11.
4. 5, 5, 2, 2, 2.
5. 5, 5, 5, 5.
6. 5, 3, 3, 3, 7.
7. 5, 7, 3, 11.
8. 3, 3, 2, 163.
9. 5, 5, 7, 3, 3.
2, 2.

Art. 91.

2. 14.
3. 32.
4. 4.
5. 7.
6. 12.
7. 3.
9. 14.
10. 4.
11. 63.
12. 75.
13. 45.
14. 4.
15. 216.
16. 45.
17. 182.
18. 63.
19. 28.
20. 72.
21. 42.
22. 47.
23. 17.
24. 87.
25. 124; 2.
26. 2 acres.
27. 23 feet.
28. 28 bu.
29. 5; 9;
11 horses.

Art. 96.

2. 1008.
3. 1188.
4. 432.
5. 126.
6. 720.
7. 450.
8. 720.
9. 3240.
10. 1125.
11. 480.
12. 16800.
13. 720.
14. 1500.
15. 525.
16. 315.
17. $60.
18. 180 ft.
19. 384.
20. 63.
21. $4536.
22. 720 bu.
23. 560 acres.

Art. 106.

2. $\frac{33}{35}$.
3. $\frac{45}{68}$.
4. $\frac{42}{84}$.
5. $\frac{55}{78}$.
6. $\frac{90}{108}$.
7. $\frac{168}{256}$.
9. $\frac{21}{25}$.
10. $\frac{7}{9}$.
11. $\frac{3}{8}$.
12. $\frac{3}{8}$.
13. $\frac{1}{8}$.
14. $\frac{3}{8}$.
15. $\frac{2}{11}$.
16. $\frac{17}{18}$.
17. $\frac{7}{8}$.
18. $\frac{4}{5}$.
19. $\frac{3}{4}$.
20. $\frac{71}{198}$.
21. $\frac{7}{8}$.
22. $\frac{13}{30}$.
23. $\frac{117}{256}$.
24. $\frac{273}{656}$.
25. $\frac{21}{32}$; $\frac{2}{15}$.
26. $\frac{7}{8}$.
27. $\frac{168}{256}$ greater.
28. $\frac{2}{3}$; $\frac{7}{9}$; $\frac{2}{5}$; $\frac{2}{17}$.

Art. 108.

3. $\frac{176}{4}$ bu.
4. $\frac{47}{8}$ yr.
5. $\frac{298}{7}$ wk.
6. $\frac{1248}{16}$ lb.
7. $\frac{284}{13}$.
8. $\frac{4087}{18}$.
9. $\frac{7691}{25}$.
10. $\frac{1467}{17}$.
11. $\frac{2012}{30}$.
12. $\frac{20887}{50}$.
13. $\frac{186517}{152}$.
14. $\frac{74874}{50}$.
15. $\frac{28034}{75}$.

Art. 110.

2. $6\frac{1}{16}$ lb.
3. $\$16\frac{7}{25}$.
4. $13\frac{1}{5}$ rd.
5. 93.
6. $20\frac{7}{12}$.
7. 28.
8. $56\frac{13}{25}$.
9. $56\frac{1}{24}$.
10. 89.
11. $128\frac{2}{35}$.
12. $100\frac{13}{35}$.
13. 52.
14. $68\frac{1}{5}$.
15. $515\frac{4}{11}$.
16. $676\frac{1}{4}$.

Art. 114.

2. $\frac{27}{63}$; $\frac{35}{63}$.
3. $\frac{35}{60}$; $\frac{48}{60}$.
4. $\frac{72}{168}$; $\frac{105}{168}$;
$\frac{112}{168}$.
5. $\frac{21}{48}$; $\frac{32}{48}$; $\frac{18}{48}$.

6. $\frac{63}{70}$; $\frac{50}{70}$; $\frac{35}{70}$.
7. $\frac{39}{234}$; $\frac{52}{234}$; $\frac{36}{234}$.
9. $\frac{42}{48}$; $\frac{28}{48}$; $\frac{24}{48}$.
10. $\frac{24}{78}$; $\frac{45}{78}$; $\frac{14}{78}$.
11. $\frac{160}{168}$; $\frac{27}{168}$; $\frac{102}{168}$.
12. $\frac{105}{140}$; $\frac{280}{140}$; $\frac{84}{140}$; $\frac{238}{140}$.
13. $\frac{125}{20}$; $\frac{7}{20}$; $\frac{140}{20}$; $\frac{30}{20}$.
14. $\frac{360}{960}$; $\frac{200}{960}$; $\frac{210}{960}$; $\frac{2668}{960}$.

Art. 116.

1. $\frac{21}{40}$.
2. $\frac{4}{27}$.
3. $14\frac{5}{24}$.
4. $\$19\frac{7}{16}$.
5. $\frac{7}{8}$.
6. $1\frac{23}{24}$.
7. $15\frac{1}{4}$.
8. $10\frac{4}{7}$.
9. $5\frac{5}{32}$.
10. $33\frac{35}{36}$.
11. $27\frac{3}{8}$.
12. $6\frac{35}{36}$.
13. $\$16\frac{11}{30}$.
14. $1\frac{19}{30}$.
15. $5\frac{39}{40}$.
16. $82\frac{29}{36}$.
17. $262\frac{9}{35}$.
18. $\frac{31}{36}$.
19. $\frac{11}{12}$.
20. $1\frac{13}{18}$.
21. $12\frac{9}{11}$.
22. $7\frac{25}{28}$.
23. $10\frac{7}{18}$.
24. $\$114\frac{1}{48}$.
25. $187\frac{7}{8}$ yd.; $\$666\frac{15}{16}$.
26. $\$16\frac{17}{40}$.
27. $\$1\frac{37}{48}$.
28. $1\frac{17}{168}$.
29. $6\frac{2}{33}$.
30. $34\frac{13}{14}$.
31. $68\frac{11}{30}$.
32. $158\frac{13}{37}$.
33. 329.

Art. 120.

7. $7\frac{4}{7}$.
8. $2\frac{3}{7}$.
9. $1\frac{3}{5}$.
10. $18\frac{1}{2}$.
11. $\frac{3}{8}$.
12. $16\frac{5}{6}$.
13. $96\frac{3}{13}$.
14. $241\frac{2}{3}$.
15. $166\frac{2}{13}$.
16. $116\frac{2}{3}$.
17. $7263\frac{1}{2}$.
18. $22387\frac{1}{3}$.
19. $\frac{28}{135}$.
20. $\frac{23}{36}$.
21. $\frac{3}{8}$.
22. $\frac{1}{6}$.
23. $\frac{17}{36}$.
24. $\frac{3}{7}$.
25. $\frac{9}{28}$.
26. $1\frac{5}{32}$.
27. $4\frac{3}{10}$.
28. $6\frac{23}{27}$.
29. $7\frac{2}{3}$.
30. $8\frac{23}{24}$.
31. $\frac{19}{33}$.
32. 10.
33. $1\frac{1}{4}$.
34. $12\frac{1}{3}$.
35. $5\frac{9}{16}$.
36. $\frac{3}{8}$.
37. $\$\frac{7}{24}$.
38. $40\frac{5}{8}$ cts.
39. $328\frac{1}{3}$ cts.
40. $\$2\frac{29}{40}$.
41. $\$114\frac{3}{8}$.
42. $278\frac{4}{5}$ cts.
43. \$10.50.
44. $\$1.72\frac{1}{2}$.
45. $\$.67\frac{1}{2}$.
46. $\$69\frac{3}{8}$.
47. $\$21\frac{31}{32}$.
48. \$345.
49. $\$12\frac{3}{20}$.
50. $\$63\frac{9}{80}$.
51. $\$145\frac{3}{5}$.
52. $\$23\frac{1}{3}$.
53. $\$11.65\frac{1}{2}$.
54. $\$2293\frac{3}{4}$.
55. \$7196.
56. \$5734.
57. $\$47\frac{17}{24}$.
58. $\$\frac{13}{32}$.
59. $\$28\frac{1}{3}$.
60. $\$199\frac{33}{50}$.
61. $\$73\frac{7}{40}$.
62. $\$110\frac{23}{32}$.
63. $\$25\frac{2}{15}$.
64. $30\frac{5}{8}$ bbl.
65. $424\frac{16}{81}$.

Art. 124.

7. $\frac{3}{43}$.
8. $\frac{9}{40}$.
9. $\frac{1}{310}$.
10. 36.
11. $\frac{8}{675}$.
12. $1\frac{5}{7}$.
13. $7\frac{3}{5}$.
14. $6\frac{3}{7}$.
15. $14\frac{27}{128}$.
16. $\frac{35}{36}$.
17. $5\frac{23}{33}$.
18. $17\frac{25}{51}$.
19. $\frac{5}{8}$.
20. $7\frac{4}{15}$.
21. $\frac{1}{3}$.
22. $1\frac{1}{4}$.
23. $1\frac{107}{336}$.
24. $1\frac{5}{7}$.
25. $2\frac{3}{32}$ rd.
26. $\$\frac{2}{3}$.
27. \$10450.
28. \$147.
29. $\$85\frac{3}{8}$.
30. $\$10\frac{19}{48}$.
31. $11\frac{13}{17}$ cd.; $44\frac{2}{17}$ cd.
32. $21\frac{3}{8}$ bu.; $46\frac{1}{8}$ bu.
33. $\frac{5}{6}$.
34. $1\frac{1}{3}$.
35. $\frac{7}{10}$.
36. $\frac{1}{3}$.
37. $\frac{3}{16}$.
38. $\frac{3}{8}$.
39. $2\frac{3}{81}$.
40. $\frac{13}{49}$.
41. $\frac{3}{7}$.
42. $1\frac{4}{7}$.
43. $7\frac{7}{10}$.
44. $3\frac{3}{8}$.
45. $1\frac{5}{7}$.
46. $5\frac{11}{18}$:
47. 7.
48. $13\frac{17}{27}$ lb.
49. 24 tons; $37\frac{1}{3}$ tons.
50. $9\frac{4}{11}$ yd.;

$26\frac{3}{11}$ yd.;
$69\frac{2}{11}$ yd.
51. $293\frac{1}{3}$ lb.
53. $1\frac{1}{6}$.
54. $\frac{5}{8}$.
55. $1\frac{1}{3}$.
56. $4\frac{1}{2}$.
57. 2.
58. $1\frac{17}{18}$.
59. $17\frac{11}{16}$ days
60. $6\frac{33}{49}$ bu.
61. $1\frac{5}{7}$ yd.
62. $1\frac{3}{20}$; $\frac{7}{20}$;
$\frac{3}{10}$; $1\frac{7}{8}$.
63. $1\frac{27}{40}$; $\frac{3}{40}$;
$\frac{7}{10}$; $1\frac{3}{35}$.
64. $1\frac{29}{42}$; $\frac{1}{42}$;
$\frac{5}{7}$; $\frac{35}{36}$.
65. $\frac{49}{72}$; $\frac{17}{72}$;
$\frac{11}{108}$; $2\frac{1}{16}$.
66. $1\frac{53}{84}$; $\frac{25}{84}$;
$\frac{9}{14}$; $\frac{56}{81}$.
67. $1\frac{13}{48}$; $\frac{19}{48}$;
$\frac{35}{96}$; $\frac{21}{40}$.
68. $1\frac{19}{30}$; $\frac{7}{30}$;
$\frac{49}{75}$; $1\frac{1}{3}$.
69. $\frac{57}{120}$; $\frac{7}{120}$;
$\frac{1}{18}$; $1\frac{7}{25}$.
70. $5\frac{7}{24}$; $3\frac{23}{24}$;
$3\frac{1}{12}$; $6\frac{15}{16}$.
71. 4; $2\frac{1}{2}$;
$2\frac{7}{16}$; $4\frac{1}{3}$.
72. $5\frac{5}{6}$; $1\frac{1}{6}$;
$8\frac{1}{6}$; $1\frac{1}{2}$.
73. $7\frac{13}{28}$; $1\frac{27}{28}$;
$12\frac{27}{28}$; $1\frac{5}{7}$.
74. $3\frac{1}{18}$; $1\frac{17}{18}$;
$1\frac{7}{18}$; $\frac{2}{9}$.
5. $13\frac{1}{3}$; $2\frac{2}{3}$;
$42\frac{2}{3}$; $\frac{2}{3}$.
76. $22\frac{13}{30}$; $21\frac{3}{80}$;
$12\frac{3}{20}$; $38\frac{2}{3}$.
77. $25\frac{5}{8}$; $23\frac{7}{8}$;
$21\frac{31}{32}$; $28\frac{3}{7}$;
78. $30\frac{7}{24}$; $26\frac{23}{24}$;
$47\frac{17}{24}$; $17\frac{7}{40}$.
79. $99\frac{1}{3}$; $25\frac{4}{5}$;
$2293\frac{3}{4}$; $1\frac{258}{367}$.
80. $29\frac{7}{10}$; $3\frac{3}{10}$;
$217\frac{1}{3}$; $1\frac{1}{4}$
81. $5\frac{1}{3}$; $1\frac{1}{2}$;
7; $1\frac{3}{4}$.
82. 13; 6;
$33\frac{1}{4}$; $2\frac{5}{7}$.
83. $1\frac{1}{3}$, $\frac{2}{3}$;
$\frac{1}{3}$; $\frac{1}{3}$.
84. $\frac{61}{72}$; $\frac{29}{72}$;
$\frac{5}{36}$; $2\frac{13}{16}$.

Art. 127.

1. $\frac{525}{25}$; $\frac{1700}{50}$;
$\frac{1350}{18}$.
2. $\frac{15}{40}$; $\frac{24}{40}$; $\frac{65}{40}$.
3. $10\frac{5}{28}$.
4. $4\frac{8}{15}$.
5. $\frac{3}{14}$.
6. $3\frac{17}{30}$.
7. 20; $1\frac{7}{8}$.
8. 140 bu.
9. \$5.89$\frac{1}{2}$.
10. \$4055$\frac{5}{8}$.
11. \$4612.
12. $\frac{1}{2}$.
13. $\frac{32}{55}$.
14. 60 ft.
15. $17\frac{1}{3}$ lb.
16. $\frac{23}{150}$.
17. $57\frac{3}{5}$.
18. $4\frac{7}{8}$ tons.
19. \$.75.
20. \$264$\frac{1}{6}$.
21. $11\frac{1}{4}$.
22. $13\frac{13}{14}$ yd.
23. $2\frac{1}{56}$; $\frac{15}{56}$; 1.
24. $10\frac{4}{5}$ lb.
25. $3\frac{3}{7}$ da.
26. \$120.
27. $19\frac{1}{2}$ bbl.
28. $2\frac{1}{4}$.
29. \$120.
30. $7\frac{1}{2}$ times.
31. $\frac{5}{104}$ increased.
32. $\frac{5}{84}$ diminish'd.
33. \$12601.
34. \$4151.91.
35. \$5141$\frac{1}{3}$.
36. \$44$\frac{1}{2}$ lost.
37. $3\frac{13}{27}$.
38. $\frac{28}{55}$.
39. $33\frac{7}{27}$.
40. $22\frac{13}{16}$.
41. 189.
42. $\frac{315}{961}$.
43. $21\frac{7}{8}$.
44. 9.

Art. 148.

1. 61500 cts.
2. 2406 cts.
3. 9206 m.
4. 860 m.
5. 763 m.
6. 475 m.
7. \$34.67.
8. \$104 08.
9. \$46.725.
10. \$302.
11. $76\frac{2}{11}$ cts.
12. \$5.607.
13. \$30.09.
14. 85 cts.
15. .23.
16. .102.
17. .9.
18. .07.
19. .70000;
.05000;
.30400;
20. 2.5000;
.1070;
.0008.
21. 4.0000;
2.1700;
.1360;
.0408.
23. $\frac{4}{25}$.
24. $\frac{1}{8}$.
25. $\frac{5}{8}$.
26. \$$\frac{3}{4}$.
27. \$$\frac{3}{8}$.
28. \$$\frac{131}{200}$.
29. $\frac{3}{125}$.
30. $\frac{9}{16}$.
31. $\frac{5}{16}$.
32. \$$\frac{7}{8}$.
33. $\frac{1}{1250}$.
34. $\frac{15}{16}$.
35. $8\frac{1}{4}$.
36. \$12$\frac{3}{4}$;
$25\frac{1}{200}$;
\$36$\frac{1}{3}$.
38. \$.75.
39. \$.875.
40. \$.8.
41. \$.31$\frac{1}{4}$.
42. .56.
43. .60625.
44. .003$\frac{1}{3}$.
45. .392.
46. .7.
47. .032.
48. 1.875.
49. .066.
50. \$5.6.
51. \$12.625.
52. \$42.20.
53. \$36.9375.

Art. 151.

3. 25.6178.
4. $\$196.788$.
5. $\$274.715$.
6. $\$84.655$.
7. $\$.495$.
8. $\$.25$.
9. $\$.0625$.
10. 50.44.
11. 6.3045.
12. 3.999976.
13. 9.99.
14. 1.873125.
15. $\$594$.
16. $\$706.63$.
17. $\$2.82$.
18. $\$70.03$.
19. $\$6.46$.
20. 254.785 tons.
21. $\$1427.12\frac{1}{2}$.
22. 31.625 yd.
23. 51.180606.
24. 2194.85 acres.
25. 23.8642 bbl.
26. $\$4149.125$
27. $\$19.77$.
28. $\$7062.15$.
29. 387.33 rd.
30. $\$9.40$.
31. 289.7892.
32. $\$62.33$.
33. $\$4.1375$.
34. $\$95$.
35. $\$47.07$.

Art. 153.

2. 15.544.
3. 235.06466.
4. .0273238.
5. 5.4008.
6. $\$14.70$.
7. 2.934.
8. 104.182.
9. .0570.
10. $\$3.125$.
11. .8448.
12. $\$.84217$.
13. .021252.
14. $\$.162$.
15. 4.36692.
16. .8823.
17. .74772.
18. $\$105.188$.
19. $\$36.0062$.
20. 572.8.
21. 620.7.
22. 1.375.
23. 676.
24. 20.496.
25. .04765625.
26. 42.
27. .0006076.
28. $\$1.50$.
29. $\$4.08$.
30. 311.6.
31. $\$133.56$.
32. $\$3467.25$
33. $\$962.50$.
34. $\$63.56$.
35. $\$78.125$.
36. $\$83\ 69$.
37. $\$1125$.
38. $\$446.25$.
39. $\$438$.
40. $\$14891.93$.
41. $\$32.93$.

Art. 155.

2. $22.66\frac{2}{3}$.
3. 4500.
4. .2.
5. $\$1.25$.
6. 36.4.
7. 4602.
8. 100.
9. .01.
10. .1.
11. .6455.
12. $\$8000$.
13. .5.
14. 1.76.
15. .31.
16. .16.
17. .0175.
18. .0105.
19. $\$2.125$.
20. 110.
21. $\$.72$.
22. 100000.
23. $\$.50$.
24. $\$.08$.
25. $\$1.17$.
26. $\$31.565$.
27. $\$.875$.
28. $\$.113$.
29. $\$.25$.
30. $\$.06\frac{1}{4}$.
31. $\$.09$.
32. 12 coats.
33. 136 bbl.
34. $\$5.85$.
35. 28.8 bu.
36. $\$10.875$;
$\$14.625$.
$\$23.06\frac{1}{4}$.
37. $\$65.28$.
38. .4.
39. $\$193.75$.
40. 44.32.
41. 1.7.
42. $\$32\ 375$.
43. 56 lb.
44. $\$7.45$.
45. 337.574;
323.676;
104.976;
1000.
46. 76.104;
.221;
.00207;
$25.55\frac{5}{9}$.
47. 1679.7756;
947.24;
4531.52;
200.
48. 87.5936;
.0004;
.0000078;
$1.15\frac{5}{13}$.
49. 244.765;
.975;
.33615;
$4.61\frac{1}{9}$.
50. 566.1735;
506.0765;
24.5470625;
$10435.5\frac{3}{7}$.
51. 253.4163;
6.1588;
83.281048;
2.78.
52. 5.99721;
3.59994;
.000216;
60000.
53. 2.25;
42.625;
$1.4\frac{1}{11}$.
54. 75.415;
17.019;
$336.17\frac{7}{8}$.
55. 722.9844;
11.2788;
$46346.15\frac{5}{13}$.
56. 87.412;
7.7;
$994.3\frac{2}{11}$.
57. 241.75;
181.875;
$323.3\frac{1}{3}$.
58. 23.25;
764.859375;
$2.2653\frac{3}{49}$.
59. 196.5375;
4783.78125;
$9.9743\frac{11}{78}$.
60. 2.38315;
.01673105;
341.45.
61. .831.
62. 1554.
63. 688.
64. 1449.21.
65. 295.

Art. 162.

2. $\$322.50$.
3. $\$7060$.
4. $\$255$.
5. $\$250$;
$\$187.50$.
6. $\$315.50$;
$\$625.00$;
$\$802.50$.
7. $\$172$.
8. $\$19.83\frac{1}{3}$.
10. $\$856$.
11. [illegible]

$360;
432.
12. $810;
$840;
$864;
$960.

Art. 163.

1. 954 baskets;
636 "
3. 1440 P. ap.;
1200 "
960 "
4, 5600 pounds:
4200 "
3500 "
2800 "
5. 603.
6. 900 yards;
1350 "
1800 "

Art. 164.

2. $200.88.
3. $104.93.
4. $18.49.
5. $34.95.
6. $29.70.
7. $56.28.
8. $8.78.
9. $328.99.
10. $64.58.
11. $664.06¼.

Art. 165.

2. $2.87.
3. $4.76.
4. $2.92.
5. $14.62.
6. $10.75.
7. $75.82.
8. $39.65.
9. $20.17.

Art. 166.

1. $\frac{16}{100}$; $\frac{625}{1000}$; $\frac{5625}{10000}$; $\frac{8}{10000}$.
2. 1.464375;
2.522375.
3. $21.125.
4. .8 yd.
5. 23 bu. each.
6. $21.375.
7. $376.25.
8. $60.
9. $3228.34.
10. $65.41.
11, $59.57.
12. $.50.
13. $122.50.
14. $534.60.
15. $55992.31.
16. $32.07.
17. $201.91.
18. $94 81.
19. $212.75.
20. $36.86.
21. 6⅜ M.
22. 18 yd.
23. 188.2 acres.
24. 192.25 lb.
25. $.75.

Art. 172.

1. $117.25.
2. $1489.46.
3. $1480.84.
4. $124.005.
5. $102.86.
6. $16.625.
7. $99.93.
8. $717.93.
9. $621.98.
10. $114.75.
11. $728.60.
12. $191.85.

Art. 197.

2. 45515 gr.
3. 105948 oz.
4. 68245 min.
5. 63964 ft.
6. 2046 in.
7. 43695 sq. ft.
8. 14918 lb.
9. 5480 pwt.
10. 525600 min.
11. 9696 cu. ft.
12. 6216 pt.
13. 792 qt.
14. 200 qui.
15. 864 doz.
16. 1008 gi.
17. $1045.50.
18. 960 rd.
19. 568 bbl.
20. 80 boxes.
21. $29.25.
22. 2160 sh.
23. $432.
24. 2419200 sec.
25. 36 bottles.
26. $17.64
27. 1920 rd.
28. 5760 sq. rd.
29. $9152
30. 876576 hr.
31. 1485 vols.
32. 256 pp.
33. 3625 lb.
34. 8344 lb.
35. 2950 lb.
36. 32620 lb.
37. 6325 lb.
38. 3150 lb., N.Y.
39. 7545 lb.
40. 461824 lb.
41. 1568 lb.
42. 1800 lb.

Art. 198.

2. 14 hr. 24 min.
3. 7 oz. 4 pwt.
4. 3⅔ qr. yd.
5. 3 wk. 1 da. 9 hr. 36 min.
6. 183 sq. rd. 10 sq. yd. 108 sq. in.
7. 1 da. 9 hr. 36 min.
8. 14 ft. 5¼ in.
9. 3 gal. 3 qt. 1 pt. 2 gi.
10. 22 hr. 4 min. 48 sec.
11. 41 lb. 8.96 oz.
12. 2 pk. 4 qt.
13. 2 rd. 9 ft. 2.88 in.
14. 4 da. 14 hr. 42 min. 43.2 sec.
15. 14.4 oz.

Art. 199.

2. 3 hhd.
3. 49 bu. 3 pk. 7 qt. 1 pt.
4. 365 da. 5 hr. 48 min. 46 sec.
5. 14 lb. 10 oz. 18 pwt. 22 gr.
6. 25 T. 15 cwt. 70 lb.
7. 25 cwt. 37 lb. 15 oz.
8. 12 lb. 6 oz.
9. 201 bu.
10. 336 gro. 1 doz. 4.
11. 227⅚ doz.
12. 251 score.
13. 22 reams 7 qui. 10 sh.
14. 151 bun. 8 qui.
15. 34 yd. 1 ft. 6 in.
16. 4 mi.

17. $40\frac{5}{8}$ yd.
18. 160 acres.
19. 8 sq. rd. 25 sq. yd. 4 sq. ft. 112 sq. in.
20. 29 cu. yd. 3 cu. ft.
21. 342 Cd. 84 cu. ft.
22. 6 cd. ft.
23. 31.72 quintals.
24. 77 pk. 2 qt.
25. 4 lb. 1 oz. 3 pwt. 5 gr.
26. 8 wk. 4 da.
27. 7 hr. 57 min. 15 sec.
28. 15 mo.
29. 36 reams 15 qui. 7 sheets.
30. 250 bu.
31. 846 bu.
32. 304 bbl. 16 gal. 3 qt.
33. $318.24.
34. $469.44.
35. $1900.80.
36. $201.60.
37. $10.92.
38. $69.
39. $220.50.
40. $5445.
41. $7.
42. $135.
43. $30.
44. $28.80.
45. $64.
46. $90.
47. $57.60.
48. $5.625.
49. $48.
50. $87.82.
51. 8 gal. 3 qt. 1 pt.
52. $34.30.

Art. 200.

2. $\frac{4}{5}$.
3. $\frac{3}{10}$.
4. $\frac{7}{64}$.
5. $\frac{1}{4}$.
6. $\frac{3}{8}$.
7. $\frac{5}{8}$.
8. $\frac{3}{10}$.
9. $\frac{1}{8}$.
10. .28125.
11. .26166+.
12. .32926+.
13. .01.
14. .15625.
15. .4375.
16. .5.
17. .3125.
18. $\frac{23}{48}$.
19 $\frac{1}{8}$.
20. $\frac{211}{1521}$.
21. $\frac{13}{78}$.
22. $\frac{25}{36}$.
23. $\frac{3}{16}$.

Art. 201.

3. 190 bu. 45 lb.
4. $130.48.
5. $61.50.
6. 10 gro. 8 doz. 6.
7. 15 hr. 28 min.
8. 60 gal. 1 qt.
9. 22 cwt. 84 lb. $14\frac{3}{7}$ oz.
10. 19 Cd. 3 cd. ft. 13 cu. ft.
11. 22 yd. 2 ft. 10 in.
12. 2 hhd. 17 gal. 2 qt. 3 gi.

Art. 202.

5. 71 lb. 8 oz.
6. 25 Cd. 6 cd. ft. 4 cu. ft.
7. 4 T. 16 cwt. 5 lb. 12 oz.
8. 13 sq. yd. 3 sq. ft. 132 sq. in.
9. 8 cwt. 41 lb. 10 oz.
10. 5 lb. 3 oz. 10 pwt.
11. 1 wk. 6 da. 5 hr. 17 min. 16.8 sec.
12. 20 hr. 24 min.
13. 4 qt.
14. 137 gal. 1 qt.
15. 8 gal. 3 qt.
16. 30 Cd. 5 cd. ft. 14 cu. ft.

Art. 203.

2. 4 yr. 3 mo. 9 da.
3. 9 yr. 8 mo. 29 da.
4. 10 yr. 3 mo. 5 da. 5 hr.
6. 175 da.
7. 258 da.
8. 13 yr. 4 mo. 21 da.

Art. 204.

2. 31 hhd. 40 gal. 3 qt.;
50 hhd. 40 gal.;
94 hhd. 59 gal. 1 qt.
3. 17 rd. 1 yd. 1 ft. 6 in.;
51 rd. 4 yd. 1 ft. 6 in.;
4. 66 cu. yd. 26 cu. ft. 184 cu. in.
133 cu. yd. 25 cu. ft. 368 cu. in.
5. 2 da. 20 hr. 34 min.
8 da. 13 hr. 42 min.
6. 476 Cd. 3 cd. ft. 8 cu. ft.;
685 Cd. 2 cd. ft.
7. 1 lb. 7 oz. 12 pwt.
8. 56 T. 8 cwt.
9. 432 lb.
10. 2 hr. 9 min. 36 sec.
11. 10 lb.
12. 7 hhd. 55 gal. 0 qt. 1 pt.
13. 1 mi. 140 rd. 13 ft. 2.4 in.
14. 37 A. 140 sq. rd.
15. 11 ft. 10.56 in.
16. 3.24 gr.
17. 12 mo., or 1 yr.
18. $196.80.
19. $125.25.

Art. 205.

2. 41 gal. 3 qt. 1 pt.
3. 54 yd. 2 ft. $2\frac{1}{4}$ in.

4. 49 gal. 2 qt. 1 pt.
5. 21 bu. 1 pk. 5 qt. 1 pt.
6. 6 T. 10 cwt. 75 lb.
7. 5 Cd. 3 cd. ft. 11 cu. ft.
8. 31 bu. 1 pk. 5 qt. 1 pt.; 28 bu. 1 pk. 0 qt. 1.9 pt.; 23 bu. 2 pk. 2 qt. $\frac{1}{4}$ pt.
9. 12 yd. 0 ft. $5\frac{55}{63}$ in.; 6 yd. 0 ft. $2\frac{53}{63}$ in.
10. 1 Cd. 2 cd. ft. 5 cu. ft.
11. 1 oz. 17 pwt. 4 gr.
12. 880 rails.
13. 8 cwt. 64 lb. $12\frac{2}{3}$ oz.
14. 41 mi. 213 rd. 5 ft. 6 in.
15. 165 A. 25 sq. rd. 24.4 sq. yd. nearly.

Art. 218.

3. 54 ft.
4. 1080 tiles.
5. $608.40.
6. $65.48.
7. $22\frac{1}{2}$ acres.
8. $312\frac{1}{2}$ sq. ft.
9. $2568.75.
10. 32 rd.
11. $\$43.87\frac{1}{2}$.
12. 33 yd.
13. $15.50.
14. $42\frac{1}{2}$ yd.
15. $111.10.
16. $21.29\frac{11}{12}$ squares.
17. $139 57.
18. 72 planks.
19. $198.
20. 840 sods.
21. 11316 shingles.
22. $25.55.
23. $447.99.
24. $122\frac{1}{2}$ A.
25. $11.70.

Art. 224.

3. 19760 cu. in.
4. $8\frac{1}{2}$ ft.
5. 221 cu. ft.
6. 208 cu. yd.
7. 3 cu. yd. 26 cu. ft. 297 cu. in.
8. 7 cu. yd. 11 cu. ft. 200 cu. in.
9. 5 cu. yd. 25 cu, ft.
10. 1 hr. 12 min.
11. $12\frac{3}{16}$ Cd.
12. 8 ft.
13. $13.18.
14. $166.60.
15. 2 Cd.
16. $49\frac{1}{11}$ perches.
17. $410.16.
18. $50.90.
19. 8 ft.
20. $1607.82.
21. $379.08.
22. 108 cans.
23. $\$471.66\frac{2}{3}$.
24. 12 min.
25. $663\frac{3}{11}$ perches.
26. $3276.
27. 2142 cu. ft.; $333.20.

Art. 226.

3. $13\frac{1}{3}$ board ft.
4. $15\frac{7}{12}$ "
5. $36\frac{2}{3}$ "
6. $22\frac{1}{2}$ "
7. $31\frac{5}{12}$ "
8. 35 "
9. 25 "
10. 288 "
11. 72 "
13. $4.20.
14. $15.75.
15. $6.14.
16. $1.06.
17. $62\frac{2}{3}$ board ft.
18. $9.90.
19. $9.43.
20. 315 board ft. $26\frac{1}{4}$ cu. ft.
21. $191.52.
22. $274.82.

Art. 229.

3. $374\frac{2}{77}$ gal.
4. 6 bbl.
5. $54\frac{6}{7}$ hhd.
6. $1968\frac{3}{4}$ lb.
7. $3500\frac{68}{77}$ gal.
8. $14.96.
9. $149\frac{47}{77}$ gal.; 1250 lb.
10. $218\frac{61}{82}$ cu. ft.
11. 3383.1 cu. ft.

Art. 230.

6. 320 bu.
7. 320 cu. ft.
8. $138\frac{2}{3}$ bu.
9. 15 ft.
10. 192 bu.
11. 8 ft.
12. $108\frac{3}{8}$ bu.
13. 56.7 bu. apples; 72 bu. barley.
14. $1920.
15. $173.25.
16. $205.06.
17. $255.02.
18. $39\frac{2}{45}$ bbl.
19. $697\frac{1}{2}$ tons.
20. 537.1 bu.
21. 238.14 bu.
22. $20.68.
23. $8\frac{1}{2}$ tons.
24. $27.
25. 270 tons; $1485.
26. $14.75.
27. 82.56 tons.
28. 1.792 tons.
29. $\$193.33\frac{1}{3}$.
30. $826.20.

Art. 242.

2. $243\frac{1}{4}$ lb.
3. $6321.
4. 14.346 rd.
5. $6014.40.
6. $3180.01.
7. 2 mi. 306 rd.
8. $386\frac{2}{3}$ ft.
9. 2912 bu.
10. $175.
11. $205.49.
12. 21.6 bu.
13. $437\frac{1}{2}$ lb.
14. 123 men.
15. $100\frac{8}{15}$ lb.
16. $264\frac{2}{3}$ lb.
17. $9896.25.
18. $831.725.
19. $7035.
20. $944.

Art. 244.

2. 20%; 40%.
3. $12\frac{1}{2}$%.
4. $5\frac{1}{3}$%; $12\frac{1}{2}$%.
5. $62\frac{1}{2}$%.
6. $78\frac{4}{13}$%.
7. $12\frac{1}{2}$%.
8. 75%.
9. 5%.
10. 16%.
11. 8%.
12. $87\frac{1}{2}$%.
13. $15\frac{1}{3}$%.
14. $\frac{1}{2}$%.
15. 6%; 6%.
16. 75%; 20%.
17. $46\frac{2}{3}$%.
18. $96\frac{1}{2}$%.
19. 50%.
20. 65%.
21. 7%; $4\frac{12}{43}$%.
22. $17\frac{1}{2}$%.

Art. 246.

2. 130 gal.
3. $300.
4. 3360 bbl.
5. $625.
6. 50 yd.
7. $6500.
8. 314.
9. 264 lb.
10. $31.375.
11. 2000 ft.
12. $2476.80.
13. $2708.
14. $12465.
15. 4500 bu.
16. 78133.33\frac{1}{3}$.

Art. 248.

2. 2500.
3. $6000.
5. 500.
6. 1717.28.
7. 800.
8. 175 A.
9. 42.3 cwt.
10. 1200 bu.
11. $450.
12. $7400.
13. 57 ft.
14. 1242 mi.
15. $485.
16. $325.40.
17. $400.
18. $600.
19. $2400.
20. $342\frac{6}{7}$ A.

Art. 252.

1. 833.62\frac{1}{2}$.
2. $576; $174.
3. $1351.78;
$232.72.
4. $250.17;
$115.58.
5. $204.28;
$55.72.
6. $8.125.
7. $5.45.
8. $5\frac{40}{100}$+.

Art. 256.

2. 4.37\frac{1}{2}$.
3. $3498.90.
4. $584.77.
5. $2094
6. $3962.40.
7. 1909.12\frac{1}{2}$.
8. $95.83.
9. 4.56\frac{3}{4}$ s. p.;
71.63wh.gain.
10. $10 loss.
11. $1050 gain.
12. $9\frac{3}{4}$ cts.
15. $137.89.
16. $2996.
17. $1177.05.
18. $2752.50.
19. $4.375;
$2.80.
20. $.53$\frac{1}{8}$;
1.06\frac{1}{4}$.
21. $11\frac{2}{3}$ cts.
22. $2.85.

Art. 258.

2. 25%.
3. 40%.
4. $14\frac{2}{7}$%.
5. 16%.
6. $18\frac{3}{4}$%.
7. $14\frac{2}{7}$%.
8. 23%.
9. 50%.
10. $87\frac{1}{2}$%.

Art. 260.

2. $9375.
3. $8.80.
4. $1.50.
5. $14.14.
7. $2.25.
8. $86.25.
9. $6.86.
10. $.75.
11. $4.91.

Art. 261.

2. $150.
3. $96.
4. $27500.

Art. 265.

2. $116.25.
3. $76.17.
4. $378.13.
5. $82.11.
6. $379.40.
7. $285.19.
8. $96.90.

Art. 266.

2. $1\frac{1}{3}$%.
3. $2\frac{1}{2}$%.
4. 5%.
5. $6\frac{1}{4}$%.

Art. 267.

2. $16800.
3. $3500.
4. $3400.
5. $15760.

Art. 268.

2. $4400.
3. $6480.
4. $9860.
5. 256 bbl.

Art. 271.

2. $72 75.
3. $33.60
4. $90.
5. $285.

Art. 272.

2. $1\frac{3}{4}$%.
3. $\frac{5}{8}$%.
4. $1\frac{3}{8}$%.

Art. 273.

2. $13600.
3. $8960.
4. $60000.
5. $15600.
6. 24500.

Art. 277.

2. 5¼ mills.
3, $95.25.
4. $108.87½.
5. $13.09.
7. $11350.

Art. 284.

2. $388.50.
3. $154.35.
4. $170.10.
5. $842.40.
6. $1234.375.
7. $1465.10.
8. $132.
9. $1706.25.

Art. 285.

1. $67 50 ; 12%.
2. $4948.12½.
3. $3.84.
4. $22.50.
5. $1636.
6. 25%.
7. $2.50 ; 28%.
8. $200.
9. $200 lost.
10. $48.
11. ⅝%.
12. $1.06¼.
13. $138.
14. $8.25.
15. 60%.
16. $18.42 gain'd.
17. $8700, cost.
18. $452.81.
19. 4% lost.
20. $2370.87.
21. $4080.50.
22. $.31¼.
23. $405.

Art. 294.

3. $12.58,
at 6% ;
$8.39, at 4%.
4. $92.53,
at 5% ;
$148.04,
at 8%.
5. $269.47,
at 7% ;
$288.72,
at 7½%.
6. $3.46, at 6% ;
$4.03, at 7% ;
$4.32, at 7½%.
7. $115.70,
at 5% ;
$185.12,
at 8% ;
$208.26,
at 9%.
8. $421.99.
9. $85.72.
10. $79.28.
11. $421.87½.
12. $13227.50.

Art. 296.

2. $9.09.
3. $9.28.
4. $36.47.
5. $22 70.
6. $3.84.
7. $38.34.
8. $2.57.
9. $6.84.
10. $16.39.
11. $242.94.
12. $318.
13. $269.34.
14. $4.73 ;
$2.48 ;
$6.98 ;
$9 ;
$3.53.
15. $4.47.
16. $2.32.
17. $52.
18. $57.39.
19. $5.78.
20. $34.94.
21. $472.14.
22. $1542.50.
23. $136.29.

Art. 298.

4. $8.29.
5. $81.89.
6. $63.75.
7. $227.11.
8. $51.67.
9. $11.
10. $8.40.
11. $1.72.
12. $6.91.
13. $54.17.
14. $91.86.
15. $515.50.
16. $2579.86.
17. $1519.
18. $411.96.
19. $1255.35.
20. $1020.48.
21. $890.10.
22. $1350.40.
23. $582 gain.
24. $5037.22.
25. $828.07.
26. $1120.69.
27. $1868.58, loss
28. $54, loss.

Art. 300.

4. $669.12,
at 6% ;
$334.56,
at 12%.
5. $6936.09,
at 7% ;
$6069.08,
at 8%.
6. $1403.08.
7. $1500.
8. 8%.
9. 7½%.
10. 7%.
11. 2 mo. 12 da.
12. 2 yr. 9 mo.
13. 8 mo. 12 da.
14. 1 yr. 2 mo.
15. 3 yr. 1 mo. 15 da.
16. $19500.
17. 6½%.
18. $33700.
19. $1520.91.
20. 4 yr. 2 mo.
21. 7½%.
25. 4 mo.
26. 10%.
27. 5½%.
28. $25000.

Art. 302.

2. $371.60, amt.;
$59.60, comp. int.
$935.89, amt.;
$135.89, comp. int.
3. $777.92, amt.;
$137.92, comp. int.
$466.854, amt.;
$90.854, comp. int.
4. $1330.086, amt.;
$130.086 comp. int.
$1538.65, amt.
$338.65, comp. int.
5. $443.49, amt.;
$43.49, comp. int.
6. $2164.86, amt.;
$164.86. comp. int.
7. $2653.41, amt.;
$153.41, comp. int.

Art. 310.

2. $600, p. w.; $14.40, dis.
3. $2500, p. w.; $140, dis.
4. $461.17, p.w.; $18.83, dis.
5. $43 65.
6. $4.52, loss.
7. $945.40.
8. $$.83\frac{1}{3}$.
9. $3677.75.

Art. 318.

2. $6.65, dis.; $361.35, pro.
3. $6.27, dis.; $591.23, pro.
4. $11.70, dis.; $672.90, pro.
5. $19.44, dis.; $1060.56, pro.
6. $15.02, dis.; $859.98, pro.
7. $13.54, dis.; $1236.46, pro
8. $18, dis.; $462, pro.
9. $21.93, dis.; $918.07, pro.
10. $20.51.
11. $1472.88.
12. $32, dis.; $448, pro.
13. $.90, diff.
14. $552.44.
15. $8.55, dis.; $891.45, pro.
16. $9.22, dis.; $670.78, pro.
17. $10.20, dis.; $1239.79, pro.
18. $8.75, dis.; $1491.25, pro.
19. $3.50, dis.; $372.10, pro.
20. $1613.64.
21. Mat. Dec. 15; 80 da., term of dis.; $1276.43, pro.
22. Mat. June 18; 59 da., term of dis.; $500.18, pro.
23. Mat. Apr. 8; 46 da., term of dis.; $942.53, pro.

Art. 319.

2. $1278.42.
3. $281.94.
4. $1205.52.
5. $360.
6. $2990.54.
7. $240.
8. $564.92.
9. Mat. May 12; 63 da., term of dis.; $646 79, face.
10. Mat. July 18: 60 da., term of dis.; $383.84, face.
11. Mat. May 18; 66 da., term of dis.; $1278.76, face.
12. $584.20.
13. $194.64.

Art. 323.

3. $506.37.
4. $366.98.
5. $366.09.
6. $1382.33.
7. $173.75.
8. $216.29.
9. $1642.31.

Art. 324.

2. $312.36.
3. $355.15.

Art. 338.

2. $24468.75.
3. $16856.25.
4. $1740, gain.

Art. 339.

2. 136 shares, at 10% dis. 102 shares, at 20% prem.
3. 220 shares.
4. 204 shares.

Art. 340.

2. $25500.
3. $21100.

Art. 341.

2. $700.
3. $2530.
4. $21.45, loss.

Art. 342.

2. $5\frac{5}{9}\%$ at 108. $7\frac{1}{7}\%$ at 84.
3. $8\frac{8}{9}\%$.
4. 8% bonds $\frac{20}{33}\%$ better; 6% bonds $\frac{5}{14}\%$ better.
5. 6% bonds $\frac{10}{21}\%$ better.

Art. 350.

2. $1601.93.
3. $856.80.
4. $491.77.
5. $1135.40.
6. $1194.
7. $2436.60.
8. $555.89.
9. $775.67.
10. $202.91.
11. $4764.84.
12. $5153.24.
13. $6388.80.
14. $4791.60.

Art. 351.

3. $829.63.
4. $1664.60.
5. $2629.58.
6. $979.85.
7. $1809.03.
8. $763.16.
9. $1873.13.
10. $316.45.

Art. 356.

2. 3 mo. 25 da.
3. 6 mo. 26 da., time of Cr.; June 27, '81, Eq. time.
4. May 5, 1883.
5. 5 yr. 20 da. from date of last paym't.
7. Nov. 26, Eq. time.
8. 73 da., term of Cr. Feb. 26, Eq. time.
9. Mar. 7, Eq. time; $1178.01, cash value.

Art. 357.

2. Aug. 19, '85, Eq. time.
3. June 7, 1884.
4. June 27, '82; Dis., $149.28
5. Apr. 23, 1884.

Art. 372.

1. 130.
2. 1728.
3. 48.
4. 160.
5. 125.
6. $601.20.
7. $12\frac{15}{26}$ yd.
8. 27.08.
10. $3\frac{3}{7}$ yd.

11. 60.
12. 240.
13. 9.
14. 2.

Art. 373.

3. $100.
4. 43$\frac{1}{8}$ T.
5. $157.50.
6. 22$\frac{1}{4}$ bbl.
7. $4.27.
8. $21.
9. $5446.62.
10. $10.50.
11. $184 24.
12. 72 bu.
13. 24 men.
14. $1875.
15. 30 mo.
16. $240.
17. $52.79.
18. 210 A.
19. $202.50.
20. 900 tiles.

Art. 378.

4. 560 bu.
5. 220$\frac{4}{5}$ Cd.
6. $10958.90.
7. 114$\frac{3}{7}$ yd.
8. $63.
9. 2 oz. 10 pwt.
10. 20 men.
11. $185.
12. $6428.57.
13. 2$\frac{2}{5}$ da.
14. $709.87.
15. $89.60.
16. $120.
17. $100.
18. 9 men.
19. 8.116 ft.
20. 59$\frac{3}{8}$ da.
21. $7320.
22. $1728.
23. 3291$\frac{3}{7}$ yd.
24. 14$\frac{2}{7}$ da.
25. 70 lb.

Art. 384.

3. $336, A's;
   $504, B's;
   $840, C's.
4. $96, A's;
   $80, B's;
   $48, C's.
5. $2089.06;
   $1044.53.
6. $6000;
   $8402.25;
   $5055.75;
   $3042.
7. $120.45, A's;
   $240.90, B's;
   $602.25, C's.

Art. 385.

2. $2400,
   Smith's;
   2666\frac{2}{3}$,
   Bank's;
   2933\frac{1}{3}$,
   Crane's.
3. $102, A's;
   $148.40, B's;
   $96.80, C's.
4. $22 50,
   A pays;
   $15, B pays;
   $18, C pays.
5. 30 bu., A's;
   27 bu., B's;
   36 bu., C's.
6. $604.71, A's;
   $167.49, B's.

Art. 389.

2. 1369; 1764;
   3136; 5625.
3. 3375; 5832;
   74088;
   157464.
4. 3969;
   110592;
   1048576;
   248832.
8. 1.1025;
   .0064;
   23.04;
   5.5696;
   .000049.
9. 645.16.
10. 1191016.
11. 1958$\frac{1}{16}$.
12. $\frac{14641}{50625}$.
13. $\frac{32}{16807}$.
14. $\frac{15625}{64}$.
15. .00116964.
16. .000000015625.
17. 46733.803208.
18. 33169$\frac{1}{4}$.
19. 16.6056$\frac{1}{4}$.
20. 24.76099.

Art. 390.

3. 2304.
4. 3136.
5. 9604.
6. 15625.
7. 11025.
8. 50625.
9. 88809.
10. 116964.

Art. 391.

3. 39304.
4. 91125.
5. 262144.
6. 373248.
7. 1953125.
8. 6028568.
9. 13144256.
10. 3.511808.
11. 830.584.
12. .421875.
13. 669921.875.
14. .000250047.
15. 42.875000.
16. 95757$\frac{39}{64}$.
17. $\frac{343}{512}$.
18. $\frac{15625}{32768}$.
19. $\frac{27000}{79507}$.
20. 1520875.
21. 2023$\frac{27}{125}$.
22. 5.887.
23. 640000.
24. 2540.0390625.
25. 125.

Art. 396.

3. 36.
4. 49.
5. 54.
6. 75.
7. 107.
8. 144.
9. 11.2.
10. 14.4.
11. 3.05.
12. 145.
13. 109.
14. 324.
15. 516.
16. 103.9.
17. 14.0048+.
18. .035.
19. $\frac{25}{81}$.
20. $\frac{84}{95}$.
21. 4.218+.
22. 2.68+.
23. 36.37.
24. 64.
25. 1.
26. 1.4142.
27. 1.7320.
28. 2.8284.
29. 3.4641.
30. 5.0990.
31. 1008 ft.
32. 52 rd.
33. 200.56.
34. 145$\frac{1}{4}$ rd.
35. $187.20.
36. 235 men.

Art. 398.

1. 20.
2. 21 ft.
3. 45 yd.
4. 25 ft. 7.34 in.
5. 39 ft.
6. 360 ft. 6.66 in.

Art. 400.

3. 25.
4. 55.
5. 73.
6. 83.
7. 354.
8. 1598.
9. $\frac{10}{11}$.
10. $\frac{24}{25}$.
11. 1.42+.
12. 5.5.
13. .45.
14. 2.84.
15. 1.259.
16. 1.5874.
17. 1.8171.
18. 2.0800.
19. 27.15.
20. 60.8.
21. 3 ft.
22. 12150 sq. ft.
23. 5 ft. 8 in.+.

Art. 401.

1. $94.67.
2. .1.
3. 560.
4. $4200.
5. $4.50.
6. 560 A.
7. $1.15.
8. $82.32.
9. 1280 rd.
10. $79.64.
11. $16.75.
12. 8 A. 109.9 sq. rd.
13. $1.69.
14. 44.6 pch.
15. 1100 lb.
16. $1584.
17. $450.
18. 21 da. 15 hr. 44 min.
19. 6 ft. 5.8 in.
20. 14$\frac{230}{539}$ hhd.
21. .15.
22. 1$\frac{1}{5}$ mo.
23. $8248.80.
24. 12$\frac{1}{2}$%.
25. $2291.44.
26. .48.
27. 35.35 ft.
28. 25 ft.
29. $142.60.
30. $16153.84.
31. $896.95.
32. 6$\frac{2}{3}$ rd.
33. 93$\frac{1}{3}$ ft.
34. 7 mo. 15 da.
35. 333.33\frac{1}{3}$.
36. $95.
37. 320$\frac{1}{18}$ sq. ft.
38. $63.28.
39. $21500.
40. $575.99.
41. $652.86.
42. $455.
43. $1.20.
44. $32.77.
45. $193.60.
46. $.60.
47. $18375.
48. 6%.
49. $1000, B; $1500, C.
50. 1.69.
51. $48.60.
52. $1113.
53. $264.25.
54. 6 ft.
55. $318.77.
56. $15.25.
57. $308.08.
58. 9$\frac{9}{10}$ da.
59. 640 stones.
60. $66.63.
61. 20%.
62. $439.84.
63. $389.61.
64. 600 bu. w't; 180 bbl. pork; 183$\frac{38}{49}$ bbl. flour.
65. $2031.25.
66. 1590$\frac{6}{11}$ gal.
67. $308.38, g'n.
68. 2310 tons.
69. $133.35.
70. $1210.59.
71. $80.44.
72. $1408.08.
73. $430.15.
74. 600 men.
75. 160 rd.
76. 14 ft.
77. $3905.
78. 7454 rails; $372.71, cost.
79. 3 yr. 4 mo.
80. 2$\frac{1}{3}$ da.
81. 4 yr. 2 mo.
82. $2362.50.
83. 37$\frac{1}{2}$%.
84. $126.17.
85. 156 ft.
86. $616.
87. 15362$\frac{2}{25}$ ft.
88. $390.19.
89. $79.60.
90. 31$\frac{6}{10}$%.
91. $250000.
92. 80 rd.
93. $2037.75.
94. $1.80.
95. 1944 tiles.
96. $1242, gain; 30$\frac{15}{22}$% "
97. 15$\frac{12}{23}$%.
98. $1672.53.
99. $225, A's; $330, B's; $270, C's.
100. 173.569 bu.
101. $71.25.
102. 360 acres.
103. $39.75.
104. 373.336 cu.ft.
105. $22.32.
106. $5.91.
107. $14400.
108. 662$\frac{2}{3}$ sq. yd.
109. 62$\frac{1}{10}$ min.
110. $480, gain; $36 an acre.
111. $300.
112. $11773.83.
113. 9 men.
114. $1.60.
115. $125.
116. 17.88 ft.
117. $532.18.
118. $90.
119. $282.24, simp. int. $2202.24, amount; $295.56, comp. int.; $2215.56, comp. amt.; $1678.93+, pres. w'th; $246.07, true disc't; $283.20, b'k disc't; $1636.80, proceeds; $2252.20, face.
120. $2.16.
121. .023%; $58.13.
122. $850.72, loss.
123. 2266.66\frac{2}{3}$, cost; 2436.66\frac{2}{3}$, sold for; 2833.33\frac{1}{3}$, asking price.

ADDENDA.

Art. 410.

2. $\frac{5}{11}$.
3. $\frac{4}{9}$.
4. $\frac{11}{37}$.
5. $\frac{33}{37}$.
6. $\frac{13}{37}$.
7. $\frac{42}{101}$.
8. $\frac{65}{101}$.
9. $\frac{31}{71}$.
10. $\frac{35}{37}$; $\frac{447}{37}$.
12. $\frac{28}{45}$.
13. $\frac{11}{225}$.
14. $\frac{113}{11225}$.
15. $\frac{178}{275}$.
16. $\frac{533}{666}$.
17. $\frac{5}{336}$.
18. $7\frac{263}{495}$.
19. $2\frac{553}{999}$.
20. $5\frac{5}{13}$.
21. $\frac{42}{325}$.
22. $\frac{13}{37}$.
23. $7\frac{5}{396}$.

Art. 423.

1. 29106 l.
2. ℈4785.
3. f ʒ 4800.
4. 332 in.
5. 1116610″.
6. 7551250 sq. l.
7. 17520 hr.
8. 59066″.
9. 5° 48′ 7.2″.
10. ℥ 8 ʒ 1 ℈ $1\frac{4}{11}$.
11. 2 mi. 99 rd. 11 ft. 2.64 in.
12. f ʒ 5 ♏ 36.
13. 42′ 54″.
14. O. 3 f ℥ 3 ʒ 1 ♏ 36.
15. ℥ 11 ʒ 2.
16. 6 S. 22° 30′.
17. f ʒ 3 ♏ 35.
18. 3 ft. 9 in.
19. 12° 51′ $25\frac{5}{7}$″.
20. 9 oz. 3 pwt. 14.4 gr.
21. 3963.76 ft.
22. 424.98775 A.

Art. 424.

1. 1 mi. 4 ch. 2 rd. 20 l.
2. 239 fath. 1 ft.
3. 557° 33′ 20″.
4. Cong. 6 O. 4 f ℥ 11 f ʒ 4.
5. 87°, 50 naut. miles.
6. ℔ 314 ℥ 5 ʒ 3 ℈ 1.
7. f ℥ $\frac{7}{16}$.
8. $\frac{1}{440}$ mi.
9. $\frac{1}{44}$ ch.
10. .5825 + lea.
11. .20025 mi.
12. .3.
13. 1 da. 4 hr. 7 min. 30 sec.
14. $2275.
15. 960 rd.
16. 7° 3′.

Art. 426.

1. 80 A.; $\frac{1}{8}$ sec.
2. 5760 rails; $230.40.
3. $340.
4. 240 A.; $\frac{3}{8}$ sec.
5. 120 A. left; $27.20, gain.
6. 420 A. left; $635, gain.

Art. 436.

2. 107° 19′ $48\frac{1}{7}$″.
3. 122° 26′ 45″.

Art. 437.

2. 5 hr. 57 min. 49 sec.
3. 5 hr. 59 min. 51 sec.
4. 1 hr. 3 min. 58 sec.
5. 1 hr. 13 min. $32\frac{2}{3}$ sec.
6. 51 min. 18 sec.
7. 1 hr. 40 min. 8 sec.
8. 6 hr. 28 min. 27 sec.
9. 1 hr. 33 min. 27 sec.
10. 5 hr. 6 min. $15\frac{1}{15}$ sec., Cincinnati; 4 hr. 53 min. 43 sec., Chi.; 4 hr. 43 min. 13 sec., St. L.
11. 12 hr. 7 min. 41 sec. night, Berlin; 4 hr. 53 min. $46\frac{1}{3}$ sec. P.M., St. Paul.
12. 5 hr. 5 min. 32 sec. later.

Art. 438.

2. $120.
3. $10.58.
4. $.04.
5. $82.36.
6. $10.96.

Art. 439.

2. $464.10.
3. $7308.
4. $11.30.
5. $1161.04.
6. $674.16.

Art. 440.

1. $925.73.

Art. 446.

2. $96.12.
4. $2937.26.
5. $1050.10.
6. $612.77.
7. $1248.11.
8. $1877.44.
9. $693.77.
10. $529.10.
11. $4210.63.

Art. 447.

4. £1543 4s. 2d.
5. 1664.13 marks.
6. 25360.46 fr.
7. 8825.1 guild.
8. $17066\frac{2}{3}$ marks.
9. 31888.83 fr.
10. 12918.75 fr.
11. $606.875.

Art. 454.

1. 103.
2. 8.
3. 15.
4. $415.
5. 3.
6. 7%.
7. 10.
8. 6.
9. 11.
10. 222.
11. 78.
12. $32448.

Art. 457.

1. 3.
2. 6144.
3. 4.
4. 7.
5. 765.

6. $5314.40.
7. $2187.

Art. 465.

2. $42\frac{3}{16}$ sq. ft.
3. 22 A. 6 ch.
   13.45 P.
4. $449.07.
5. $147.

Art. 466.

2. $1\frac{1}{2}$ ft.
3. 13 in.
4. 28 rd.
5. 672 rd. $5\frac{3}{17}$ yd.
6. $8\frac{1}{3}$ ch.

Art. 467.

2. 111.8+ ft.
3. 3 sq. ft. 1.7
   sq. in.
4. 13 A. 41.76 P.

Art. 468.

2. 84 sq. ft.
3. $5\frac{1}{4}$ acres.
4. $5940.
5. 1 acre.
6. $28\frac{2}{3}$ board ft.

Art. 469.

2. 11178 sq. ft.
3. 2 acres.

Art. 470.

2. 381.625 sq. ft.
3. $17.82\frac{1}{8}$ A.

Art. 471.

3. 15 ft. 10 9 in.
4. 5 ft. 10.6 in.
5. 5 ft.
6. 7 ft. 3.96 in.

Art. 472.

4. 318.3 A.
5. 114.59 A.
6. 3.509 A.

Art. 473.

3. 7 rd.
4. 10.01 yd.;
   31.447 yd.
5. 19.098 ft.;
   59.998 ft.

Art. 479.

4. 207.34 sq. ft.
5. $168\frac{7}{8}$ sq. ft.
6. 263.8944 sq.
   feet.
7. 301.176 sq. ft.

Art. 480.

3. 31.808 cu. ft.
4. $274\frac{5}{8}$ cu. ft.
5. $27.
6. $53.63.

Art. 481.

3. 429.25 sq. ft.
4. 512.9 sq. ft.

Art. 482.

3. 39.27 cu. ft.
4. $29.23.
5. 192000 cu. ft.,
   22284.6 sq. ft.

Art. 483.

3. $256\frac{2}{3}$ sq. yd.

Art. 484.

2. 58.1196 cu. ft.
3. 38.125 cu. ft.
4. 64.9881 cu. ft.

Art. 485.

2. 28.2744 sq. ft.
3. 12.5664 sq. ft.

Art. 486.

2. 14137.2 cu. ft.
3. 523.6 cu. yd.

Art. 487.

2. 99.144 gal.
3. 120.0948 gal.

Art. 505.

5. 178.597 m.
6. 80.635 m.
7. 10.50412 Km.
8. 4.8486 Km.
9. 12.80 m.
10. $1.92.
11. 24.87475 Km.
12. 147.2 Km.
13. 150000 m.
14. 84 sq. m.
15. 326468 ca.
16. 1512 sq. Dm.
17. 72 m.
18. 35.664 cu. m.
19. 38.4 cu. m.
20. 432.96 cu. m.
21. 39.066.
    $107.415.
22. 2677.02 l.
23. 14430 l.
24. 20 l.
25. 75 l.
26. 90 Hl.
27. 16500 Kl.;
    16500000 g.;
    165000000 dg.
28. $400.

Art. 508.

1. 80.38+ ft.;
   15.48 ft.
2. 3.728 mi.;
   248 56 mi.
3. 68.58 m.;
   16.91+ m.
4. 10.773 A.;
   1723.68 sq. rd.
5. 29.999+ a.
6. .5287 Hl.
7. 2239.44 g.;
   2721.6 g.
8. 81.42685
   sq. m.
9. 160.003 Hl.;
   $38.03 gain.
10. 88.9056 Kg.
11. $288.75.
12. $1860.
13. $531.29.
14. $1666\frac{2}{3}$ m.
15. $210.38.
16. $185.69.
17. 437.7 tons.
18. $133.52.
19. 45 92 bu.
20. 2803.38 liters.
21. 3.6 M. tons.

Art. 509.

1. 3207 arpents
   354 sq. toises
   20 sq. ft. 24
   sq. in.; or,
   3083 A. 73 P.
   7.93 sq. yd.

Art. 510.

1. 42 sq. lea. 3 lab.
   62500 sq. v.;
   or, 291 sq. mi.
   295 A. 44 P.
   19 sq. yd. 3
   sq. ft. 34.56
   sq. in.

Art. 511.

1. $1.25.
2. $87.815.
3. $4.50.
4. $21.125.
5. $31.66+.
6. 11 lb. 8 oz. 7 pwt. 7 gr.
7. 1 A. 60 P.
8. $82.50.
9. 2970 vol.
10. $2.50.
11. 1 yr. 8 mo. 9 da. 14 min. 59 sec.
13. 5 cu. yd. 7 cu. ft. 756 cu. in.
14. 8 hr. 8 min.
15. 2 hr. 8 min. 36 sec. P. M.
16. $800.92.
17. $34.13+.
18. $50.91.
19. 424.98775 A.
20. $55.76.
21. $5640.58+.
22. $3\frac{3}{5}$ ft.
23. $173.25.
24. $31.52.
25. $2016.
26. $51.
27. $59.57.
28. $23.68.
29. 1.464375.
30. $\frac{1}{80}$, increased.
31. $\frac{8}{85}$, dimin.
32. $13\frac{1}{3}$ da.
33. $9\frac{43}{48}$ bu.
34. $\frac{817}{1628}$.
35. 135 lb.
36. $1584.
37. $5.25.
38. 2.07887.
39. $1744.06.
40. $295.
41. 960 rd.
42. $2.54.
43. $53.166.
44. $199.25.
45. $111.94.
46. £60.
47. 7880 ft. 5'.
48. 1320 pickets. $36.30.
49. $.83$\frac{1}{3}$.
50. $1260.
51. 1155 boards.
52. $526\frac{2}{3}$ sq. yd.
53. $159.365.
54. $1250.
55. 57 men.
56. $1600.
57. $33\frac{1}{3}$%.
58. $7\frac{1}{2}$%.
59. $17\frac{2}{3}$ da.
60. 20%.
61. $42.875.
62. $3\frac{3}{16}$%, rate. $511.86, N's tax.
63. 4 yr. 3 mo. 6 da.
64. 792 sq. ft.
65. 70% paid.
$2800, A's sh.
$3500, B's "
$4900, C's "
66. $5750.
67. $68.75.
68. $9.59.
69. $1.62 gain.
70. $2; $3; $9.60, etc.
71. $14350.
72. $6\frac{2}{3}$%.
73. 80 cans.
74. $680\frac{8}{11}$ gal. $5687\frac{1}{2}$ lb.
75. $10203.947.
76. 62 da., term of credit. Mar. 3, time of payment.
77. $2530.
78. 2400 bbl.
79. $1288.53.
80. 24 men.
81. $32894.73.
82. 1.06\frac{2}{3}$.
83. $571.62.
84. 35.35 ft.
85. 25 ft.
86. 29.15 ft.
87. $315.79; $473.69; $710.52.
88. $900, J'ly 28.
89. $.97875.
90. $5384.12.
91. $830.415.
92. $31.81 gain.
93. 281.9681 Hl.; $137.73 gain.
94. $292\frac{1}{2}$.
95. $318.81.
96. 154 ft.
97. 162 shares.
98. 8000 bu.
99. $5785.20; $5142.40.
100. $564.575.
101. $806.20.
102. $110.25.
103. $121.57.
104. 938.66\frac{2}{3}$.
105. 50%.
106. $14\frac{2}{7}$ yr.
107. $187.29+.
108. $503.22.
109. $23436.
110. $1295.51.
111. $2106, G's; $1300, D's; $982.80, C's.
112. 21 da.
113. $21\frac{7}{8}$%.
114. $1875.
115. 2 yr. 6 mo.
116. $38.65.
117. $5236.169.
118. $652.86.
119. 106.66\frac{2}{3}$.
120. $604.71, A's; $167.49, B's.
121. 7.23 A.
122. $12.
123. $593.70.
124. $194.63.
125. 11.31 rd. and 8 rd.
126. $4000.
127. $220 less.
128. $294.94.
129. $339.05.
130. $607.20.
131. 15 ft.
132. 371200 A.
133. $940.37 pro.; 81 da., term disc't.
134. $1295.82, proceeds; 79 da., term of disc't.
135. $1281.77, pr.; 30 da., term disc't.
136. $13.50.
137. 93\frac{1}{3}$ gain in wearing the $56 suit.
138. $532.18.
139. $10839.83.
140. 27 da.
141. $235\frac{7}{8}$ cu. ft.
142. $838.26.
143. $45\frac{63}{100}$% gain.
144. 5 ft. 8 in.
145. 20 ft.
146. 5 mo. 4 da.
147. $1501.20.
148. $43.65.
149. $931.203.
150. $29082.617.
151. $2735.828.
152. $39.156, A's; $46.987, B's; $62.25, C's; $70.48, D's.
153. 8 ft.
154. 2 ft.
155. 12150 sq. ft.
156. 40 yr.
157. *l*, 72; *a*, 30.
158. $6\frac{7}{8}$.
159. 466\frac{2}{3}$, A's; 833\frac{1}{3}$, B's; $10 per bbl.
160. $1.60.
161. $2023.32.
162. $100.
163. .0125, rate; $95.25, A's t.
164. $7350.75.